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Modelling and control aspects of specific mobile robot

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1. Introduction

Usually, mechanical solution of mobile robot namely "two-wheel differential drive mobile robot" has minimum three wheels. The two "drive wheels" have a common horizontal axis, fixed on the robot body. One or more free wheels (or "castor" wheels) assure the robot equilibrium [1]. Therefore, while three wheels introduce isostatic equilibrium for the robot body, more that three wheels introduce hyper static equilibrium, which ensures a better stability on complex trajectories including curved segments [2]. Each castor wheel is independently mounted on a vertical non driven axis of the body and it is automatically and free aligned on the route as a result of the forces developed by the two "drive wheels". The entire control of the mobile robot on trajectories is assured controlling angular velocities of the two drive wheels [3]. There are three fundamental cases.

- If the angular velocities are identical, as values and relative senses, the robot makes a spin motion; the spin motion produces the robot body rotation around a vertical axis passing through the geometrical symmetry point (or the centre of gravity). There is a particularity of this mechanical configuration, because only the two-wheel differential drive mobile robot can do this type of motion.
- If the angular velocities are identical as values but opposite as senses, the robot makes a linear motion; the direction of the linear motion, forward or backwards, depends on the senses of the driven wheels angular velocities.
- If the angular velocities are different as value, the robot makes a curved motion. Of course, the characteristics of the curve motion, i.e. the curvature coefficient k of the curve-segment trajectory depends on the differences between the values and senses of the two drive wheels.

This mechanical solution of mobile robot namely "two-wheel differential drive mobile robot" is extensively used in practice now. The explanation is that it assures a good balance between large capabilities in locomotion (or tracking possibilities) and mechanical complexity (or construction costs) [4]. So, as we see above, the two-wheel differential drive mobile robot is the single structure which can make spin motion.

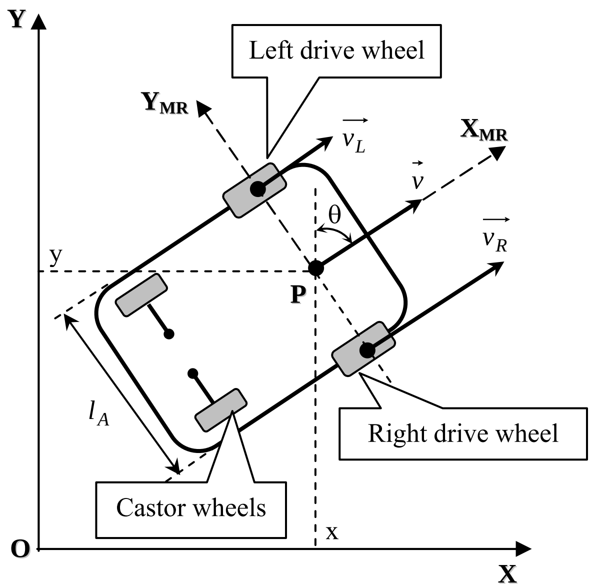

Fig. 1 Coordinate systems and notations used for the two-wheel differential drive mobile robot

2. Models for the two-wheeled differential drive mobile robot

To characterize the current localization of the mobile robot in its operational space of evolution, we must define at first its position and orientation.

The position of the mobile robot on a plane surface is given by the vector (x,y), which contains Cartesian coordinates of its characteristic point P (see Fig. 1). This characteristic point P is placed in the middle of the common axis of the driven wheels.

As we can see in Fig. 1, the orientation (or direction) of the mobile robot is given by the angle θ between instant linear velocity of the mobile robot $\vec{v}$ (or the X_{RM} axis) and local vertical axis.

The instant linear velocity of the mobile robot is noted by $\vec{v}$ and is attached and defined relatively to the characteristic point *P*.

As Eq. (1) denotes, this velocity is the result of linear velocities of the left driven wheel $\vec{v}_L$ and the right driven wheel $\vec{v}_R$ respectively. These two drive velocities $\vec{v}_L$ and $\vec{v}_R$ are permanently two parallel vectors and, in the same time, they are permanently perpendicular on the common mechanical axis of these two driven wheels

$$v = \frac{v_L + v_R}{2} \tag{1}$$

Eqs. (2) and (3) give the two Cartesian components of linear velocity

$$\left.\begin{aligned} v_x &= \dot{x} = v \sin\theta \\ v_y &= \dot{y} = v \cos\theta \end{aligned}\right\} \tag{2}$$

The position, orientation and linear velocities of the two driven wheels define the robot state as a five elements vector

$$(x, y, \theta, v_L, v_R)^T \tag{3}$$

The input vector contains two accelerations of the left $\vec{a}_S$ and the right $\vec{a}_D$ driven wheels respectively. Inserting Eq. (1) into Eq. (2), the next Eq. (4) are obtained. They give finally the first two state equations (for linear velocity components of the mobile robot)

$$\left.\begin{aligned} \dot{x} &= \frac{v_L + v_R}{2} \sin\theta \\ \dot{y} &= \frac{v_L + v_R}{2} \cos\theta \end{aligned}\right\} \tag{4}$$

If we note by (x_L, y_L, x_R, y_R) Cartesian positions of the driven wheels in the global references attached to the operational space, we can write the next two equations

$$\left.\begin{aligned} x_L - x_R &= -l_A \cos\theta \\ y_L - y_R &= l_A \sin\theta \end{aligned}\right\} \tag{5}$$

and respectively the associate equations

$$\left.\begin{aligned} \dot{x}_L - \dot{x}_R &= l_A \dot{\theta} \sin\theta \\ \dot{y}_L - \dot{y}_R &= l_A \dot{\theta} \cos\theta \end{aligned}\right\} \tag{6}$$

Whereas the vectors for linear velocity of the wheels $\overrightarrow{v_S}$ and $\overrightarrow{v_R}$ are orthogonal on the common axis of the driven wheels (see Fig. 1), we can write the third state Eq. (7), representing angular velocity of the robot

$$\dot{\theta} = \frac{v_L - v_R}{l_A} \tag{7}$$

The last two state equations denoting linear accelerations of the two drive wheels are evident

$$\left.\begin{aligned} \dot{v}_L &= a_L \\ \dot{v}_R &= a_R \end{aligned}\right\} \tag{8}$$

The curvature coefficient *k* associated for a specific trajectory-segment is defined as inverse ratio of the radius of that trajectory–segment. The equation for the curvature can be obtained because radius of the trajectory–segment can be written as the ratio of linear and angular velocities of the robot body.

Therefore, dividing Eq. (7) by Eq. (1) we obtain the equation for the curvature coefficient *k* of a segment-trajectory as

$$k = \frac{1}{\rho} = \frac{\dot{\theta}}{v} = \frac{v_L - v_R}{v_L + v_R} \cdot \frac{2}{l_A} \tag{9}$$

As Eqs. (4) is nonlinear, we must introduce some assumptions to obtain a linear model for the mobile robot. There are some different solutions. A possible method is to introduce the hypothesis that the two instant drive wheel accelerations, a_L and respectively a_R, are equals in module. If their sense is the same, the mobile robot executes a linear motion and if it is opposite, the mobile robot executes a special curve namely "clotoide" [5].

The most common actuator used to energize locomotion system of the mobile robots is direct current (DC) motor. An associated encoder, as common speed and position sensor, is currently attached. In the same normal hypothesis (electrical constants have the lower values mechanical constants), the DC servomotor is a first order system with the transfer function

$$H_S(s) = \frac{\omega(s)}{U(s)} = \frac{K}{1 + Ts} \tag{10}$$

where ω represents angular velocity of the DC servomotor and *U* is the applied voltage.

So, considering two DC servomotors, as right (*R*) and left (*L*) actuators for the two driven wheels of the mobile robot, and the associated simplest transfer function

$$\left.\begin{aligned} H_L(s) &= \frac{v_L}{v_{Lc}} = \frac{K_L}{1 + T_L s} \\ H_R(s) &= \frac{v_R}{v_{Rc}} = \frac{K_R}{1 + T_R s} \end{aligned}\right\} \tag{11}$$

we can obtain, finally, a first kinematical model for the two-wheel differential drive mobile robot, which is depicted in Fig. 2.

If our target is to simplify the mathematical model, we can introduce the evident assumption that the two DC servomotors are practically identical in their behavior.

So, in addition, some equalities between the parameters of their transfer Eq. (11) can be written as

$$\left.\begin{aligned} K_L &= K_R = K_a \\ T_L &= T_R = T \end{aligned}\right\} \tag{12}$$

Using the Eqs. (1) and (7), we can obtain (after same bloc-diagram reductions and associate transformations) a new bloc-diagram [6]. Fig. 3 shows this new bloc-diagram.

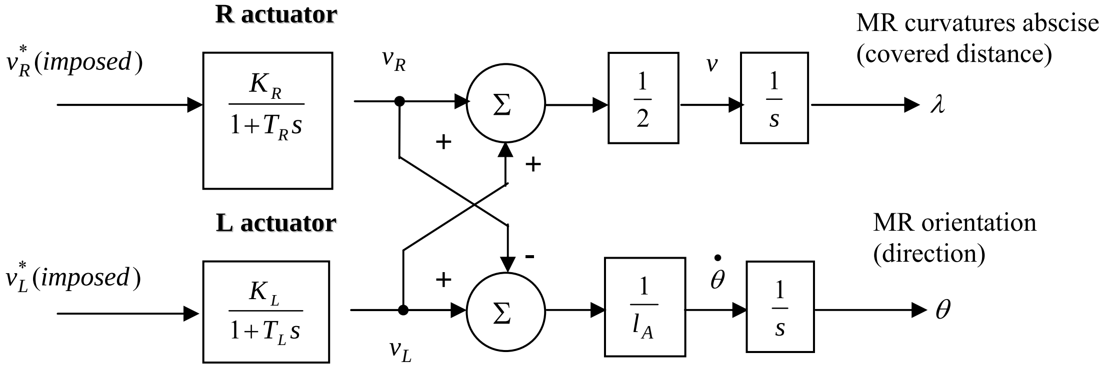

Fig. 2 A primary model for the two-wheel differential drive mobile robot, considering two DC servo-motors as actuators in the locomotion system

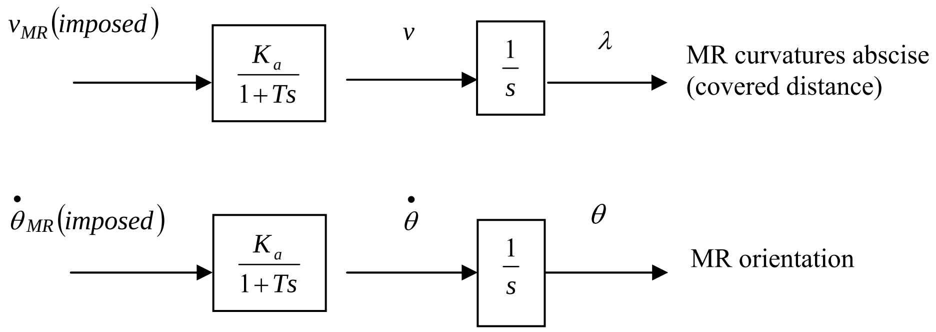

Fig. 3 The simplified model for the two-wheel differential drive mobile robot, considering the same behavior for the two actuators of the locomotion system

This new control diagram is still not satisfactory. The basic explanation is that substantial tracking errors can occur between an imposed (or desired) trajectory for the mobile robot and the real trajectory developed by them. In fact, regarding the posture of the mobile robot and its trajectory, we can divide these errors into three categories:

- tangential error;
- lateral (or normal) error;
- orientation error.

If these errors exceed acceptable and predefined limits, impacts between the mobile robot and different obstacles placed in the operational space can occur and, as result, the entire functionality of the mobile robot is affected.

This is the reason to introduce two closed loops control. The first one is for the curvatures abscise λ (or covered distance by the robot) and the second is for the robot orientation.

Each of them uses a classical proportional integral derivative (PID) controller, mathematically depicted by the next Eq. (13) and respectively (14)

$$\dot{\theta}_c = K_{P\theta}\Delta\theta + K_{D\theta}\Delta\dot{\theta} + K_{I\theta}\int \Delta\theta dt \tag{13}$$

$$\dot{\bar{x}}_c = K_P\Delta\bar{x} + K_D\Delta\dot{\bar{x}} + K_I\int \Delta\bar{x}dt \tag{14}$$

where $\dot{\theta}_c$ is represents the imposed angular velocity, $\Delta\theta$ is represents orientation (or direction) error, $\dot{\bar{x}}_c$ is represents imposed linear velocity and $\Delta\dot{\bar{x}}$ is represents position error of the mobile robot.

3. Control solution for the mobile robot

Figs. 4 and 5 present the final solutions proposed to control the two-wheel differential drive mobile robot. Fig. 4 presents the closed loop position control of the two-wheel differential drive mobile robot. Fig. 5 includes the proposed closed loop control for this type mobile robot position.

4. Control stability

To evaluate the stability of the proposed control we consider a simplified situation. The diagram is depicted in Fig. 6 and contains only a single channel $\left(x^d \to x\right)$ (see Fig. 5) while the influence of the second channel $\left(y^d \to y\right)$ is integrated into the perturbation $\Lambda(s)$.

The open lop transfer function is presented in the Eq. (15)

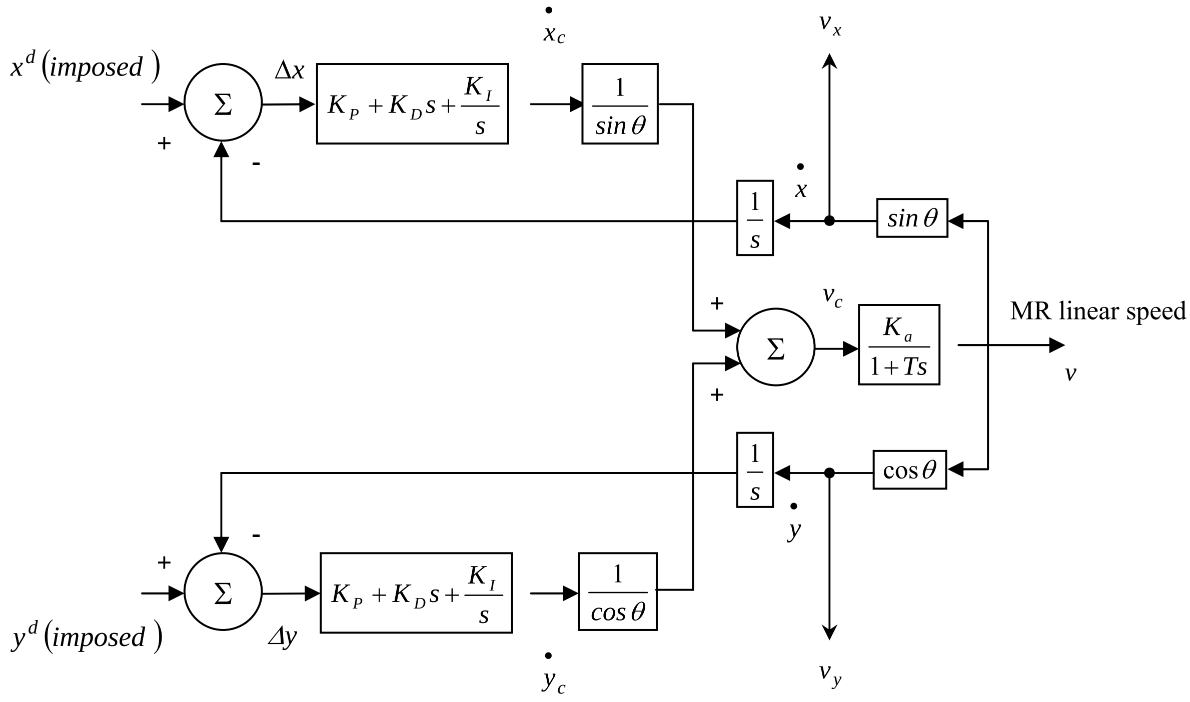

Fig. 4 Position control for the two-wheel differential drive mobile robot

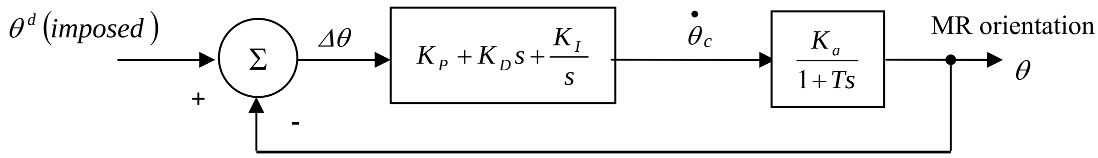

Fig. 5 Orientation or direction control for the two-wheel differential drive mobile robot

$$H(j\omega) = \frac{K_a\left[K_I + \omega^2\left(TK_P - K_D\right)\right]}{\omega^2\left(1+T^2\omega^2\right)} - j\frac{K_a\left[\left(K_P - TK_I\right) + K_D T\omega^2\right]}{\omega\left(1+T^2\omega^2\right)} \quad (15)$$

where K_P, K_I and K_D are respectively proportional parameter, integration parameter and derivative parameter for the PID controllers used in the diagrams of Fig. 4 and 5.

Now, concerning the stability for the proposed control solutions, we can consider the next different basics cases.

The first case is: $K_P - TK_I \geq 0$. Using Nyquist criteria, the final conclusion is that stability is assured if the parameter $K_D > 0$. If the proportional parameter $K_P = TK_I$ and the derivative parameter $K_D = 0$, some oscillations with constant amplitude are produced.

The second case is $K_P - TK_I < 0$. In this situation, using Nyquist criteria, the system is stable if the point M_0 is placed in the left of the point $(-1, j0)$ in the root locus method diagram depicted in Fig. 7. If the system is stable, the residual error is zero for an input step of position or a step of velocity and constant for an input step of acceleration.

Concerning perturbation, the residual error is zero for an input step of position and constant for an input step of velocity.

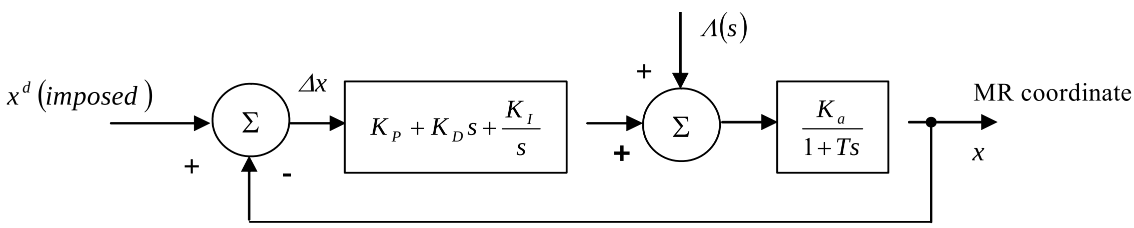

Fig. 6 Simplified model of the two-wheel differential drive mobile robot for stability analyzes

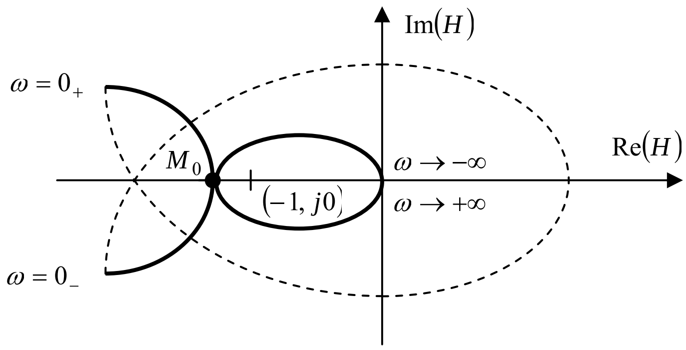

Fig. 7 Root locus method diagram for stability analysis of the two-wheel differential drive mobile robot

5. Conclusions

This paper presents some results regarding mathematical models for one kind of mobile robots, namely two-wheel differential drive mobile robot. This is one of the most common mechanical structures now utilized in mobile robotics.

The closed loop control diagrams for position control and respectively for direction control in tracking along imposed trajectories are developed and also analyzed.

Finally, for these control solutions, the paper presents an analysis regarding the stability for different classical type of inputs as step of position, step of velocity and step of acceleration.

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SPECIFINIŲ MOBILIŲJŲ ROBOTŲ MODELIAVIMO IR VALDYMO ASPEKTAI

R e z i u m ė

Darbe pristatoma keletas dviratės diferencialinės pavaros mobiliųjų robotų valdymo ir matematinio modeliavimo sprendimų. Šios klasės robotai dabar yra vieni iš plačiausiai naudojamų mechaninių įrenginių mobiliųjų robotų technikos praktikoje. Nagrinėjamos ir pateikiamos nustatyta trajektorija judančio roboto uždaro ciklo pozicijos ir krypties valdymo schemos. Šiems valdymo sprendimams pagrįsti skirtingomis sąlygomis atlikta keletas stabilumo tyrimų.

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MODELLING AND CONTROL ASPECTS FOR A TYPE OF MOBILE ROBOT

S u m m a r y

This work presents some considerations regarding mathematical models and control solutions for a class of mobile robots namely two-wheel differential drive mobile robots, one of the most utilized mechanical structures now in mobile robotics practice. The closed loop control diagrams for position control and respectively for direction control in tracking along imposed trajectories are also analyzed and included in this paper. For these control solutions, the paper presents therefore some analyses regarding the stability in different circumstances.

М. Нитулеску, В. Стоиан

АСПЕКТЫ МОДЕЛИРОВАНИЯ И УПРАВЛЕНИЯ МОБИЛЬНЫХ РОБОТОВ ОПРЕДЕЛЕННОГО ТИПА

Р е з ю м е

Представлено несколько решений по управлению и математическому моделированию определенного типа мобильных роботов, а конкретно двухколесных дифференциальных приводов для мобильных роботов, которые считаются в настоящее время чаще всего используемыми механическими устройствами в практике мобильных роботов. Проведен анализ и приведены схемы управления замкнутого цикла для управления позицией и направлением по ходу заданной траектории. Для предложенных решений по управлению выполнен ряд исследований стабильности в разных условиях.

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