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A contact problem on partial interaction of faces of a variable thickness slot under the influence of temperature field

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1. Introduction

One of the effective means of braking of cracks (slots) may be temperature and thermoelastic fields [1, 2]. In the fracture mechanics the problem of "healing" of a crack existing in the construction has an important value. Influence of heat source decreases [3] deformation of the extended plane in the direction perpendicular to the crack (slot), owing to which the stress intensity factor in the vicinity of the crack end decreases.

Under certain ratio of physical and geometrical parameters of the sheet element and heat source, in the sheet element there will arise contractive stresses, zones where the slot will make a contact at some area and at this area of slot faces there will arise contact stresses. Thus, the construction failure may be prevented by creating thermal fields along the way of the slot growth. Creation of thermal fields is justified by the ease of obtaining and comprehensive nature of influence on failure process. Furthermore, technical ease of obtaining in the extended object of temperature and thermoelastic field any in size and distribution creates wide possibilities of change of direction and braking of crack propagation. Goal of the paper is to develop a mathematical model of slot braking by means of temperature fields.

2. Problem statement

Assume that an unbounded elastic isotropic plane is weakened by a variable $h(x)$ thickness rectilinear slot comparable with elastic deformations. At infinity the plane is subjected to uniform tension along the ordinate axis $\sigma_y^\infty = \sigma_0$. The slot faces are free from external loads. For braking the growth of the slot along the way of its growth, the zone of compressive stresses are formed by heating the domain $S = S_1 + S_2$ to temperature T_0 by the heat source. The following assumptions were accepted: all thermoelastic characteristics of the plane's material are temperature independent: the plate's material is a homogeneous isotropic medium. It is assumed that at initial time $t = 0$ the arbitrary domain $S = S_1 + S_2$ along the way of the growth of the slot in the plane instantly heats up to constant temperature $T = T_0$. The remaining part of the plane at initial moment has the temperature $T = 0$.

For many metallic materials (steels, alluminium alloy and so on) at temperature change to 300-400ºC the dependence of thermoelastic characteristics weakly changes according to temperature. This fact was experimentally established [4, 5]. Thus for all structural materials there exists such a temperature range in which the assumption on steadiness of characteristics of the material is correct, it is established on the basis of temperature dependence of the modulus of elasticity. The experiments [1] show that by heating the track of the crack way to 70-100ºC, retardation and braking of the crack is observed. We can cite other papers [6, 7], where there is a positive answer on the observable effects of partial closing of the crack. Behavior of stresses near the ends of the cracks is investigated and stress intensity factors are determined in the paper [7]. It is established that at some values of the problem parameters the stress intensity factors turn to be negative. This means that the crack faces make a contact. Existence of negative stress intensity factors at least near the crack end reduces to necessity of account of partial contact of faces in some vicinity of the crack end.

We assume that in the deformation process the slot's faces in the vicinities of apexes make a contact in the areas (a, α_1) and (β_1, b) (Fig. 1). It is assumed that each contact area consists of areas of faces stick (a, c_1) and (d_1, b) and two areas (c_1, α_1) and (β_1, d_1) on which slipping if possible. Denote by L_1' the set of stick areas; by L_2' the set of slippage areas; by L_3' the area of the slots faces free from load. When determining the temperature field, for simplifying the problem its disturbance is not taken into account because of existence of a slot.

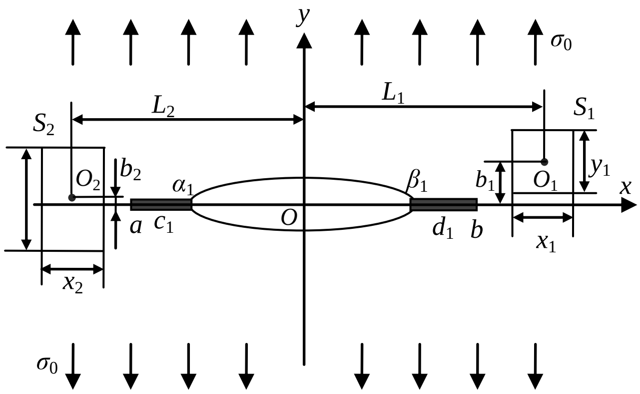

Fig. 1 Design scheme of the problem

The boundary conditions on the slot faces for the considered contact problem with stresses disappearing at infinity are written subject to superposition principle in the form:

$$\sigma_y^+ = \sigma_y^-, \quad \tau_{xy}^+ = \tau_{xy}^- \text{ on } L = L_1' + L_2' + L_3', \qquad (1)$$

$$\frac{\partial}{\partial x}(v^+ - v^-) = -h'(x) \text{ on } L_1' + L_2', \tag{2}$$

$$\sigma_y^+ = -\sigma_0 \text{ on } L,$$
$$\frac{\partial}{\partial x}(u^+ - u^-) = 0 \text{ on } L_1', \tag{3}$$
$$\tau_{xy}^+ = f\sigma_y^- \text{ on } L_2',$$
$$\tau_{xy}^+ = 0 \text{ on } L_3'.$$

Here it is accepted that on the slippage areas there hold the dry friction forces (the friction law is accepted in the Coulomb form); $f(x)$ is the friction coefficient; $(u^+ - u^-)$ is tangential $(v^+ - v^-)$ is normal component of the slop faces opening. The sizes of contact zones are not known in advance and should be determined.

A model of a contact with friction and stick was first considered by L.A. Galin [8]. Recent years a number of papers devoted to investigation of bodies with cracks with regard to cohesive forces between the faces and possibility of their contact were published [9-16].

3. Method of solution

Represent the stress state in the plane with a slot in the form:

$$\sigma_x = \sigma_{x_0} + \sigma_{x_1}, \quad \sigma_y = \sigma_{y_0} + \sigma_{y_1},$$
$$\tau_{xy} = \tau_{xy_0} + \tau_{xy_1}, \tag{4}$$

where σ_{x_0}, σ_{y_0}, τ_{xy_0} are the solution of the thermoelasticity problem for a slotless plane.

For finding the σ_{x_0}, σ_{y_0}, τ_{xy_0} stress state we solve the thermoelasticity problem for an continuous plane. At first we determine temperature distribution in a plane. For that we solve a problem of heat conduction theory:

$$\frac{\partial T}{\partial t} = a\Delta T, \quad T = \begin{cases} T_0 & (x, y \in S) \\ 0 & (x, y \notin S) \end{cases} \quad \text{for } t = 0,$$

where Δ is the Laplace operator; a is the thermal conductivity of the material.

Let for definiteness, the domain $S = S_1 + S_2$ heated by heat source represent a set of two rectangles. After solving the thermoelasticity problem for an entire plane, we find:

$$\sigma_{y_0} = \sum_{k=1}^{n} \sigma_{y_{0k}}, \quad \tau_{xy_0} = \sum_{k=1}^{n} \tau_{xy_{0k}}, \quad n=2; \tag{5}$$

$$\sigma_{y_{0k}} = -\frac{\mu(1+\nu)\alpha T_0}{4\sqrt{\pi}} \Big\{ 4\sqrt{\pi} A(x,y) + \frac{4}{\sqrt{\pi}} \Big[arctg\frac{y_{1*}}{x_{1*}} + arctg\frac{y_{2*}}{x_{2*}} + arctg\frac{y_{2*}}{x_{1*}} + arctg\frac{y_{2*}}{x_{1*}} - \int_0^t \frac{1}{\tau\sqrt{a\tau}} \Big[x_{1*} exp\Big(-\frac{x_{1*}^2}{4a\tau}\Big) + x_{2*} exp\Big(-\frac{x_{2*}^2}{4a\tau}\Big)\Big]\Big[Erf\Big(\frac{y_{1*}}{2\sqrt{a\tau}}\Big) + Erf\Big(\frac{y_{2*}}{2\sqrt{a\tau}}\Big)\Big] d\tau \Big\};$$

$$\tau_{xy_{0k}} = -\frac{\mu(1+\nu)\alpha T_0}{2\pi} \Big\{ \ln\frac{x_{2*}^2 + y_{1*}^2}{x_{2*}^2 + y_{2*}^2} + \ln\frac{x_{1*}^2 + y_{2*}^2}{x_{1*}^2 + y_{1*}^2} - \int_0^t \frac{1}{\tau} \Big[exp\Big(-\frac{x_{1*}^2}{4a\tau}\Big) - exp\Big(-\frac{x_{2*}^2}{4a\tau}\Big)\Big]\Big[exp\Big(-\frac{y_{1*}^2}{4a\tau}\Big) - exp\Big(-\frac{y_{2*}^2}{4a\tau}\Big)\Big] d\tau \Big\},$$

where $A(x,y) = \begin{cases} 1 & (x, y \in S_k) \\ 0 & (x, y \notin S_k) \end{cases}$; $y_{1*} = y - b_k + y_k$; $y_{2*} = y_k + b_k - y$; $x_{1*} = x - L_k + x_k$; $x_{2*} = x_k + L_k - x$; μ are the shear modulus of the plate material; ν is the Poisson ratio; α is the coefficient of linear temperature expansion.

Express the stress tensor components σ_{x_1}, σ_{y_1}, τ_{xy_1} and displacement vectors u_1, v_1 by two piecewise analytic complex variable functions $\Phi(z)$ and $\Omega(z)$ [17]:

$$\left.\begin{aligned} &\sigma_{y_1} - i\tau_{xy_1} = \Phi(z) + \Omega(\bar z) + (z - \bar z)\overline{\Phi'(z)} \\ &2\mu\frac{\partial}{\partial x}(u_1 + iv_1) = \kappa\Phi(z) - \Omega(\bar z) - (z - \bar z)\overline{\Phi'(z)} \end{aligned}\right\}, \tag{6}$$

where κ is the Muskhelishvili constant.

Following N.I. Muskhelishvili [17], based on boundary conditions (1), (2), with respect to formulas (5), (6) we arrive at the linear conjugation problem with discontinuous coefficients:

$$\left.\begin{aligned} &[\Omega + \overline{\Omega}]^+ + [\Omega + \overline{\Omega}]^- = 2f_0(x) \quad \text{on } L, \\ &[\Omega + \overline{\Omega}]^+ + [\Omega + \overline{\Omega}]^- = -4\pi i\gamma h'(x) \quad \text{on } L_1' + L_2', \end{aligned}\right\} \tag{7}$$

where $f_0(x) = -\sigma_0 - \sigma_{y0}$ on $L_1' + L_2' + L_3'$, $\gamma = \dfrac{\mu}{\pi(1+\kappa)}$.

The solution of this problem is of the form:

$$\Omega(z) + \overline{\Omega}(z) = \frac{X(z)}{2\pi i}\int_L \frac{f_0(t)\,dt}{X^+(t)(t-z)} - 2\gamma X(z)\int_{L_1+L_2} \frac{h'(t)\,dt}{X^+(t)(t-z)}. \tag{8}$$

here $X(z) = \sqrt{(z-a)(z-b)(z-\alpha_1)(z-\beta_1)}$.

From the conditions at infinity, the constants α_1 and β_1 satisfy the conditions:

$$2\gamma \int_{L_1+L_2} \frac{h'(t)t^k dt}{X^+(t)} + \int_L \frac{f_0(t)t^k dt}{X^+(t)} = 0 \qquad k=0,1 \tag{9}$$

From boundary condition (3) we get [17] a problem of linear conjugation of boundary values:

$$\Omega^+ - \Omega^- = -2\pi i\gamma h'(x) \quad \text{on} \quad L_1',$$
$$\Omega^+ + \Omega^- = (1 - if(x))g(x) \quad \text{on} \quad L_2',$$
$$\Omega^+ + \Omega^- = f_1(x) \quad \text{on} \quad L,$$

here $f_1(x) = -\sigma_0 - (\sigma_{y_0} - i\tau_{xy_0})$ on L,

$$g(x) = -2\gamma X^+(x) \int_{L_1+L_2} \frac{h'(t)dt}{X^+(t)(t-x)} -$$
$$-2X^+(x)\int_L \frac{f_0(t)dt}{X^+(t)(t-x)}.$$

From the solution of this linear conjugation problem, we determine the function $\Omega(z)$:

$$\Omega(z) = \frac{X_1(z)}{2\pi i}\left\{-2\pi i\gamma \int_{L_1} \frac{h'(t)dt}{X_1^+(t)(t-z)} + \right.$$
$$\left. + \int_{L_2} \frac{(1-if(t))g(t)dt}{X_1^+(t)(t-z)} + \int_L \frac{f_1(t)dt}{X_1^+(t)(t-z)}\right\},$$
$$X_1(z) = \sqrt{(z-a)(z-b)(z-c_1)(z-d_1)}.$$

The constants c_1 and d_1 satisfy the following equations:

$$\gamma \int_{L_1} \frac{h'(t)t^k dt}{X_1^+(t)} - \frac{1}{2\pi i}\int_{L_2} \frac{(1-if(t))g(t)t^k dt}{X_1^+(t)} -$$
$$-\frac{1}{2\pi i}\int_L \frac{f_1(t)t^k dt}{X_1^+(t)} = 0 \qquad k = 0{,}1. \qquad (11)$$

For determining the stick zones we have the complete system of equations.

From the boundary condition (1) we find:

$$\Phi(z) = \Omega(z).$$

For contact stresses we get

$$\left.\begin{aligned} \sigma_y^+ = g(x) \quad \text{on } L_1' + L_2', \quad \tau_{xy}^+ = f(x)\sigma_y^+ \text{ on } L_2' \\ 2i\tau_{xy}^+ = \left[\overline{\Omega} - \Omega\right]^+ + \left[\overline{\Omega} - \Omega\right]^- \quad \text{on } L_1' \end{aligned}\right\}. \qquad (12)$$

We calculate the necessary integrals containing the functions $X^+(t)$ and $X_1^+(t)$ by the method suggested by N.I. Muskhelishvili [17, §110].

4. Analysis of the results of simulation

Analysis of partial closing of a variable width slot is reduced to parametric investigation according to formulas (12) at different laws of distribution of temperature fields and stresses in a plane, geometric parameters and also mechanical constant of the material. Normal and tangential stresses in the contact zone and also the sizes of stick and slippage zones are determined directly by means of calculations from the obtained formulas.

The graphs of distribution of contact stresses along the right contact zone for a slot whose width changes according to linear law are depicted in Figs. 2 and 3. As calculations the dimensionless coordinates $2x = b + \beta_1 + (b - \beta_1)x'$ were used. At calculations the following values of free parameters were used: $f = 0{,}2$; $\nu = 0{,}3$; $t_* = 4at/L_1^2 = 10$; $x_1/L_1 = 0{,}5$; $y_1/L_1 = 0{,}2$; $b_1/L_1 = 0{,}1$ where L_1 and b_1 are the coordinates of the center of the domain S_1; $(b-a)/R = 0{,}05$; $(b-a)/R = 0{,}02$ (the curves *1*, *2*). Here R is the typical linear size of the medium.

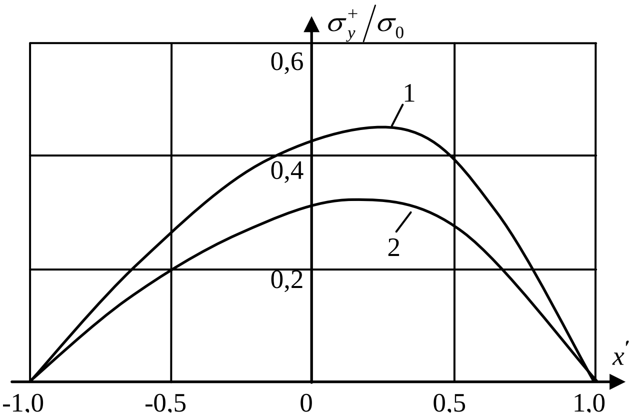

Fig. 2 Distribution of normal dimensionless contact stresses along the right contact zone

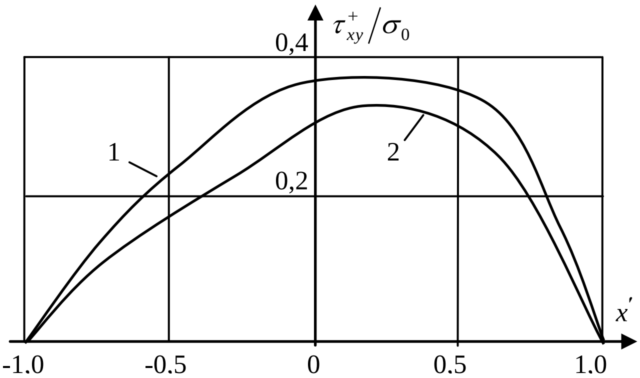

Fig. 3 Distribution of tangential dimensionless contact stresses along the right contact zone

Theoretical and experimental investigations show that the temperature field created in the course of some limited time braking and partial closing of the slot is insuperable barrier [1] on the way of its growth.

The next temperature field relief ($t \to \infty$) will gradually lower the value of compressive stresses and efficiency of partial closing of the slot. The stress intensity factors, having achieved the zero value when the shell is closed, gradually will grow to the value stipulated by the mechanical load.

Under the action of temperature field, the maximum tensile stress decreases and turns to the direction of heat source. This reduces [1, 18] to displacement of plane of fault observable in experiments. After temperature field relief this circumstance will promote increase of the external load necessary for the growth of the slot.

5. Conclusions

An effective scheme of analysis of a partially closed slot of variable thickness in the plane under the action of external tensile loads is suggested. Based on the obtained results we can consider that the temperature field created in the vicinity of the slot apex is a barrier on its propagation way. Account of the disturbed temperature field will amplify the braking efficiency of the induced temperature field of stresses.

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A CONTACT PROBLEM ON PARTIAL INTERACTION OF FACES OF A VARIABLE THICKNESS SLOT UNDER THE INFLUENCE OF TEMPERATURE FIELD

S u m m a r y

Temperature variations near the end of a variable thickness slot, comparable with elastic deformations are considered. A problem of equilibrium of a slot with partially contacting faces under the action of external tensile loads, induced temperature field and forces on contacting surfaces of the slot is reduced to the problem of linear conjunction of analytic functions. Herewith it is assumed that on some part of the contact there arises stick of faces, and on the other part there may be slippage. Normal and tangential stresses, values of the sizes of the end contact zone are found.

Keywords: variable thickness slot, temperature field, contact stick zone, slippage zone, contact stresses.

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