ISSN 1392–1207. MECHANIKA. 2015 Volume 21(4): 277–284

Study of non-steady process in "Single-acting piston pump – damless pitched-blade hydroturbine" hydrodynamic system

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crossref http://dx.doi.org/10.5755/j01.mech.21.3.10163

1. Introduction

Hydropower engineering constitutes a significant part of the overall energy system and relates to renewable energy sources. Currently a surge of public interest occurs to the field of engineering using renewable energy sources. The most efficient power plants based on renewable energy sources are the hydroturbines of the various types. This research is concerned with the study of the interaction of single-acting crank pump with the damless pitch-blade hydroturbine exciting its vibrations with the limited power. With this view we develop a new mathematical model describing the process of interaction of single-acting crank pump with the damless pitch-blade hydroturbine. Connexity of processes occurring in the single-acting crank pump and in the source of energy – in the hydroturbine leads to the qualitatively new effects in their dynamics which cannot be detected through examining the problem in formulation of the ideal excitation. A new model of the motion effect of the single-acting crank pump to the power source – damless hydroturbine is also studied. The special attention is paid to the study of effect of parameters of the single-acting crank pump to the hydroturbine motion modes.

In the papers [1] fundamentals of computation theory and theory of operation of crank piston pumps are illuminated. They show typical pump designs, considered design features and strength calculation of details of hydraulic block.

In the paper [2] torque and power of damless hydroturbine blades in the beginning of the dive and the possibility of their maximum values depending on the position of the blades oriented with respect to the axis of rotation of the hydroturbine were investigated.

In the paper [3], the study of the dynamics of single-acting crank pump driven by damless hydroturbine is given. The dynamic and mathematical models were composed and the motion equations of damless hydroturbine at a load of single-acting crank pump were obtained.

Paper [4] an original rotary heat pump suggested by the authors is described in the presented paper. All thermo-dynamic processes of working fluid (evaporation – compression – condensation) based on the principle of heat pipe are provided in one volume of mushroom form closed vessel. The transportation of external heat flows is combined in the unit as well. The heat necessary for evaporation is taken from external heat sources, and heat produced by working fluid at condensation is transferred by disk type ventilators. The three components of a new type are applied in the suggested heat pump: hydro-dynamic pump, disk ventilator, and regeneration heat exchanger.

Paper [5] Part IV is focused on improvements in the modeling and analysis of the phenomena bound to the pressure distribution around the gears in order to achieve results closer to the measured values. As a matter of fact, the simulation results have shown that a variable meshing stiffness has a notable contribution on the dynamic behavior of the pump but this is not as important as the pressure phenomena. As a consequence, the original model was modified with the aim at improving the calculation of pressure forces and torques. The improved pressure formulation includes several phenomena not considered in the previous one, such as the variable pressure evolution at input and output ports, as well as an accurate description of the trapped volume and its connections with high and low pressure chambers. The importance of these improvements are highlighted by comparison with experimental results, showing satisfactory matching

2. Statement of the problem

Design scheme of the crank piston pump, driven by a damless hydraulic hydroturbine, is shown in Fig. 1.

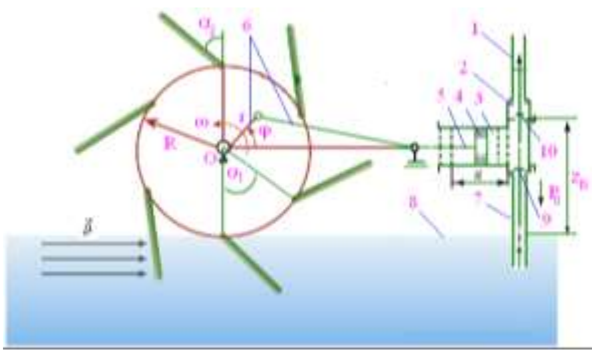

Fig. 1 A dynamic model of single-acting crank pump

In cylinder *3*, there is a piston *4*, abutting with its side surface to the cylinder wall. The piston of the hydraulic hydroturbine gets its moving by means of the crank mechanism *6* and stock *5*. The valve box *2*, in which there are suction valve *9* and discharge valve *10*, is attached (or integrally molded) to the cylinder. The valve box connects suction *7* and pressure *1* pipes. Pump draws fluid/liquid from a reservoir *8*. By z and z_1 geometric suction height and pump discharge are identified. Valves of the pump are self-acting and leakin one direction - upwards. Under the impact of the impeller *12*, with the flow of river waters, hydraulic hydroturbine impellers perform rotational motions, characterized by the angle of rotation φ of the crank to the right; the piston is moving inside a cylinder reciprocating, per stroke $S = 2r$, (r – crank radius). Paper [2] determines the characteristics of the drive torque of a hydraulic hydroturbine with six flat and radial blades.

Averaged moments of the driving forces (for such damless hydraulic hydroturbines) have the form:

$$M = a - b\dot{\varphi}, \tag{1}$$

where $a = \frac{\rho Q_0^2}{h}\left(3W - T_1 \cdot \mu_{11} + D_1\mu_{12}\right)$; $b = \frac{\rho Q_0^2}{h}\left(E_1\lambda_{11} + K_1\lambda_{21}\right)$; $W = \frac{1}{2}\frac{R}{l}\cos\alpha_2 + \frac{1}{4}$; $C = \frac{R^2}{l^2} + \frac{R}{l}\cos\alpha_2 + \frac{1}{3}$; $Q = \vartheta hl$;

$$E_1 = \frac{lC}{\vartheta}\left[\cos\alpha_2 + \cos(\alpha_1+\alpha_2) + \cos(2\alpha_1+\alpha_2)\right];\ K_1 = \frac{lC}{\vartheta}\left[\sin\alpha_2 + \sin(\alpha_1+\alpha_2) + \sin(2\alpha_1+\alpha_2)\right];$$

$$T_1 = \frac{1}{2}\frac{R}{l}\left[\sin\alpha_2 + \sin(2\alpha_1+\alpha_2) + \sin(4\alpha_1+\alpha_2)\right] + \frac{1}{4}\left[\sin 2\alpha_2 + \sin 2(\alpha_1+\alpha_2) + \sin 2(2\alpha_1+\alpha_2)\right];$$

$$D_1 = \frac{1}{2}\frac{R}{l}\left[\cos\alpha_2 + \cos(2\alpha_1+\alpha_2) + \cos(4\alpha_1+\alpha_2)\right] + \frac{1}{4}\left[\cos 2\alpha_2 + \cos 2(\alpha_1+\alpha_2) + \cos 2(2\alpha_1+\alpha_2)\right];$$

$$\mu_{11} = \frac{1}{\pi - 2\alpha_1 - \alpha_2}\int_0^{\pi-2\alpha_1-\alpha_2} \sin 2\varphi d\varphi;\ \mu_{21} = \frac{1}{\pi - 2\alpha_1 - \alpha_2}\int_0^{\pi-2\alpha_1-\alpha_2} \cos 2\varphi d\varphi;$$

$$\lambda_{11} = \frac{1}{\pi - 2\alpha_1 - \alpha_2}\int_0^{\pi-2\alpha_1-\alpha_2} \sin\varphi d\varphi;\ \lambda_{21} = \frac{1}{\pi - 2\alpha_1 - \alpha_2}\int_0^{\pi-2\alpha_1-\alpha_2} \cos\varphi d\varphi.$$

Q_0 is fluid flow at the working part of the blade; h is hydraulic hydroturbine blade width; R is radius of a base (disk) of the hydraulic hydroturbine supporting cylinder, for which the blades are attached; l is blade length, α_1 is the angle between the blades; α_2 is the angle of blades inclination, ϑ is the rate of fluid flow. Other parameters are shown in the scheme of hydraulic hydroturbine.

The positive directions of the crank rotation are rotations, taken anticlockwise.

Let us make the following prepositions, simplifications and assumptions: dead volume is missing; sucked liquids are incompressible; there is no friction in the mechanical components of the pump.

We formulate the objectives of the study for the dynamics of the crank piston pump, driven by a hydraulic hydroturbine with radial vanes, as follows: to find regularities characterizing the operation of a piston pump, driven by a damless hydraulic hydroturbine.

3. Kinematic characteristics of the crank pump piston motion

At the crank pumps, the law of the piston motion is caused by the kinematics of the crank mechanism.
If we neglect the influence of the finite length of the piston-rod, i.e.to consider the length of the piston-rod is many times greater than the radius of the crank, then the path traveled by the piston can be associated with an crank angle φ (angle of rotation of the hydraulic hydroturbine) with the following relation (Fig. 1).

$$x = r(1 - \cos\varphi) \tag{2}$$

Velocity and acceleration of the piston are determined by:

$$\left.\begin{aligned} u &= \frac{dx}{dt} = r\omega\sin\varphi; \\ j &= \frac{du}{dt} = r\omega^2\cos\varphi, \end{aligned}\right\} \tag{3}$$

where $\omega = \frac{d\varphi}{dt}$ – angular speed of rotation of the crank (hydro hydroturbine).

In a properly operating pump, fluid continuously follows the piston, not coming off from it. Pump flow at the infinitesimal movement of the piston will be

$$dq = Fdx = Fudt,$$

where F is the piston area. From this we can see that, since the piston area F remains constant, the pump flow varies according to the same law as the piston speed.

4. The equations of the system motion

The equations of the motion for a damless hydraulic hydroturbine with radial blades, when connecting a crank piston pump to them, is determined in [3] and have the following form:

– for a period of suction:

$$\ddot{\varphi} + \left[(\varepsilon_2 - \varepsilon_1)\sin 2\varphi + \mu_1 \sin^3\varphi\right]\dot{\varphi}^2 + \omega_{OB}^2 \sin\varphi = \frac{M}{J}; \tag{4}$$

– for a discharge period:

$$\ddot{\varphi} + \left[(\varepsilon_2 - \varepsilon_1)\sin 2\varphi + \mu_1 \sin^3\varphi\right]\dot{\varphi}^2 + \omega_{OH}^2 \sin\varphi = \frac{M}{J}, \tag{5}$$

where $\varepsilon_1 = \frac{M_s r^2}{8J}$; $\varepsilon_2 = \frac{m_b r(L_B + D)}{4J}$; $\varepsilon_3 = \frac{m_b r(L_H + D)}{4J}$; $\omega_{OB}^2 = \frac{m_b g(z_B + D + h_B)}{2J}$; $\omega_{OH}^2 = \frac{m_b g(z_H + D + h_H)}{2J}$;

$$J = \frac{1}{2}m_0R^2 + 6\rho_1 hdl\left(R^2 + \frac{1}{3}l^2 + Rl\cos\alpha_2\right);\quad \mu_B = \frac{m_b r^2}{4J};$$

$$\mu_H = \left(1 - \frac{F^2}{f_e^2}\right)\frac{m_b r^2}{4J}\,;\ m_b = 2\rho F r\,;$$

J is moment of inertia of hydraulic hydroturbine; m_0 is mass of a hydraulic hydroturbine bearing cylinder; m_b is mass of a fluid in the pump cylinder; M_s is mass of the piston-rod; D is diameter of the pump piston; d is blade thickness, ρ is density of the fluid; ρ_1 is density of the blade material; g free-fall acceleration; h_B, h_H is pressure expended to overcome the resistance of the suction and discharge valves; L_B, L_H is reduced length of the suction and discharge piping; z_B is the elevation of the piston in its lowermost position above the liquid level in the receiving reservoir; z_H is elevation of the drain-hole of the discharge pipe at the lower piston position, f_e is sectional area of drain-hole of the discharge pipe.

Let us suppose that $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are values of the same order of magnitude and neglect resistances in suction and discharge valves, as well as coefficients $\varepsilon_2 - \varepsilon_1, \varepsilon_3 - \varepsilon_1$.

Taking into account the characteristics of the hydraulic hydroturbine drive torque (1), we write the equation of the system motion in the form:

– for a period of suction:

$$\ddot{\varphi} + \frac{b}{J}\dot{\varphi} + \mu_B \sin^3\varphi \cdot \dot{\varphi}^2 + \omega_{OB}^2 \sin\varphi = \frac{a}{J}; \tag{6}$$

– for a discharge period:

$$\ddot{\varphi} + \frac{b}{J}\dot{\varphi} + \mu_H \sin^3\varphi \cdot \dot{\varphi}^2 + \omega_{OH}^2 \sin\varphi = \frac{a}{J}. \tag{7}$$

By substituting $d\tau = \omega_{OB}dt,\ d\tau = \omega_{OH}dt$, we transform the equations of motion for the system (6) to dimensionless form:

$$\frac{d^2\varphi}{d\tau^2} + \frac{b}{J\omega_{OB}}\frac{d\varphi}{d\tau} + \mu_B \sin^3\varphi\left(\frac{d\varphi}{d\tau}\right)^2 + \sin\varphi = \frac{a}{J\omega_{OB}^2}, \tag{8}$$

where $\varphi \subseteq [0,\pi] \cup [2\pi,3\pi] \cup [4\pi,5\pi] ... [2n\pi,(2n+1)\pi]$, $n = 0,1,2,3...$;

$$\frac{d^2\varphi}{d\tau^2} + \frac{b}{J\omega_{OH}}\frac{d\varphi}{d\tau} + \mu_H \sin^3\varphi\left(\frac{d\varphi}{d\tau}\right)^2 + \sin\varphi = \frac{a}{J\omega_{OH}^2}, \tag{9}$$

where $\varphi \subseteq [\pi,2\pi] \cup [3\pi,4\pi] \cup [5\pi,6\pi] ... [(2n-1)\pi,2n\pi]$, $n = 1,2,3...$.

The differential Eqs. (8) and (9) differ from each other by the coefficients, but have the same form; therefore it is sufficient to solve one of them. Eqs. (8) and (9) are essentially nonlinear, since the nonlinear term is included in the equation without a small parameter.

5. Solutions of the equations of a motion by the asymptotic methods

We now turn to the solution of the equations of a motion for the system (9) by the asymptotic method [6] Static equilibrium of the system (8), (9) is defined by the equation:

$$\sin\varphi = \frac{a}{J\omega_{OB}^2}\,;\ \sin\varphi = \frac{a}{J\omega_{OH}^2}. \tag{10}$$

Now we find the conditions for the excitation of the system rotational motion as:

$$\frac{a}{J\omega_{OB}^2} \ge 1\,;\ \frac{a}{J\omega_{OH}^2} \ge 1. \tag{11}$$

The limits of the excitation area of rotational motion of the hydroturbine are plotted by the Eq. (11) and shown in Fig. 2. It may be noted that in Fig. 2 the surfaces of excitation area of rotational motion of the hydroturbine are plotted on river flow rate parameters Q_0, the hydro turbine impeller radius and the hydraulic pump discharge head z_H.

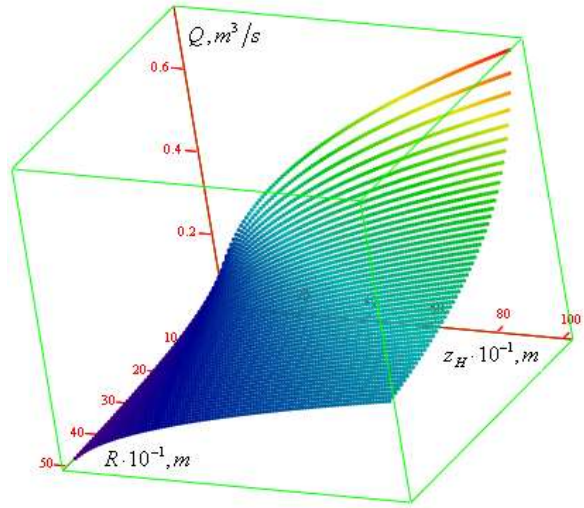

Fig. 2 The limits of the excitation area of rotational motion of the hydroturbine

Substituting:

$$\varphi = z;\ \dot{z} = \Omega + x \tag{12}$$

transform the Eq. (8) to the form:

$$\dot{x} = -\frac{b}{J\omega_{OB}}x - \mu_B \sin^3 z(\Omega + x)^2 - \sin z;\ \dot{z} = \Omega + x, \tag{13}$$

where $\Omega = a/b\omega_{OB} = \Omega_0/\omega_{OB}$.

Let us suppose that $\tau = \mu^2 s$, where $\mu = J\omega_{OB}/b$. Then the system of Eq. (13) is brought to the form:

$$\left.\begin{aligned} x' &= -\mu x - \mu^2\mu_B \sin^3 z(\Omega + x)^2 - \mu^2 \sin z; \\ z' &= \mu^2\frac{\Omega_0}{\omega_{OB}} + \mu^2 x, \end{aligned}\right\} \tag{14}$$

with notation $x' = \dfrac{dx}{ds}$.

For system (14) we agree to consider the following Cauchy problem:

$$\text{at } \; s = 0;\ z = 0;\ z' = 0. \tag{15}$$

Following the work [12], we bring:

$$\left.\begin{aligned} x &= \bar{x} + \mu u_1(\bar{x},\bar{z}) + \mu^2 u_2(\bar{x},\bar{z}) + \mu^3 u_3(\bar{x},\bar{z}) + \dots; \\ z &= \bar{z} + \mu \vartheta_1(\bar{x},\bar{z}) + \mu^2 \vartheta_2(\bar{x},\bar{z}) + \mu^3 \vartheta_3(\bar{x},\bar{z}) + \dots, \end{aligned}\right\} \tag{16}$$

where functions $\bar{x}, \bar{z}$ satisfy the following equations:

$$\left.\begin{aligned} \dot{\bar{x}} &= \mu A_1(\bar{x}) + \mu^2 A_2(\bar{x}) + \mu^3 A_3(\bar{x}) + \dots \ ; \\ \dot{\bar{z}} &= \mu^2 \frac{\Omega_0}{\omega_{0B}} + \mu B_1(\bar{x}) + \mu^2 B_2(\bar{x}) + \mu^3 B(\bar{x}) + \dots \ . \end{aligned}\right\} \tag{17}$$

We will consider that u_i, υ_i, $\frac{\partial u_i}{\partial \bar{x}}$ and $\frac{\partial \upsilon_i}{\partial \bar{x}}$ have limitations. Further, with a view to satisfy the conditions of (15), we will assume:

$$u_i(\bar{x},0) = \upsilon_i(\bar{x},0) = 0;\ \frac{\partial u_i(\bar{x},0)}{\partial \bar{x}} = \frac{\partial \upsilon_i(\bar{x},0)}{\partial \bar{x}} = 0. \tag{18}$$

Taking into account the Eq. (17), we expand the function $\sin z$, $\sin z$ in the small parameters μ in a Taylor series:

$$\sin z = \sin\bar{z} + \mu\vartheta_1 \cos\bar{z} + \frac{1}{2}\mu^2\left[-\vartheta_1^2 \sin\bar{z} + 2\vartheta_2 \cos\bar{z}\right] + \\ + \frac{1}{6}\mu^3\left[-\vartheta_1^3 \cos\bar{z} - 6\vartheta_1\vartheta_2 \sin\bar{z} + 6\vartheta_3 \cos\bar{z}\right];$$

$$\sin^3 z = \frac{1}{4}\left(3\sin\bar{z} - \sin 3\bar{z}\right) + \frac{3}{4}\mu\vartheta_1\left(\cos\bar{z} - \cos 3\bar{z}\right) - \\ - \frac{3}{4}\mu^2\left[\frac{1}{2}\vartheta_1^2\left(\sin\bar{z} - 3\sin 3\bar{z}\right) + \vartheta_2\left(\cos\bar{z} + \cos 3\bar{z}\right)\right].$$

Equations for determining the functions u_i, ϑ_i, A_i and B_i will be:

$$\left.\begin{aligned}
&1.\quad A_1 = -\bar{x}; \\
&2.\quad B_i = 0; \\
&3.\quad \bar{x}\frac{du_1}{d\bar{x}} - u_1 = A_2 + \sin\bar{z} + \frac{3}{4}\mu_B\left(\Omega + \bar{x}\right)^2 \sin\bar{z} + \frac{3}{4}\mu_B\left(\Omega + \bar{x}\right)^2 \sin 3\bar{z}; \\
&4.\quad B_2 - \bar{x}\frac{d\vartheta_1}{d\bar{x}} = \bar{x}; \\
&5.\quad -\bar{x}\frac{du_2}{d\bar{x}} + u_2 = -A_3 - \frac{du_1}{d\bar{x}}A_2 - \frac{\Omega_0}{\omega_{0B}}\frac{du_1}{d\bar{z}} - \frac{du_1}{d\bar{z}}B_2 - \vartheta_1\cos\bar{z} - \frac{3}{4}\mu_B\left(\Omega + \bar{x}\right)^2\vartheta_1\cos\bar{z} - \frac{3}{4}\mu_B\left(\Omega + \bar{x}\right)^2\vartheta_1\cos 3\bar{z} \\
&\qquad - \frac{3}{2}\mu_B\left(\Omega + \bar{x}\right)u_1\sin\bar{z} - \frac{3}{2}\mu_B\left(\Omega + \bar{x}\right)u_1\sin 3\bar{z}; \\
&6.\quad \bar{x}\frac{d\vartheta_2}{d\bar{x}} = B_3 + \frac{d\vartheta_1}{d\bar{x}}A_2 + \frac{\Omega_0}{\omega_{0B}}\frac{d\vartheta_1}{d\bar{z}} + \frac{d\vartheta_1}{d\bar{z}}B_2 - u_1.
\end{aligned}\right\} \tag{19}$$

Let us consider the third equation of the system (19). From the conditions (18) we will find $A_2 = 0$.

Substituting $\psi = u_1 + \sin\bar{z}$ we bring the equation (19.3) to the form:

$$\bar{x}\frac{d\psi}{d\bar{x}} - \psi = \frac{3}{4}\mu_B\left(\sin\bar{z} + \sin 3\bar{z}\right)\left(\Omega + \bar{x}\right)^2.$$

Presenting $\psi = \bar{x}\theta$, we find:

$$\frac{d\theta}{d\bar{x}} = \frac{3}{4}\mu_B\left(\sin\bar{z} + \sin 3\bar{z}\right)\frac{\left(\Omega + \bar{x}\right)^2}{\bar{x}^2}.$$

Consequently, at the initial conditions $\bar{z} = 0$; $u_1(\bar{x},0) = 0$ solution of the equation (19.3) will have the form:

$$u_1(\bar{x},\bar{z}) = -\sin\bar{z} - \frac{3}{4}\mu_B\left(\Omega^2 - 2\Omega\bar{x}\ln\bar{x} - \bar{x}^2\right)\times \\ \times\left(\sin\bar{z} + \sin 3\bar{z}\right). \tag{20}$$

Repeating the arguments for the fourth equation of the system (19), we get:

$$B_2 = \bar{x}\ ; \tag{21}$$

and, hence

$$\vartheta_1 = 0\,. \tag{22}$$

Using the results (20), (21) and (22), we shall rewrite the fifth equation of system (19):

$$\bar{x}\frac{du_2}{d\bar{x}} - u_2 = A_3 + \left(\frac{\Omega_0}{\omega_{0B}} + \bar{x}\right)\frac{du_1}{d\bar{z}} + \frac{3}{4}\mu_B\left(\Omega + \bar{x}\right)u_1\times \\ \times\left(\sin\bar{z} + 2\sin 3\bar{z}\right),$$

where

$$A_3 = \left(\frac{\Omega_0}{\omega_{0B}} + \bar{x}\right)\left[1 + 3\mu_B\left(\Omega^2 - 2\Omega\bar{x}\ln\bar{x} - \bar{x}^2\right)\right] \tag{23}$$

and subsequently:

$$\bar{x}\frac{du_2}{d\bar{x}}-u_2=\left(\frac{\Omega_0}{\omega_{0B}}+\bar{x}\right)(1-\cos\bar{z})+3\mu_B\left(\frac{\Omega_0}{\omega_{0B}}+\bar{x}\right)\left(\Omega^2-2\Omega\bar{x}\ln\bar{x}-\bar{x}^2\right)\left(1-\frac{1}{4}\cos\bar{z}-\frac{3}{4}\cos 3\bar{z}\right)-$$
$$-\frac{3}{4}\mu_B\left(\Omega+\bar{x}\right)\left(\sin^2\bar{z}+\sin\bar{z}\sin 3\bar{z}\right)-\frac{9}{8}\mu_B^2\left(\Omega+\bar{x}\right)\left(\Omega^2-2\Omega\bar{x}\ln\bar{x}-\bar{x}^2\right)\left(\sin\bar{z}+\sin 3\bar{z}\right)^2.$$

Substituting $u_2=\bar{x}\psi_1$, we find:

$$\frac{d\psi_1}{d\bar{x}}=\left(\frac{\Omega_0}{\omega_{0B}}\frac{1}{\bar{x}^2}+\frac{1}{\bar{x}}\right)(1-\cos\bar{z})+3\mu_B\left(\frac{\Omega_0}{\omega_{0B}}\frac{1}{\bar{x}^2}+\frac{1}{\bar{x}}\right)\cdot\left(\Omega^2-2\Omega\bar{x}\ln\bar{x}-\bar{x}^2\right)\left(1-\frac{1}{4}\cos\bar{z}-\frac{3}{4}\cos 3\bar{z}\right)-$$
$$-\frac{3}{4}\mu_B\left(\frac{\Omega}{\bar{x}^2}+\frac{1}{\bar{x}}\right)\left(\sin^2\bar{z}+\sin\bar{z}\sin 3\bar{z}\right)-\frac{9}{8}\mu_B^2\left(\frac{\Omega}{\bar{x}^2}+\frac{1}{\bar{x}}\right)\left(\Omega^2-2\Omega\bar{x}\ln\bar{x}-\bar{x}^2\right)\left(\sin\bar{z}+\sin 3\bar{z}\right)^2.$$

Then, for the function u_2 we get:

$$u_2(\bar{x},\bar{z})=-\left(\Omega-\bar{x}\ln\bar{x}\right)\left[\left(1-\cos\bar{z}-\frac{3}{4}\mu_B\sin\bar{z}\right)\left(\sin\bar{z}+\sin 3\bar{z}\right)\right]-$$
$$-3\mu_B\left[\Omega^3-\Omega^2\bar{x}\ln\bar{x}+\Omega^2\bar{x}\ln^2\bar{x}+2\Omega\bar{x}^2\left(\ln\bar{x}-1\right)+\frac{1}{2}\bar{x}^3\right]\times\left[\left(1-\frac{1}{4}\cos\bar{z}-\frac{3}{4}\cos 3\bar{z}\right)-\frac{3}{4}\mu_B\left(\sin\bar{z}+\sin 3\bar{z}\right)^2\right]. \quad (24)$$

Rewrite, finally, the last equation of the system (19), taking into account the results obtained:

$$\bar{x}\frac{\partial\vartheta_2}{\partial\bar{x}}=B_3-u_1.$$

Where from

$$B_3=0 \quad (25)$$

and thus, we obtain:

$$\vartheta_2=\ln\bar{x}\sin\bar{z}-\frac{3}{4}\mu_B\left[\Omega^2\ln\bar{x}-2\Omega\bar{x}\left(\ln\bar{x}-1\right)-\frac{1}{2}\bar{x}^2\right]\times$$
$$\times\left(\sin\bar{z}+\sin 3\bar{z}\right). \quad (26)$$

Continuing this procedure, it is easy to find A_1 etc. Restricting in our calculations, to the terms of order $O\left(\mu^2\right)$, we shall bring the equation (16) and (17) to the form:

$$x=\bar{x}-\mu\sin\bar{z}-\frac{3}{4}\mu\mu_B\left(\Omega^2-2\Omega\bar{x}\ln\bar{x}-\bar{x}^2\right)\left(\sin\bar{z}+\sin 3\bar{z}\right)-\mu^2\left(\Omega-\bar{x}\ln\bar{x}\right)\left[1-\cos\bar{z}-\frac{3}{4}\mu_B\sin\bar{z}\left(\sin\bar{z}+\sin 3\bar{z}\right)\right]-$$
$$-3\mu^2\mu_B\left[\Omega^3-\Omega^2\bar{x}\ln\bar{x}+\Omega^2\bar{x}\ln^2\bar{x}+2\Omega\bar{x}^2\ln\bar{x}-2\Omega\bar{x}^2+\frac{1}{2}\bar{x}^3\right]\left[1-\frac{1}{4}\cos\bar{z}-\frac{3}{4}\cos 3\bar{z}-\frac{3}{8}\mu_B\left(\sin\bar{z}+\sin 3\bar{z}\right)^2\right];$$

$$z=\bar{z}+\mu^2\left\{\ln\bar{x}\sin\bar{z}-\frac{3}{4}\mu_B\left[\Omega^2\ln\bar{x}-2\Omega\bar{x}\left(\ln\bar{x}-1\right)-\frac{1}{2}\bar{x}^2\right]\left(\sin\bar{z}+\sin 3\bar{z}\right)\right\};$$

$$\left.\begin{aligned}\bar{x}'&=-\mu\bar{x};\\ \bar{z}'&=\mu^2\left(\frac{\Omega_0}{\omega_{OB}}+\bar{x}\right).\end{aligned}\right\} \quad (27)$$

Where the following relationship shall be hold true:

$$\begin{cases}\ln\bar{x}, & if\quad \bar{x}>0\\ \ln(-\bar{x}), & if\quad \bar{x}<0\end{cases}$$

Integrating the third and fourth of the systems Eqs. (27) and considering the initial conditions (15), we-get:

$$\left.\begin{aligned}\bar{x}&=-\frac{\Omega_0}{\omega_{OB}}e^{-\mu s};\\ \bar{z}&=\mu^2\frac{\Omega_0}{\omega_{OB}}s-\mu\frac{\Omega_o}{\omega_{OB}}\left(1-e^{-\mu s}\right).\end{aligned}\right\} \quad (28)$$

Therefore, the solution of differential Eq. (8) shall be as follows:

$$\bar{x}_B=-\frac{\Omega_0}{\omega_{OB}}e^{-\mu S};$$

$$\bar{z}_B=\mu^2\frac{\Omega_0}{\omega_{OB}}s-\mu\frac{\Omega_o}{\omega_{OB}}\left(1-e^{-\mu S}\right).$$

$$x_B = \overline{x}_B - \mu \sin \overline{z}_B - \frac{3}{4}\mu\mu_B\left(\Omega^2 - 2\Omega\overline{x}_B \ln\overline{x}_B - \overline{x}_B{}^2\right)\left(\sin \overline{z}_B + \sin 3\overline{z}_B\right) - \mu^2\left(\Omega - \overline{x}_B \ln\overline{x}_B\right)\times$$
$$\times\left[1 - \cos\overline{z}_B - \frac{3}{4}\mu_B \sin \overline{z}_B\left(\sin \overline{z}_B + \sin 3\overline{z}_B\right)\right] - 3\mu^2\mu_B \times\left[\Omega^3 - \Omega^2\overline{x}_B \ln\overline{x}_B + \Omega^2\overline{x}_B \ln^2 \overline{x}_B + 2\Omega\overline{x}_B{}^2 \ln\overline{x}_B - 2\Omega\overline{x}_B{}^2 + \frac{1}{2}\overline{x}_B{}^3\right]\times$$
$$\times\left[1 - \frac{1}{4}\cos \overline{z}_B - \frac{3}{4}\cos 3\overline{z}_B - \frac{3}{8}\mu_B\left(\sin \overline{z}_B + \sin 3\overline{z}_B\right)^2\right];$$

$$\left.\begin{aligned} z_B &= \overline{z}_B + \mu^2\left\{\ln \overline{x} \sin \overline{z}_B - \frac{3}{4}\mu_B\left[\Omega^2 \ln \overline{x} - 2\Omega\overline{x}\left(\ln \overline{x} - 1\right) - \frac{1}{2}\overline{x}^2\right]\times\left(\sin \overline{z}_B + \sin 3\overline{z}_B\right)\right\}; \\ z_B' &= \mu^2\left(\frac{\Omega_0}{\omega_{OB}} + x_B\right). \end{aligned}\right\} \quad (29)$$

Repeating the line of reasoning for solution of the differential Eq. (8), we will develop the solution of Eq. (9) at the following initial conditions:

$$s = s_\pi,\ z = \pi,\ z' = z_B'\left(s_\pi\right),\ \overline{x} = \overline{x}_B\left(s_\pi\right),$$

$$u_i\left(\overline{x},\pi\right) = \upsilon_i\left(\overline{x},\pi\right) = 0;\ \frac{\partial u_i\left(\overline{x},\pi\right)}{\partial\overline{x}} = \frac{\partial\upsilon_i\left(\overline{x},\pi\right)}{\partial\overline{x}} = 0.$$

The solutions of the differential Eq. (9) may be represented as:

$$\overline{x}_H = \overline{x}_B\left(s_\pi\right)e^{-\mu\left(S - S_\pi\right)},$$
$$\overline{z}_H = \mu^2\frac{\Omega_0}{\omega_{OH}}\left(s - s_\pi\right) - \mu\overline{x}_B\left(s_\pi\right)\left(1 - e^{-\mu\left(S - S_\pi\right)}\right) + \pi.$$

$$x_H = \overline{x}_H - \mu \sin \overline{z}_H - \frac{3}{4}\mu\mu_H\left(\Omega^2 - 2\Omega\overline{x}_H \ln\overline{x}_H - \overline{x}_H{}^2\right)\left(\sin \overline{z}_H + \sin 3\overline{z}_H\right) - \mu^2\left(\Omega - \overline{x}_H \ln\overline{x}_H\right)\times$$
$$\times\left[1 - \cos \overline{z}_H - \frac{3}{4}\mu_H \sin \overline{z}_H\left(\sin \overline{z}_H + \sin 3\overline{z}_H\right)\right] - 3\mu^2\mu_H\left[\Omega^3 - \Omega^2\overline{x}_H \ln\overline{x}_H + \Omega^2\overline{x}_H \ln^2 \overline{x}_H + 2\Omega\overline{x}_H{}^2 \ln\overline{x}_H - 2\Omega\overline{x}_H{}^2 + \frac{1}{2}\overline{x}_H{}^3\right]\times$$
$$\times\left[1 - \frac{1}{4}\cos \overline{z}_H - \frac{3}{4}\cos 3\overline{z}_H - \frac{3}{8}\mu_H\left(\sin \overline{z}_H + \sin 3\overline{z}_H\right)^2\right];$$

$$\left.\begin{aligned} z_H &= \overline{z}_H + \mu^2\left\{\ln \overline{x} \sin \overline{z}_H - \frac{3}{4}\mu_B\left[\Omega^2 \ln \overline{x} - 2\Omega\overline{x}\left(\ln \overline{x} - 1\right) - \frac{1}{2}\overline{x}^2\right]\left(\sin \overline{z}_H + \sin 3\overline{z}_H\right)\right\}; \\ z_H' &= \mu^2\left(\frac{\Omega_0}{\omega_{OH}} + x_H\right). \end{aligned}\right\} \quad (30)$$

6. Results and analysis

Coming back to variables $t = \frac{\mu^2}{\omega_{OB}}s$, $t = \frac{\mu^2}{\omega_{OH}}s$, $\varphi = z_B$, $\varphi = z_H$ and based on formulas (29) and (30) the graphs of dependence of the angular speed against the rotation angle of the damless hydroturbine were plotted, Fig. 3. Calculations were made in the following values of parameters:

$R = 13\cdot10$ m; $l = 1$ m; $h = 1$ m; $d = 2\cdot10$; $\alpha = \pi/3$;
$\alpha = \pi/6$; $\vartheta = 1$m/s; $\rho = 2.7\cdot10$ kg/m ; $m = 28$ kg;
$F = 0.031$ m ; $f = F/\sqrt{2}$.

In Fig. 3, a line 1 describes the dependence of the angular speed against the rotation angle of the hydroturbine in the inoperative pump (no-load operation of the hydroturbine), and lines 2 and 3 characterize the change in the angular speed of the hydroturbine when the modes of suction $\left(0 \le \varphi \le \pi\right)$, $\left(2\pi \le \varphi \le 3\pi\right)$, $\left(4\pi \le \varphi \le 5\pi\right)$ and pumping $\left(\pi \le \varphi \le 2\pi\right)$, $\left(3\pi \le \varphi \le 4\pi\right)$ of the piston pump, respectively.

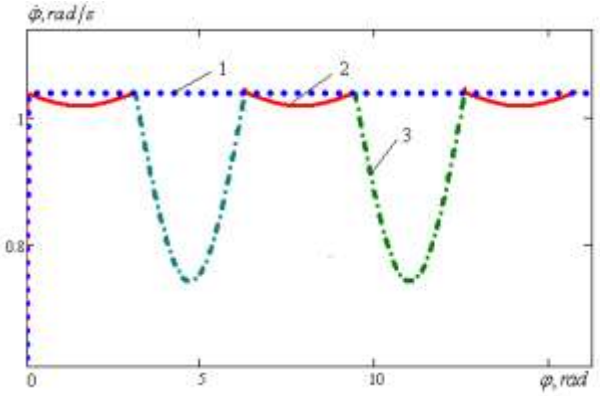

Fig. 3 Graph of dependence of the angular speed on the rotation angle of the damless hydroturbine in the different operating modes of the piston pump

Dependence of the angular speed against the rotation angle of the hydroturbine at the different values. z_H is levation of pressure pipe and $S = 2r$, stroke length of the piston pump are shown in the Figs. 4 and 5, respectively. In the Fig. 4 the graphs of solution obtained by the asymptotic method (lines *1*, *2*, *3*, *4*) and through numerical integration using Runge-Kutta scheme (line *5*). As it can be concluded from examining the line *4* in the Fig. 4, the hydroturbine ends rotation when a critical point $z_H = 20$ m of elevation of the pressure pipe is achieved. Comparison of curves *2* and *5* demonstrates the good accuracy of the analytical calculation.

The graphs in Figs. 4-9 show that the hydroturbine rotates unevenly. The values of the hydroturbine angular speed at the no-load operation correspond to its maximum values. The minimum value of the angular speed depends on the parameters of the hydroturbine and piston pump and is determined by the μ parameter.

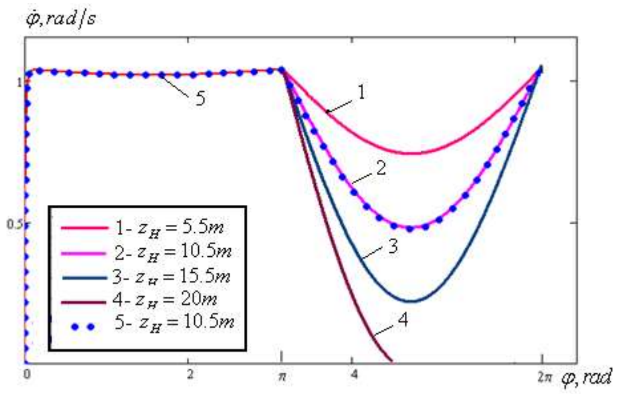

Fig. 4 Graph of dependence of the angular speed on the rotation angle of the damless hydroturbine in the different values of elevation of pressure pipe

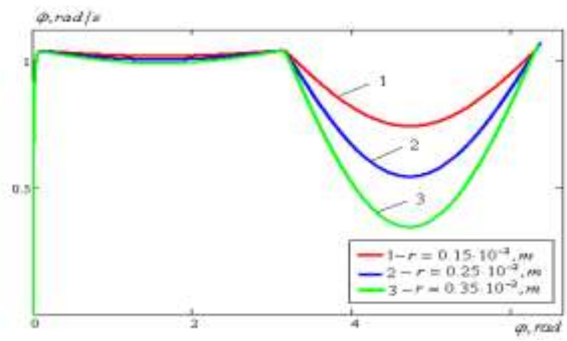

Fig. 5 Graph of dependence of the angular speed on the rotation angle of the damless hydroturbine in the different values of crank radius

The irregularity of rotation is estimated by the coefficient of irregularity:

$$\delta = \mu / \Omega_0 = J \omega_{OH} / a \, . \tag{31}$$

The δ coefficient characterizes the fluctuations in the angular speed with respect to its maximum value. The smaller the δ coefficient, the relatively smaller amplitude of vibrations, and the calmer the hydroturbine rotates.

Thus, the coefficient of irregularity depends on the parameters of the piston pump and the hydroturbine. In Fig. 6, we plot the dependence of the coefficient of irregularity δ against the *R* radius of the hydroturbine impeller, and z_H – elevation of the drain-hole of the discharge pipe. Based on the Fig. 6 we can draw the following conclusions: selecting the parameters of hydroturbine and piston pump the value of the δ coefficient of irregularity can be done acceptably small.

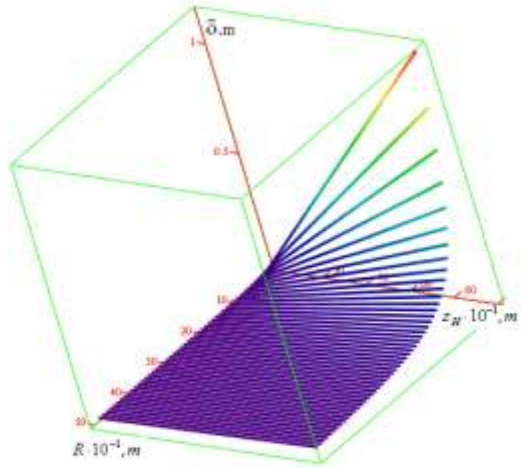

Fig. 6 Graph of dependence of the coefficient of irregularity of angular speed against hydroturbine impeller radius and elevation of pressure pipe

The maximum value of the angular speed depends only on the hydroturbine parameters. The Figs. 7-9 shows that the maximum value of the angular speed decreases with increasing of the hydroturbine impeller radius and it increases with increase in the value of river flow rate and the hydroturbine blade angle.

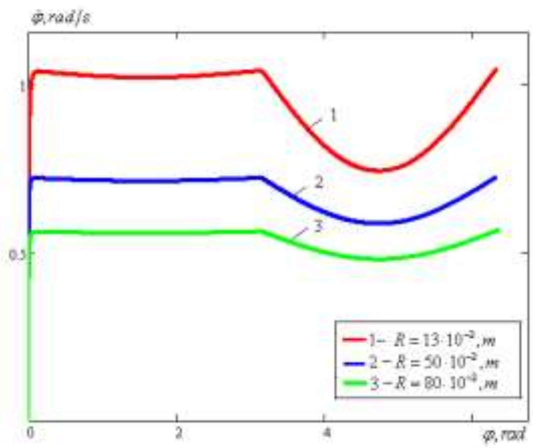

Fig. 7 Graph of dependence of the angular speed on the rotation angle of the damless hydroturbine in the different values of hydroturbine impeller radius

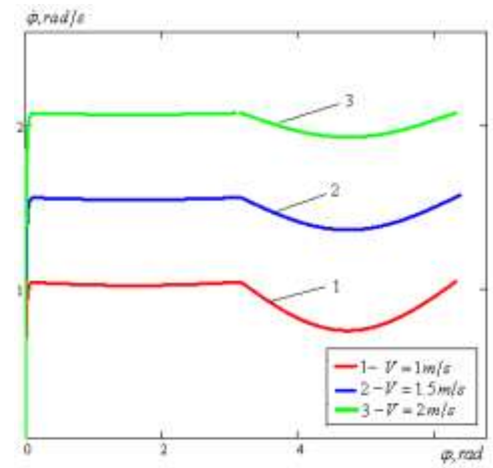

Fig. 8 Graph of dependence of the angular speed on the rotation angle of the damless hydroturbine in the different values of river flow rate

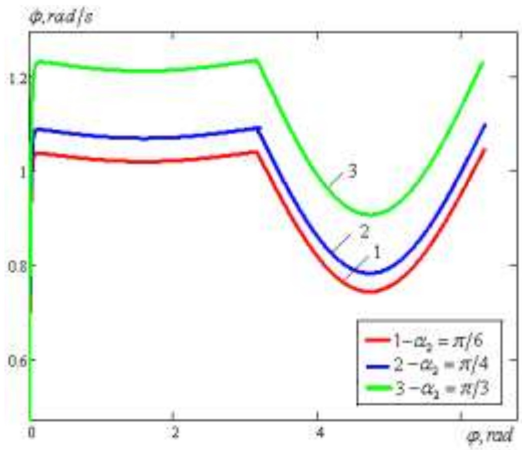

Fig. 9 Graph of dependence of the angular speed against the rotation angle of the damless hydroturbine in the different values of hydroturbine blade angle

5. Conclusions

A new model of the motion effect of the single-acting crank pump to the power source – damless hydro-turbine is developed and studied. The special attention is paid to the study of effect of parameters of the single-acting crank pump to the hydroturbine motion modes. It was established that the system motion equation is essentially nonlinear and has a large nonlinearity parameter in the first-order derivative, the hydroturbine rotates unevenly and the coefficient of irregularity of rotation of the hydro-turbine depends on the parameters of the crank-piston pump and hydroturbine.

The values of the hydroturbine angular speed at the no-load operation correspond to its maximum values. The minimum values of the angular speed depend on the parameters of the hydroturbine and piston pump, and the maximum values of the angular speed depend only on the hydroturbine parameters.

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STUDY OF NON-STEADY PROCESS IN “SINGLE-ACTING PISTON PUMP – DAMLESS PITCHED-BLADE HYDROTURBINE” HYDRODYNAMIC SYSTEM

S u m m a r y

This research is concerned with the study of the interaction of single-acting crank pump with the damless pitch-blade hydroturbine exciting its vibrations with the limited power. With this view we develop a new mathematical model describing the process of interaction of single-acting crank pump with the damless pitch-blade hydro-turbine. Connexity of processes occurring in the single-acting crank pump and in the source of energy – in the hydroturbine leads to the qualitatively new effects in their dynamics which cannot be detected through examining the problem in formulation of the ideal excitation.

A new model of the motion effect of the single-acting crank pump to the power source – damless hydro-turbine is developed and studied. The special attention is paid to the study of effect of parameters of the single-acting crank pump to the hydroturbine motion modes. It was established that the system motion equation is essentially nonlinear and has a large nonlinearity parameter in the first-order derivative, the hydroturbine rotates unevenly and the coefficient of irregularity of rotation of the hydro-turbine depends on the parameters of the crank-piston pump and hydroturbine.

Keywords: single-acting crank pump, damless hydro turbine, watercourse speed.

Received March 04, 2015
Accepted May 07, 2015