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Estimation of account the flanges in the direct calculation of stress-strain state parameters at normal sections of structural members

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1. Introduction

Most construction design and management regulations [1-4] recommend using nonlinear stress diagrams. The use of the nonlinear state diagrams enables us to calculate the actual stress-strain state parameters of structural members (of the compression and the tension zones, the crack depth, etc.) and to design structural members with greater accuracy.

Traditionally, the curriculum subject Strength of Materials taught to students deals with structural members made of elastic materials. The formulae presented in this course are not applicable for the cases when the stress-strain dependence is nonlinear (curvilinear). Taking into account the nonlinearity of the stresses is a highly topical issue [1-14], but up till now there exists no uncomplicated practical engineering method applicable in this case.

For the calculation of the stress-strain state parameters at normal sections of structural members subjected to bending, paper [12] presents a method of successive approximations (*iterative method*). The method employs *nonlinear (curvilinear)* stress diagrams. In paper [13], the analogous parmeters of members with *rectangular* cross-section are calculated directly, i.e. the calculation is iteration free. The formulae presented there may serve as a basis for a practical engineering method. The cases when the stress diagrams are not *nonlinear (curvilinear)* or when the members have cracks will be presented in further papers.

The aim of the present paper is to present a way enabling us to take into account the *flanges* of the structural members subjected to bending in the aforementioned direct calculation method presented in paper [13].

The object of the reasearch is a structural members subjected to bending with a cross-section as shown in Fig. 1. The main part of the member and the flanges may be made of different materials. The flanges and the reinforcement may be positioned both in the compression zone and in the tension zone.

2. Definitions and symbols used in the paper

Structural member (hereafter-member) is flexural, eccentrically compressed or eccentrically tensioned bar made of concrete, reinforced concrete, metal, wood or other materials;

the main material of the member is material constituting the biggest part of the member into which materials of other parts are reduced for the analysis: in case of concrete and reinforced concrete it is concrete, in case of wooden members it is wood, etc.;

effective cross-section is the part of cross-sectional area of the member that is assumed in the analysis and at the time considered is subject to the normal stress;

Z_c *and* Z_t are the zones of effective cross-sectional area of the member in compression and tension respectively; for the zone Z_c characteristics for materials in compression are applied, while for the zone Z_t – characteristics for these in tension;

member strengthening is strengthened parts of the members; e.g. stronger members imbedded into concrete members or fixed to the timber member;

member weakening is ducts in the member, e.g. for the prestressed reinforcement, etc.;

parts of the cross-section are sectors of the member layers, strengthenings (weakenings)and reinforcements in the cross-section;

first edge of the cross-section layer (marked by Index c) is the edge with the lowest arithmetical value of acting stress*;

second edge of the cross-section layer (marked by Index t) is the edge with the highest arithmetical value of acting stress*;

values of the parameters x_i, ν_i, ω_{ni}, ω_{mi} of the first edge are marked by *Index c*; while these of the second edge are marked by *Index t*;

symbol f is used to mark the parameter of strengthening (weakening), and symbol s is used to mark the parameter of reinforcement;

$d_u = x_{tu} - x_w = h$;

$0-0$ is neutral axis;

$w-w$ is axis parallel to the neutral axis; it is expedient to place it on the edge of the cross-section with the lowest arithmetical value of strain;

$a-a$ is any axis parallel to the axis $0-0$;

$\varepsilon-\varepsilon$ is any layers of the material of the member that are parallel to the axis $0-0$ and whose strain ε_ε is assumed in the equations of static equilibrium;

x_w is distance* from the neutral axis $0-0$ to the axis $w-w$;

x_{hc} stands for the depth of the compression zone of the member (thickness of the layer of the compression zone), i.e. the distance from the compressed edge of the member to the neutral axis $0-0$ (the direction of measurement is important – the positive direction is the direction of the axis x (Fig. 1); when the axis $w-w$ is in the edge subject to compression, then the depth of the compression zone $x_{hc} = 0 - x_w$;

a_a is distance from the axis $w-w$ to the axis $a-a$;

* for zone Z_c negative values are taken

a_ε is distance from the axis $w-w$ to the layers $\varepsilon-\varepsilon$ of the member;

symbols of other dimensions are shown in the Fig. 1;

E_i, E_{fi}, E_{si}, E are elasticity moduli of the layers of the main material in cross-section (e.g. layers of the concrete), strengthening (weakening) of the cross-section, reinforcement, selected materials of the equivalent cross-section; it is expedient to take E equal to the value E_i of the modulus of the main material at the most important layer;

$\alpha_{ei} = E_i / E$; $\alpha_{efi} = E_{fi} / E$; $\alpha_{esi} = E_{si} / E$;

A_{fi}, A_f, A_{fc} and the like stand for cross-sectional area of strengthening (weakening), area of weakening is negative; (see Fig. 1);

A_{si}, A_s, A_{sc} and the like stand for is cross-sectional area of reinforcement (the reinforcement may be pre-stressed); the reinforcement may be located not only in the tension zone, but also in the compression zone (see Fig. 1);

parts of cross-section is areas of the layers of the member, strengthening (weakening) and reinforcement in the cross-section;

equivalent cross-section is cross-section with areas of parts of its effective cross-section multiplied by respective coefficients α_e .

Strain diagrams (Fig. 1):

straight line $\varepsilon_0 - 0 - \varepsilon_0$ is diagram of linear strains for the material of transformed section (conforming to the hypothesis (Bernoulli's) that the plane sections remain plane);

line $\varepsilon_c - 0 - \varepsilon_t$ (not necessarily a straight line) is strain diagram for the main material of the cross-section; symbols *c* and *t* stand for compression and tension respectively;

line $\varepsilon_{sc} - 0 - \varepsilon_{st}$ (not necessarily a straight line) is reinforcement strain diagram.

ε_i, ε_f, ε_s ε_0 are strains* of layers of the main material of the cross-section, strengthening (weakening), reinforcement, selected material of transformed section respectively;

ε_m is absolute value of the strain* corresponding to the value of the maximum stress* σ_m (e.g. concrete strength* f_{cm}); linear strain* of the concrete layer *i* in compression $\varepsilon_m = \varepsilon_{0mi}$.

σ_u is stress* corresponding to the value of the limiting strain* ε_u (e.g. of concrete ε_{cu1});

σ_{pi} is pre-stress of reinforcement;

ε_{si} is strain* of reinforcement caused by external forces;

σ_{si} is stress* of reinforcement caused by external forces;

$\varepsilon_{Si} = \varepsilon_{pi} + \varepsilon_{si}$ is total strain of reinforcement measured from the initial (zero) state of the reinforcement;

$\sigma_{Si} = \sigma_{pi} + \sigma_{si}$ is total stress of the reinforcement measured from the zero state of the reinforcement;

$\overline{E}_i$, $\overline{E}_{fi}$, $\overline{E}_{si}$ are secant elasticity (strain) moduli: $\overline{E}_i = \nu_i E_i = \nu_i \alpha_{ei} E$, $\overline{E}_{fi} = \nu_{fi} E_{fi} = \nu_{fi} \alpha_{efi} E$, $\overline{E} = \nu_{Si} E_{Si} =$ $= \nu_{Si} \alpha_{esi} E$; $\varphi = \dfrac{\varepsilon_w}{x_w} = \dfrac{\varepsilon_{0i}}{x_i} = \dfrac{\varepsilon_{0fi}}{x_{fi}} = \dfrac{\varepsilon_{0si}}{x_{si}} = \dfrac{\varepsilon_{0mi}}{x_{mi}}$ (Fig. 1);

$k_\varepsilon = \dfrac{\varepsilon_\varepsilon}{\varepsilon_{0\varepsilon}}$, $k_i = \dfrac{\varepsilon_i}{\varepsilon_{0i}}$, $k_{mi} = \dfrac{\varepsilon_{mi}}{\varepsilon_{0mi}}$, $k_{fi} = \dfrac{\varepsilon_{fi}}{\varepsilon_{0fi}}$, $k_{si} = \dfrac{\varepsilon_{si}}{\varepsilon_{0si}}$.

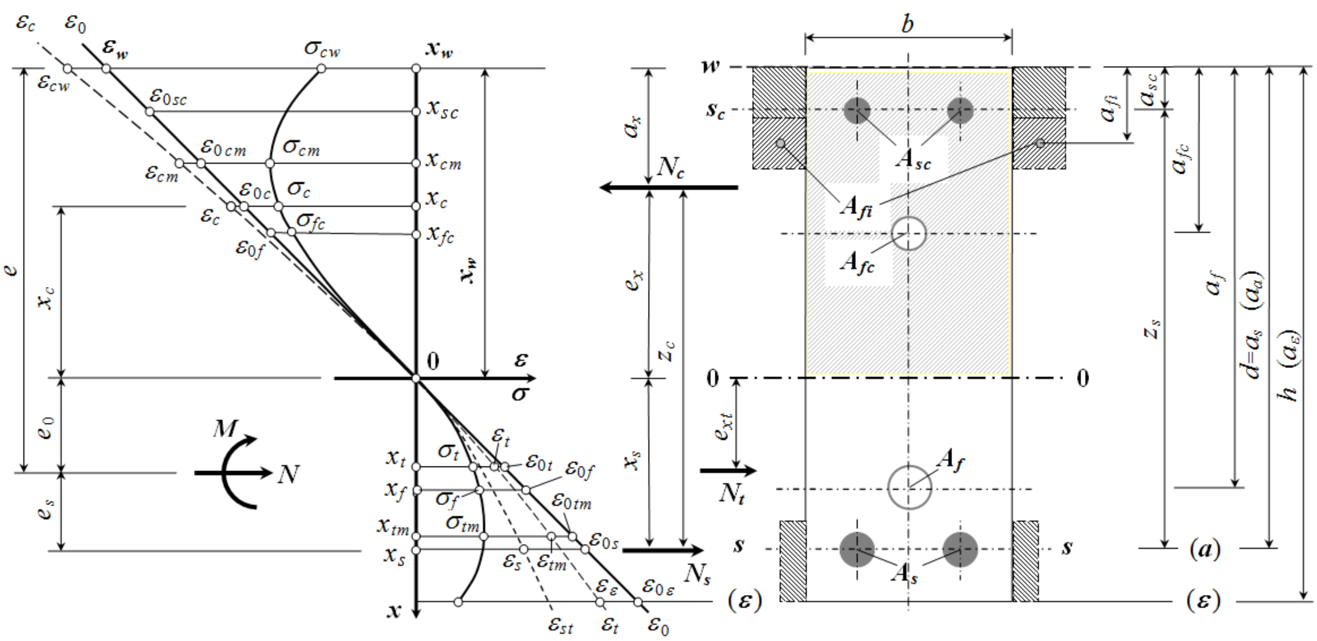

Fig. 1 Cross-section of the member and stress-strain diagrams

3. The essence of the method and its formulae

Eq. (1) of the static equilibrium of the forces and Eq. (2) the static equilibrium of the bending moments presented in paper [13] in the present paper have been adapted for the case of member *with flanges* (Fig. 1). In addition to parameters $\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}$, parameter $\Sigma k_{fi}\alpha_{efi}A_{fi}\nu_{fi}$ is used Eqs. (1) and (2). For simplicity, instead of $\sum_{i=1}^{n}$ the symbol Σ is written. With the help of this parameter the flanges or other strengthening and/or weakenings of the member are taken into account when making the calculations. Please note that $\nu_{Si} = const$, and $\nu_{fi} \neq const$. The change of ν_{fi} is taken into account with the help Eq. (17).

$$\begin{aligned}&\left(k_t\omega_{nt} - k_c\omega_{nc}\right)x_w^2 + \\ &+\left[2k_t\omega_{nt}d_u + \frac{\Sigma k_{fi}\alpha_{efi}A_{fi}\nu_{fi}}{b} + \frac{\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}}{b} + \frac{\Sigma(P_i\nu_{Si}/\nu_{pi}) + N}{bE\varepsilon_\varepsilon / k_\varepsilon}\right]x_w + \\ &+k_t\omega_{nt}d_u^2 + \frac{\Sigma k_{fi}\alpha_{efi}A_{fi}\nu_{fi}a_{fi}}{b} + \\ &+\frac{\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}a_{si}}{b} + \frac{\Sigma(P_i\nu_{Si}/\nu_{pi}) + N}{bE\varepsilon_\varepsilon / k_\varepsilon}a_\varepsilon = 0 \qquad (1)\end{aligned}$$

$$\begin{aligned}&\left[k_t\left(\omega_{nt} - \omega_{mt}\right) - k_c\left(\omega_{nc} - \omega_{mc}\right)\right]x_w^3 + \\ &+\left[k_t\left(\omega_{nt}a_a + 2\omega_{nt}d_u - 3\omega_{mt}d_u\right) - k_c\omega_{nc}a_a\right]x_w^2 + \\ &+\left\{\begin{aligned}&k_t\left(2\omega_{nt}a_a d_u + \omega_{nt}d_u^2 - 3\omega_{mt}d_u^2\right) + \\ &+\frac{\Sigma k_{fi}\alpha_{efi}A_{fi}\nu_{fi}\left(a_a - a_{fi}\right)}{b} + \\ &+\frac{\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}\left(a_a - a_{si}\right)}{b} + \\ &+\frac{\Sigma(P_i\nu_{Si}/\nu_{pi})\left(a_a - a_{si}\right) + N\left(a_a - e\right) + M}{bE\varepsilon_\varepsilon / k_\varepsilon}\end{aligned}\right\}x_w + \\ &+k_t\left(\omega_{nt}a_a d_u^2 - \omega_{mt}d_u^3\right) + \frac{\Sigma k_{fi}\alpha_{efi}A_{fi}\nu_{fi}\left(a_a - a_{fi}\right)a_{fi}}{b} + \\ &+\frac{\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}\left(a_a - a_{si}\right)a_{si}}{b} + \\ &+\frac{\Sigma\left(P_i\nu_{Si}/\nu_{pi}\right)\left(a_a - a_{si}\right) + N\left(a_a - e\right) + M}{bE\varepsilon_\varepsilon / k_\varepsilon}a_\varepsilon = 0 \qquad (2)\end{aligned}$$

If necessary, the flanges of the member may be divided into sectors of any size A_{fi}. Other types of strengthening and/or weakenings of the member (e.g. various ducts) are also treated as sectors A_{fi}. An analogous method of dividing the cross-section or its part into smaller sectors was used in paper [8]. An analogous method is offered by the regulations [3, 4]. Unfortunately, the regulations [3, 4] do not contain a practical implementation of the method.

Distances a_{fi} are assumed from axis $w-w$ to sectors A_{fi} of the layers in which the resultant force N_{fi} of the stresses σ_{fi} of the sectors is selected. The force $N_{fi} = \sigma_{fi}A_{fi}$. This force does not necessarily have to be assumed in the centre of gravity of the respective sectors. By selecting the resultant force not in the centre of the sectors it is possible to get the desired uneven distribution of the stresses in the sector. It seems that in most cases it would be worthwhile to select the centre of the sector. The change of σ_{fi} is taken into account with the help Eq. (16).

The most important parameters are denoted by Eqs. (3) - (13):

$$\nu_{Si} = \sigma_{Si} / \overline{E}_{si}\varepsilon_{Si} \qquad (3)$$

$$\nu_{pi} = \sigma_{pi} / \overline{E}_{si}\varepsilon_{pi} \qquad (4)$$

$$P_i = \sigma_{pi}A_{si} = \overline{E}_{si}A_{si}\varepsilon_{pi} = \nu_{pi}E_{si}A_{si}\varepsilon_{pi} \qquad (5)$$

$$Z_{bsi} = k_{si}\alpha_{esi}A_{si}\nu_{Si} / b \qquad (6)$$

$$Z_{fvi} = k_{fi}\alpha_{efi}A_{fi} / b \qquad (7)$$

$$Z_{bfi} = k_{fi}\alpha_{efi}A_{fi}\nu_{fi} / b = Z_{fvi}\nu_{fi} \qquad (8)$$

$$Z_E = bE / k_\varepsilon \qquad (9)$$

$$Z_{pn} = \frac{\Sigma(P_i\nu_{Si}/\nu_{pi}) + N}{Z_E} \qquad (10)$$

$$Z_{pn\varepsilon} = Z_{pn} / \varepsilon_\varepsilon \qquad (11)$$

$$Z_{pm} = \frac{\Sigma(P_i\nu_{Si}/\nu_{pi})\left(a_a - a_{si}\right) + N\left(a_a - e\right)}{Z_E} \qquad (12)$$

$$Z_{pm\varepsilon} = \left(Z_{pm} + \frac{M}{Z_E}\right)\frac{1}{\varepsilon_\varepsilon} \qquad (13)$$

For structural members without cracks $d_u = h$.

When in Eqs. (1) and (2) we use the parameters Eqs. (3) - (13) we receive Eqs. (14) and (15):

$$\begin{aligned}&\left(k_t\omega_{nt} - k_c\omega_{nc}\right)x_w^2 + \\ &+\left(2k_t\omega_{nt}d_u + \Sigma Z_{fvi}\nu_{fi} + \Sigma Z_{bsi} + Z_{pn\varepsilon}\right)x_w + \\ &+k_t\omega_{nt}d_u^2 + \Sigma Z_{fvi}\nu_{fi}a_{fi} + \Sigma Z_{bsi}a_{si} + Z_{pn\varepsilon}a_\varepsilon = 0 \qquad (14)\end{aligned}$$

$$\begin{aligned}&\left[k_t\left(\omega_{nt} - \omega_{mt}\right) - k_c\left(\omega_{nc} - \omega_{mc}\right)\right]x_w^3 + \\ &+\left\{k_t[\omega_{nt}(2d_u + a_a) - 3\omega_{mt}d_u] - k_c\omega_{nc}a_a\right\}x_w^2 + \\ &+\left\{\begin{aligned}&k_t[\omega_{nt}(d_u + 2a_a) - 3\omega_{mt}d_u]d_u + \\ &+\Sigma Z_{fvi}\nu_{fi}\left(a_a - a_{fi}\right) + \Sigma Z_{bsi}\left(a_a - a_{si}\right) + Z_{pm\varepsilon}\end{aligned}\right\}x_w + \\ &+k_t(\omega_{nt}a_a - \omega_{mt}d_u)d_u^2 + \Sigma Z_{fvi}\nu_{fi}\left(a_a - a_{fi}\right)a_{fi} + \\ &+\Sigma Z_{bsi}\left(a_a - a_{si}\right)a_{si} + Z_{pm\varepsilon}a_\varepsilon = 0 \qquad (15)\end{aligned}$$

3.1. Parameters of the nonlinear stress diagram of flanges and strengthening /weakening

In the present paper, the change of stresses σ_{fi} in sectors A_{fi} is described by functions

$$\sigma_{fi} = E_{fi}\varepsilon_{fi}(1 + c_{1fi}\eta_{fi} + c_{2fi}\eta_{fi}^2 + c_{3fi}\eta_{fi}^3 + c_{4fi}\eta_{fi}^4) = \\ = \nu_{fi}E_{fi}\varepsilon_{fi} = \nu_{fi}\sigma_{efi} \quad (16)$$

$$\nu_{fi} = 1 + c_{1fi}\eta_{fi} + c_{2fi}\eta_{fi}^2 + c_{3fi}\eta_{fi}^3 + c_{4fi}\eta_{fi}^4 \quad (17)$$

$$\eta_{fi} = \varepsilon_{fi} / \varepsilon_{fmi} \quad (18)$$

If the material of the sectors is the same as the one of the entire cross-section, then the coefficients of all the sectors α_{efi} will be the same and equal to one ($\alpha_{efi} = E_{fi} / E = 1$), $c_{jfi} = c_i$ and $\sigma_{fi} = \sigma_c$.

Paper [13] analysed the case when the stress-strain diagram $\sigma_c - \varepsilon_c$ of the material of compression and tension zones of a beam (e.g. for concrete of strength classes C08/10-C90/105) was defined by Eqs. (27) and (28) or by Eqs. (4) and (5) presented in paper [14].

The change of the coefficient ν_{fi} included into the static equilibrium equations is described by function (17). This function is rearranged in order to make x_w the variable of the function. Please note that the height x_w of the compression zone is negative.

$$\eta_{0\varepsilon fi} = \varepsilon_{0\varepsilon} / \varepsilon_{0fmi} = const \quad (19)$$

$$x_{fi} = x_w + a_{fi} \quad (20)$$

$$\frac{x_{fmi}}{a_\varepsilon + x_w} = \frac{\varepsilon_{0fmi}}{\varepsilon_{0\varepsilon}} = \frac{\varepsilon_{fmi} / k_{fmi}}{\varepsilon_\varepsilon / k_\varepsilon} = \frac{1}{\eta_{0\varepsilon fi}} \quad (21)$$

$$x_{fmi} = (a_\varepsilon + x_w) / \eta_{0\varepsilon fi} \quad (22)$$

$$\eta_{fi} = \frac{\varepsilon_{fi}}{\varepsilon_{fmi}} = \frac{\varepsilon_{0fi}}{\varepsilon_{0fmi}} = \frac{x_{fi}}{x_{tm}} = \frac{\eta_{0\varepsilon fi} x_{fi}}{a_\varepsilon + x_w} = \frac{\eta_{0\varepsilon fi}(x_w + a_{fi})}{a_\varepsilon + x_w} \quad (23)$$

$$\left.\begin{array}{l} u_{1fi} = c_{1fi}\eta_{0\varepsilon fi}, \quad u_{2fi} = c_{2fi}\eta_{0\varepsilon fi}^2 \\ u_{3fi} = c_{3fi}\eta_{0\varepsilon fi}^3, \quad u_{4fi} = c_{4fi}\eta_{0\varepsilon fi}^4 \end{array}\right\} \quad (24)$$

$$w_x = x_w + a_\varepsilon \quad (25)$$

$$v_{4x} = 1/60 w_x^4 = 1/60(x_w + a_\varepsilon)^4 = \\ = 1/60(x_w^4 + 4a_\varepsilon x_w^3 + 6a_\varepsilon^2 x_w^2 + 4a_\varepsilon^3 x_w + a_\varepsilon^4) \quad (26)$$

$$u_{4x} = v_{4x} / 7 = 1/420 w_x^4 = 1/420(x_w + a_\varepsilon)^4 = \\ = 1/420(x_w^4 + 4a_\varepsilon x_w^3 + 6a_\varepsilon^2 x_w^2 + 4a_\varepsilon^3 x_w + a_\varepsilon^4) \quad (27)$$

$$\nu_{fi} = 1 + c_{1fi}\eta_{fi} + c_{2fi}\eta_{fi}^2 + c_{3fi}\eta_{fi}^3 + c_{4fi}\eta_{fi}^4 = \\ = 1 + c_{1fi}\frac{\eta_{0\varepsilon fi}(x_w + a_{fi})}{x_w + a_\varepsilon} + c_{2fi}\frac{\eta_{0\varepsilon fi}^2(x_w + a_{fi})^2}{(x_w + a_\varepsilon)^2} + \\ + c_{3fi}\frac{\eta_{0\varepsilon fi}^3(x_w + a_{fi})^3}{(x_w + a_\varepsilon)^3} + c_{4fi}\frac{\eta_{0\varepsilon fi}^4(x_w + a_{fi})^4}{(x_w + a_\varepsilon)^4} = \\ = (w_{4fi}x_w^4 + w_{3fi}x_w^3 + w_{2fi}x_w^2 + w_{1fi}x_w + w_{0fi}) : w_x^4 \quad (28)$$

$$w_{0fi} = a_\varepsilon^4 + u_{1fi}a_{fi}a_\varepsilon^3 + u_{2fi}a_{fi}^2 a_\varepsilon^2 + \\ + u_{3fi}a_{fi}^3 a_\varepsilon + u_{4fi}a_{fi}^4 \quad (29)$$

$$w_{1fi} = 4a_\varepsilon^3 + u_{1fi}(3a_{fi} + a_\varepsilon)a_\varepsilon^2 + \\ + 2u_{2fi}a_{fi}(a_{fi} + a_\varepsilon)a_\varepsilon + \\ + u_{3fi}a_{fi}^2(a_{fi} + 3a_\varepsilon) + 4u_{4fi}a_{fi}^3 \quad (30)$$

$$w_{2fi} = 6a_\varepsilon^2 + 3u_{1fi}(a_{fi} + a_\varepsilon)a_\varepsilon + \\ + u_{2fi}(a_{fi}^2 + 4a_{fi}a_\varepsilon + a_\varepsilon^2) + \\ + 3u_{3fi}a_{fi}(a_{fi} + a_\varepsilon) + 6u_{4fi} \quad (31)$$

$$w_{3fi} = 4a_\varepsilon + u_{1fi}(a_{fi} + 3a_\varepsilon) + \\ + 2u_{2fi}(a_{fi} + a_\varepsilon) + \\ + u_{3fi}(3a_{fi} + a_\varepsilon) + 4u_{4fi}a_{fi} \quad (32)$$

$$w_{4fi} = 1 + u_{1fi} + u_{2fi} + u_{3fi} + u_{4fi} \quad (33)$$

3.2. Parameters of the nonlinear stress diagram of the compression zone [13]

$$\eta_{0\varepsilon c} = \varepsilon_{0\varepsilon} / \varepsilon_{0cm} = const \quad (34)$$

$$\eta_c = \eta_{cw} = \frac{\varepsilon_w}{\varepsilon_{0cm}} = \frac{x_w}{x_{cm}} = \frac{x_w}{a_\varepsilon + x_w}\eta_{0\varepsilon c} \quad (35)$$

$$\omega_{nc} = \frac{1}{2} + \frac{c_{1c}}{3}\eta_{cw} + \frac{c_{2c}}{4}\eta_{cw}^2 + \frac{c_{3c}}{5}\eta_{cw}^3 + \frac{c_{4c}}{6}\eta_{cw}^4 = \\ = (n_{4c}x_w^4 + n_{3c}x_w^3 + n_{2c}x_w^2 + n_{1c}x_w + n_{0c})v_{4x} \quad (36)$$

$$\omega_{mc} = \frac{1}{3} + \frac{c_{1c}}{4}\eta_{cw} + \frac{c_{2c}}{5}\eta_{cw}^2 + \frac{c_{3c}}{6}\eta_{cw}^3 + \frac{c_{4c}}{7}\eta_{cw}^4 = \\ = (m_{4c}x_w^4 + m_{3c}x_w^3 + m_{2c}x_w^2 + m_{1c}x_w + m_{0c})u_{4x} \quad (37)$$

$$\left.\begin{array}{l} u_{1c} = c_{1c}\eta_{0\varepsilon c}, \quad u_{2c} = c_{2c}\eta_{0\varepsilon c}^2 \\ u_{3c} = c_{3c}\eta_{0\varepsilon c}^3, \quad u_{4c} = c_{4c}\eta_{0\varepsilon c}^4 \end{array}\right\} \quad (38)$$

$$\left.\begin{array}{l} n_{0c} = 30a_\varepsilon^4 \\ n_{1c} = (120 + 20u_{1c})a_\varepsilon^3 \\ n_{2c} = (180 + 60u_{1c} + 15u_{2c})a_\varepsilon^2 \\ n_{3c} = (120 + 60u_{1c} + 30u_{2c} + 12u_{3c})a_\varepsilon \\ n_{4c} = 30 + 20u_{1c} + 15u_{2c} + 12u_{3c} + 10u_{4c} \end{array}\right\} \quad (39)$$

$$\left.\begin{array}{l} m_{0c} = 140a_\varepsilon^4 \\ m_{1c} = (560 + 105u_{1c})a_\varepsilon^3 \\ m_{2c} = (840 + 315u_{1c} + 84u_{2c})a_\varepsilon^2 \\ m_{3c} = (560 + 315u_{1c} + 168u_{2c} + 70u_{3c})a_\varepsilon \\ n_{4c} = 30 + 20u_{1c} + 15u_{2c} + 12u_{3c} + 10u_{4c} \\ m_{4c} = 140 + 105u_{1c} + 84u_{2c} + 70u_{3c} + 60u_{4c} \end{array}\right\} \quad (40)$$

3.3. Parameters of the nonlinear stress diagram of the tension zone [13]

$$\eta_{0\varepsilon t} = \varepsilon_{0\varepsilon} / \varepsilon_{0tm} = const \quad (41)$$

$$\eta_t = \eta_{th} = \frac{x_{th}}{x_{tm}} = \frac{x_{th}}{a_\varepsilon + x_w}\eta_{0\varepsilon t} = \frac{h + x_w}{a_\varepsilon + x_w}\eta_{0\varepsilon t} \quad (42)$$

$$\omega_{nt} = \frac{1}{2} + \frac{c_{1t}}{3}\eta_{th} + \frac{c_{2t}}{4}\eta_{th}^2 + \frac{c_{3t}}{5}\eta_{th}^3 + \frac{c_{4t}}{6}\eta_{th}^4 = \\ = (n_{4t}x_w^4 + n_{3t}x_w^3 + n_{2t}x_w^2 + n_{1t}x_w + n_{0t})v_{4x} \quad (43)$$

$$\begin{aligned}\omega_{mt} &= \frac{1}{3} + \frac{c_{1t}}{4}\eta_{th} + \frac{c_{2t}}{5}\eta_{th}^2 + \frac{c_{3t}}{6}\eta_{th}^3 + \frac{c_{4t}}{7}\eta_{th}^4 = \\ &= \left(m_{4t}x_w^4 + m_{3t}x_w^3 + m_{2t}x_w^2 + m_{1t}x_w + m_{0t}\right)u_{4x}\end{aligned} \tag{44}$$

$$\left.\begin{aligned}&u_{1t} = c_{1t}\eta_{0\varepsilon t}, \quad u_{2t} = c_{2t}\eta_{0\varepsilon t}^2 \\ &u_{3t} = c_{3t}\eta_{0\varepsilon t}^3, \quad u_{4t} = c_{4t}\eta_{0\varepsilon t}^4\end{aligned}\right\} \tag{45}$$

$$\left.\begin{aligned}n_{0t} &= 30a_\varepsilon^4 + 20u_{1t}ha_\varepsilon^3 + \\ &\quad + 15u_{2t}h^2a_\varepsilon^2 + 12u_{3t}h^3a_\varepsilon + 10u_{4t}h^4 \\ n_{1t} &= 120a_\varepsilon^3 + 20u_{1t}(3h + a_\varepsilon)a_\varepsilon^2 + \\ &\quad + 30u_{2t}h(h + a_\varepsilon)a_\varepsilon + 12u_{3t}h^2(h + 3a_\varepsilon) + \\ &\quad + 40u_{4t}h^3 \\ n_{2t} &= 180a_\varepsilon^2 + 60u_{1t}(h + a_\varepsilon)a_\varepsilon + \\ &\quad + 15u_{2t}\left(h^2 + 4ha_\varepsilon + a_\varepsilon^2\right) + 36u_{3t}h(h + a_\varepsilon) + \\ &\quad + 60u_{4t}h^2 \\ n_{3t} &= 120a_\varepsilon + 20u_{1t}(h + 3a_\varepsilon) + 30u_{2t}(h + a_\varepsilon) + \\ &\quad + 12u_{3t}(3h + a_\varepsilon) + 40u_{4t}h \\ n_{4t} &= 30 + 20u_{1t} + 15u_{2t} + 12u_{3t} + 10u_{4t}\end{aligned}\right\} \tag{46}$$

$$\left.\begin{aligned}m_{0t} &= 140a_\varepsilon^4 + 105u_{1t}ha_\varepsilon^3 + 84u_{2t}h^2a_\varepsilon^2 + \\ &\quad + 70u_{3t}h^3a_\varepsilon + 60u_{4t}h^4 \\ m_{1t} &= 560a_\varepsilon^3 + 105u_{1t}(3h + a_\varepsilon)a_\varepsilon^2 + \\ &\quad + 168u_{2t}h(h + a_\varepsilon)a_\varepsilon + \\ &\quad + 70u_{3t}h^2(h + 3a_\varepsilon) + 240u_{4t}h^3 \\ m_{2t} &= 840a_\varepsilon^2 + 315u_{1t}(h + a_\varepsilon)a_\varepsilon + \\ &\quad + 84u_{2t}\left(h^2 + 4ha_\varepsilon + a_\varepsilon^2\right) + 210u_{3t}h(h + a_\varepsilon) + \\ &\quad + 360u_{4t}h^2 \\ m_{3t} &= 560a_\varepsilon + 105u_{1t}(h + 3a_\varepsilon) + 168u_{2t}(h + a_\varepsilon) + \\ &\quad + 70u_{3t}(3h + a_\varepsilon) + 240u_{4t}h \\ m_{4t} &= 140 + 105u_{1t} + 84u_{2t} + 70u_{3t} + 60u_{4t}\end{aligned}\right\} \tag{47}$$

When we insert the values of respective parameters into Eqs. (14) and (15) we receive equations of static equilibrium: Eq. (48) of the forces and Eq. (49) of the bending moments:

$$\begin{aligned}&a_{n0} + a_{n1}x_w + a_{n2}x_w^2 + a_{n3}x_w^3 + \\ &+ a_{n4}x_w^4 + a_{n5}x_w^5 + a_{n6}x_w^6 = 0\end{aligned} \tag{48}$$

$$\begin{aligned}&a_{m0} + a_{m1}x_w + a_{m2}x_w^2 + a_{m3}x_w^3 + \\ &+ a_{m4}x_w^4 + a_{m5}x_w^5 + a_{m6}x_w^6 + a_{m7}x_w^7 = 0\end{aligned} \tag{49}$$

where

$$\begin{aligned}a_{n0} &= k_t n_{0t}h^2 + \\ &\quad + 60\left[\Sigma Z_{fvi}w_{0fi}a_{fi} + a_\varepsilon^4\left(\Sigma Z_{bsi}a_{si} + a_\varepsilon Z_{pn\varepsilon}\right)\right]\end{aligned} \tag{50}$$

$$\begin{aligned}a_{n1} &= k_t\left(2n_{0t}h + n_{1t}h^2\right) + \\ &\quad + 60\left\{\begin{aligned}&\Sigma Z_{fvi}\left(w_{0fi} + w_{1fi}a_{fi}\right) + \\ &+ a_\varepsilon^3\left[\Sigma Z_{bsi}\left(4a_{si} + a_\varepsilon\right) + 5a_\varepsilon Z_{pn\varepsilon}\right]\end{aligned}\right\}\end{aligned} \tag{51}$$

$$\begin{aligned}a_{n2} &= k_t\left(n_{0t} + 2n_{1t}h + n_{2t}h^2\right) - k_c n_{0c} + \\ &\quad + 60\left\{\begin{aligned}&\Sigma Z_{fvi}\left(w_{1fi} + w_{2fi}a_{fi}\right) + \\ &+ 2a_\varepsilon^2\left[\Sigma Z_{bsi}\left(3a_{si} + 2a_\varepsilon\right) + 5a_\varepsilon Z_{pn\varepsilon}\right]\end{aligned}\right\}\end{aligned} \tag{52}$$

$$\begin{aligned}a_{n3} &= k_t\left(n_{1t} + 2n_{2t}h + n_{3t}h^2\right) - k_c n_{1c} + \\ &\quad + 60\left\{\begin{aligned}&\Sigma Z_{fvi}\left(w_{2fi} + w_{3fi}a_{fi}\right) + \\ &+ 2a_\varepsilon\left[\Sigma Z_{bsi}\left(2a_{si} + 3a_\varepsilon\right) + 5a_\varepsilon Z_{pn\varepsilon}\right]\end{aligned}\right\}\end{aligned} \tag{53}$$

$$\begin{aligned}a_{n4} &= k_t\left(n_{2t} + 2n_{3t}h + n_{4t}h^2\right) - k_c n_{2c} + \\ &\quad + 60\left[\begin{aligned}&\Sigma Z_{fvi}\left(w_{3fi} + w_{4fi}a_{fi}\right) + \\ &+ \Sigma Z_{bsi}\left(a_{si} + 4a_\varepsilon\right) + 5a_\varepsilon Z_{pn\varepsilon}\end{aligned}\right]\end{aligned} \tag{54}$$

$$\begin{aligned}a_{n5} &= k_t\left(n_{3t} + 2n_{4t}h\right) - k_c n_{3c} + \\ &\quad + 60\left(\Sigma Z_{fvi}w_{4fi} + \Sigma Z_{bsi} + Z_{pn\varepsilon}\right)\end{aligned} \tag{55}$$

$$a_{n6} = k_t n_{4t} - k_c n_{4c} \tag{56}$$

$$\begin{aligned}a_{m0} &= k_t\left(7n_{0t}h^2a_a - m_{0t}h^3\right) + \\ &\quad + 420\left\{\begin{aligned}&\Sigma Z_{fvi}w_{0fi}a_{fi}\left(a_a - a_{fi}\right) + \\ &+ a_\varepsilon^4\left[\Sigma Z_{bsi}a_{si}\left(a_a - a_{si}\right) + a_\varepsilon Z_{pm\varepsilon}\right]\end{aligned}\right\}\end{aligned} \tag{57}$$

$$\begin{aligned}a_{m1} &= k_t\left\{\begin{aligned}&7\left[n_{0t}(2a_a + h)h + n_{1t}h^2a_a\right] - \\ &- 3m_{0t}h^2 - m_{1t}h^3\end{aligned}\right\} + \\ &\quad + 420\left\{\begin{aligned}&\Sigma Z_{fvi}\left(w_{0fi} + w_{1fi}a_{fi}\right)\left(a_a - a_{fi}\right) + \\ &+ a_\varepsilon^3\left[\begin{aligned}&\Sigma Z_{bsi}\left(a_\varepsilon + 4a_{si}\right)\left(a_a - a_{si}\right) + \\ &+ 5a_\varepsilon Z_{pm\varepsilon}\end{aligned}\right]\end{aligned}\right\}\end{aligned} \tag{58}$$

$$\begin{aligned}a_{m2} &= k_t\left\{\begin{aligned}&7\left[n_{0t}(a_a + 2h) + n_{1t}(2a_a + h)h + n_{2t}h^2a_a\right] - \\ &- 3m_{0t}h - 3m_{1t}h^2 - m_{2t}h^3\end{aligned}\right\} - \\ &\quad - 7k_c n_{0c}a_a + \\ &\quad + 420\left\{\begin{aligned}&\Sigma Z_{fvi}\left(w_{1fi} + w_{2fi}a_{fi}\right)\left(a_a - a_{fi}\right) + \\ &+ 2a_\varepsilon^2\left[\begin{aligned}&\Sigma Z_{bsi}\left(2a_\varepsilon + 3a_{si}\right)\left(a_a - a_{si}\right) + \\ &+ 5a_\varepsilon Z_{pm\varepsilon}\end{aligned}\right]\end{aligned}\right\}\end{aligned} \tag{59}$$

$$\begin{aligned}a_{m3} &= k_t\left\{\begin{aligned}&7\left[\begin{aligned}&n_{0t} + n_{1t}(a_a + 2h) + \\ &+ n_{2t}(2a_a + h)h + n_{3t}h^2a_a\end{aligned}\right] - \\ &- m_{0t} - 3m_{1t}h - 3m_{2t}h^2 - m_{3t}h^3\end{aligned}\right\} - \\ &\quad - k_c\left[7\left(n_{0c} + n_{1c}a_a\right) - m_{0c}\right] + \\ &\quad + 420\left\{\begin{aligned}&\Sigma Z_{fvi}\left(w_{2fi} + w_{3fi}a_{fi}\right)\left(a_a - a_{fi}\right) + \\ &+ 2a_\varepsilon\left[\begin{aligned}&\Sigma Z_{bsi}\left(3a_\varepsilon + 2a_{si}\right)\left(a_a - a_{si}\right) + \\ &+ 5a_\varepsilon Z_{pm\varepsilon}\end{aligned}\right]\end{aligned}\right\}\end{aligned} \tag{60}$$

$$\begin{aligned}a_{m4} &= k_t\left\{\begin{aligned}&7\left[\begin{aligned}&n_{1t} + n_{2t}(a_a + 2h) + \\ &+ n_{3t}(2a_a + h)h + n_{4t}h^2a_a\end{aligned}\right] - \\ &- m_{1t} - 3m_{2t}h - 3m_{3t}h^2 - m_{4t}h^3\end{aligned}\right\} - \\ &\quad - k_c\left[7\left(n_{1c} + n_{2c}a_a\right) - m_{1c}\right] +\end{aligned}$$

$$+420\begin{bmatrix} \Sigma Z_{fvi}\left(w_{3fi}+w_{4fi}a_{fi}\right)\left(a_a-a_{fi}\right)+ \\ +\Sigma Z_{bsi}\left(4a_{\varepsilon}+a_{si}\right)\left(a_a-a_{si}\right)+5a_{\varepsilon}Z_{pm\varepsilon} \end{bmatrix} \quad (61)$$

$$a_{n5}=k_t\begin{Bmatrix} 7\left[n_{2t}+n_{3t}\left(a_a+2h\right)+n_{4t}\left(2a_a+h\right)h\right]- \\ -m_{2t}-3m_{3t}h-3m_{4t}h^2 \end{Bmatrix}- \\ -k_c\left[7\left(n_{2c}+n_{3c}a_a\right)-m_{2c}\right]+ \\ +420\begin{bmatrix} \Sigma Z_{fvi}w_{4fi}\left(a_a-a_{fi}\right)+ \\ +\Sigma Z_{bsi}\left(a_a-a_{si}\right)+Z_{pm\varepsilon} \end{bmatrix} \quad (62)$$

$$a_{m6}=k_t\left\{7\left[n_{3t}+n_{4t}\left(a_a+2h\right)\right]-m_{3t}-3m_{4t}h\right\}- \\ -k_c\left[7\left(n_{3c}+n_{4c}a_a\right)-m_{3c}\right] \quad (63)$$

$$a_{m7}=k_t\left(7n_{4t}-m_{4t}\right)-k_c\left(7n_{4c}-m_{4c}\right) \quad (64)$$

When stresses of the tension zone are ignored, then, in the equations of static equilibrium, $k_t=0$ and it is possible to assume that the zone has cracked. In this case it is possible to calculate the failure moment M_u of the reinforced concrete member. In this case it is possible to calculate the reinforcement of the reinforced concrete member, i.e. the area of the reinforcement A_s .The yield of the reinforcement can be taken into account using coefficient ν_S .

4. Conclusions

The method and the formulae presented in the paper are applicable for the calculation of the stress-strain state parameters (the compression and the tension zones and the depth of the crack) at normal sections of flanged structural members subjected to bending. The paper presents a sufficiently simple and resonably accurate way of taking into account the impact of the flanges and/or other strengthenings (weakenings). The method employs nonlinear (curvilinear) stress diagrams. The calculation is direct (without the successive approximation cycles). The formulae may be used for practical calculations. They are not very complicated, they are clear, easy-to-program, suitable for computer-based calculations. In the general case, the members have to be without cracks. But the formulae are also applicable to calculations of parameters of members that have cracks, for their sections between the cracks. When the stresses of the tension zone of the members are ignored, the formulae are also applicable even to the sections near the crack The direct calculation is possible when we know the strain of at least one of the layers and when we know whether the flange and the reinforcement are subject to compression or to tension. In other cases, the calculations need to be repeated. The method is applicable when it is not necessary to know with accuracy in what way the stresses are distributed within the flange. The author has developed a method and formulae taking into account the nonlinear stress distribution within the flange. However, they are more complicated. They are planned to be published in forthcoming papers.

References

1. EN 1992-1-1:2004: E. Eurocode 2: Design of Concrete Structures - Part 1 - 1: General Rules and Rules for Buildings. -Brussels: European Committee for Standardization, 2004.-253p.
2. STR 2.05.05:2005 Design of Concrete and Reinforced Concrete Structures (in Lithuanian).
3. SP 52-101-2003. Concrete and reinforces concrete structures without presstresed reinforcement. -Moscow: NIIZHB, 2005.-127p. (in Russian).
4. SP 52-102-2004.Prestressed reinforced concrete structures. Moscow, 2004.-44 p. (in Russian).
5. **Zabulionis, D., Dulinskas, E.** Analysis of compression zone parameters of cross-section in flexural reinforced concrete members according to EC2 and STR 2.05.05. -Mechanika. -Kaunas: Technologija, 2008, Nr.3(71), p.12-19.
6. **Dulinskas, E., Zabulionis, D.** Analysis of equivalent substitution of rectangular stress block for nonlinear stress diagram. -Mechanika. -Kaunas: Technologija, 2007, Nr.6(68), p.26-38.
7. **Žiliukas, A., Kargaudas, V., Adamukaitis, N.** Yield stresses in compressed and bended columns and beams. -Mechanika. -Kaunas: Technologija, 2006, Nr.3(59), p.13-18.
8. **Židonis, I**. Calculation method of stress-strain of reinforced concrete members at normal sections.-Reinforced constructions: Strength of the concrete and reinforced concrete, 10.-Vilnius, 1980, p.65-72 (in Russian).
9. **Zaliesov, A.S.; Chistiakov, E.A.; Laricheva I.J.** New methods of calculating reinforced concrete members at normal sections based on deformation calculation model.-Concrete and Reinforced Concrete, 5, 1997, p.31-34 (in Russian).
10. **Raue, E.** Non-linear analysis of cross-sections by mathematical optimisation. -Bautechnik 82, Heft 11, 2005, p.796-809 (in German).
11. **Raue, E.** Non-linear analysis of composite cross-sections by non-linear optimisation. -Modern building materials, structures and techniques: Abstracts of the 9th international conference held in Vilnius on May 16–18, 2007 (full paper on enclosed CD-ROM, 9 p.*)*. Vilnius: Technika, 2007, p.434.
12. **Židonis,** I. Alternative method for the calculation of stress-strain state parameters in normal sections of structural members. -Mechanika. -Kaunas: Technologija, 2007, Nr.5(67), p.24-32.
13. **Židonis, I.** Method for a direct calculation of stress-strain state parameters in normal right-angled sections of structural members given curvilinear stress diagrams. -Mechanika. -Kaunas: Technologija, 2009, Nr.3(77), p.27-33.
14. **Židonis, I.** A simple-to-integrate formula of stress as a function of strain in concrete and its description procedure. -Mechanika. -Kaunas: Technologija, 2007, Nr.4(66), p.23-30.

I. Židonis

LENTYNŲ ĮVERTINIMAS TIESIOGIAI APSKAIČIUOJANT ĮTEMPIŲ-DEFORMACIJŲ BŪVIO PARAMETRUS KONSTRUKCINIŲ ELEMENTŲ STATMENUOSE PJŪVIUOSE

R e z i u m ė

Šis darbas yra autoriaus darbų tęsinys. Straipsnyje pateikta metodika ir formulės, skirtos lentynų ir (arba) kitokių sustiprinimų (susilpninimų) įtakai įvertinti tiesiogiai (be nuoseklaus artėjimo ciklų) apskaičiuojant sijinių konstrukcinių elementų įtempių-deformacijų būvio parametrus ašiai statmenuose (normaliniuose) pjūviuose pagal kreivines medžiagų įtempių diagramas. Formulės taikytinos elementams be plyšių (pavyzdžiui, plyšimo momentui apskaičiuoti) ir elementų su plyšiais pjūviams tarp plyšių (armuotų elementų įtempių-deformacijų būviui apskaičiuoti, nustatyti armatūros deformacijos atsilenkimą nuo plokščiųjų pjūvių). Įtempių galima ir nepaisyti. Kai elementų tempiamos zonos įtempių nepaisoma, formulės tinka net ir pjūviams ties plyšiu (irimo momentui arba armavimui apskaičiuoti). Tiesioginis skaičiavimas galimas tuomet, kai žinoma kurio nors vieno sluoksnio deformacija ir iš anksto žinoma, ar lentyna ir armatūra gniuždoma, ar tempiama. Kitais atvejais skaičiavimus tenka kartoti. Atvejai, kai įtempių diagramos ne kreivinės arba kai elementai su plyšiais, bus skelbiami kituose straipsniuose.

I. Židonis

ESTIMATION OF THE FLANGES IN THE DIRECT CALCULATION OF STRESS-STRAIN STATE PARAMETERS AT NORMAL SECTIONS OF STRUCTURAL MEMBERS

S u m m a r y

The present paper is a continuation of papers authors. The method and the formulae presented in the paper are designed for taking into account of the impact of the flanges and/or other strengthening /weakenings when making direct calculation (without the successive approximations cycles) of the stress-strain state parameters at normal sections of bar-shaped structural members according to nonlinear material stress diagrams. The formulae are applicable for members that have no cracks (e.g. for the calculation of the cracking moment) and for the sections between the cracks of the members that have cracks (for the calculation of the stress-strain state of reinforced members, for estimating the deviation of the strain of the reinforcement from the plane sections). It is also possible to ignore the stresses. When the stresses of the tensile zone of the members are ignored, the formulae are applicable even to the sections near the crack (for the calculation of the cracking moment or of the reinforcement). Direct calculation is possible when we know in advance the strain of any of the layers and when we know whether the flange and reinforcement is subject of to compression or to tension. In other cases, the calculations need to be repeated. The cases when the stress diagrams are not curvilinear and when the members have cracks will be published as separate papers.

И. Жидонис

УЧЕТ ПОЛОК ПРИ ПРЯМОМ РАСЧЕТЕ НАПРЯЖЕННО-ДЕФОРМИРОВАННОГО СОСТОЯНИЯ ПО НОРМАЛЬНЫМ СЕЧЕНИЯМ КОНСТРУКТИВНЫХ ЭЛЕМЕНТОВ

Р е з ю м е

Работа является продолжением работ автора. В статье представлены методика и формулы учета полок и (или) других усилений (ослаблений) при прямом расчете (без циклов последовательного приближения) параметров напряженно-деформированного состояния по нормальным сечениям балочных элементов конструкций при криволинейных диаграммах напряжений материалов. Формулы предназначены для элементов без трещин (например, для расчета момента трещинообразования) и для сечений между трещинами в элементах с трещинами (для расчета напряженно-деформированного состояния армированных элементов с определением отклонения деформаций арматуры от гипотезы плоских сечений). Напряжения могут и не учитываться. Если напряжения растянутой зоны элемента не учитываются, формулы пригодны даже для сечений по трещинам (для определения момента разрушения или расчета армирования). Прямой расчет возможен тогда, когда известна деформация любого одного слоя и заранее известно, что полки и арматура сжимаемы или растягиваемы. В других случаях расчет приходится повторять. Случаи, когда диаграммы напряжений не криволинейные или присутствуют элементы с трещинами, будут представлены в других статьях.

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