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Fractal dynamics of a bouncing ball on accelerating lift tabletop with both constrained to vertical motion

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1. Introduction

In semiautomated orange fruits canning factories, the concept of bouncing ball seems applicable to the raw material transformation process in which fresh fruits are passed through conveyors from the supply station (feed station) on factory floor to an elevated height where processed fruits are translated, ready for sales to customers. The motion of these oranges is similar to a ball constrained to an accelerating lift tabletop. As the orange ball drops onto the surface of the upward carrying conveyor in slow motion, its shape will change dramatically. When the orange hits the surface of the conveyor, the bottom of the orange becomes flat against the conveyor floor. Then, when it bounces up, it returns to its original shape. However, the changes in shape are due to a balance of forces and energy.

A great number of research efforts have been made on the bouncing ball problem [1, 2]. In other studies, the bouncing ball problem has been related to chaos. Tufillaro and Albano [3] study the chaotic dynamics of a bouncing ball. This study has however generated some controversies [4]. In addition, in many industries across the world, products or raw materials are transported along a vertical accelerator. A good case is the movement of oranges for processing into fruit juice on a conveyor in a food-manufacturing industry. The movement pattern of the ball-like orange has energy implications. A study on this could be useful in evaluating the amount of energy needed to drive the system. This ball moves in a way that could be described as a fractal motion. Thus, the current work is motivated in modelling the problem as a fractal concept.

2. Methodology

The starting point in the analysis is to consider Newton's law of motion, which measures the height of the lift from origin. Here, if t is given at the time counter, x the height of the lift from the origin, x_o, the initial height of the lift and a_L, the acceleration of the lift, then the current height of the lift from the origin is defined by the equation below as

$$x(t) = x_0 + \frac{1}{2} a_L t^2 \tag{1}$$

Now, considering the ball being placed on top of the table, the same equation of Newton could be used for it. By taking x_{B0} as the initial distance of the ball from the top of the lift, g as the acceleration due to gravity.

Then

$$x(t) = x_{B0} + \frac{1}{2} g t^2 \tag{2}$$

At the initial instant of time, the ball location is at the origin. Thus $x_{B0} = 0$. But the height of the free falling ball and that of the lift tabletop from origin is approximately the same at the instant of first bounce. Thus Eqs. (1) and (2) can be equated as follows: $\frac{1}{2} g t^2 - \frac{1}{2} a_L t^2 = x_0$. By re-arranging, the following is obtained: $(g - a_L)\, t^2 = 2x_0$. Since this is the time at the first instance when the ball is bounced, it is referred to as time 1, i.e. t_1. Thus, the expression for t_1 becomes

$$t_1 = \sqrt{\frac{2x_0}{(g - a_L)}} \tag{3}$$

However, the total distance moved by the ball at the first instance is

$$x_1 = x_0 + \frac{1}{2} a_L t_1^2 \tag{4}$$

Now, for simulation purposes, it may be necessary to consider intermediate positions of the bouncing ball. An important position is $\frac{1}{3}$ bounce-up position. This height is different from the previous x_1 height and is termed z_1. This is related to x_1 as in Eq. (5) as follows

$$z_1 = \frac{1}{3} x_1 \tag{5}$$

Now, if the ball location from datum after 1st bounce is given as z_{01}, the Eq. (6) is obtained as

$$z_{01} = x_1 - z_1 \tag{6}$$

Initial distance x_{B0} must be changed to z_{01}.

Given that t_{1B} is the time value on the time line or time axis when the ball first bounced off the tabletop by z_1. Thus, the total first bounce period is the difference between t_1 and t_{1B} (i.e. $(t_1 - t_{1B})$) and for a bounce height of z_1. These two quantities (i.e. $(t_1 - t_{1B})$ and z_1) are related by Newton's law for a falling ball under gravity as: $\frac{1}{2} g \left(t_1 - t_{1B}\right)^2 = z_1$. By solving for the unknown t_{1B}, this gives Eq. (7) i.e.

$$t_{1B} = t_1 - \sqrt{\frac{2z_1}{g}} \tag{7}$$

From this time location to the time location for the second ball bounce, the time interval is (t- t_{1B}) and the ball height from origin, is $x(t)$, given as

$$x(t) = x_{B0} + \frac{1}{2} g \left(t^2 - 2t\,t_{1B} + t_{1B}{}^2\right) \tag{8}$$

At this very point in time (i.e. t) the ball height approximately equal the lift tabletop height, both measured from the origin. Hence we can equate Eq. (8) to Eq. (1) to obtain

$$\frac{1}{2} g \left(t^2 - 2t\,t_{1B} + t_{1B}{}^2\right) - \frac{1}{2} a\,L\,t^2 = x_0 - x_{B0}$$

This could be further simplified to obtain

$$\left(g - a_L\right)t^2 - 2gt\,t_{1B} + g\,t_{1B}{}^2 + 2\left(x_{B0} - x_0\right) = 0$$

Solve for the unknown t (i.e. t_2 the second time measure of ball bounce off tabletop) in this expression using almighty formula

$$t_2 = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \tag{9}$$

where $A = g - a_L$; $B = -2g\,t_{1B}$ and $C = g\,t_{1B}{}^2 + 2\left(x_{B0} - x_0\right)$.

Eqs. (1) to (9) enable us to monitor the ball through its first and subsequent journey to make a bounce off the table top in an iterated fashion, adjusting and resetting variable, due follow the motion of the ball for the second, third, fourth bounces, etc. and keep record of z-height or time of bounce back for different lift acceleration. Either of these will serve as dynamic representation of the interacting lift tabletop and bouncing ball. The whole idea expressed by the nine equations above was packaged in FORTRAN codes. The graph of the output results was found to be fractal like (Figs. 1 to 3) as shown in the results section.

3. Results

The mathematical model, which is expressed in equations, needs to be tested empirically to ascertain its usefulness in practical terms. This is usually done by either manual calculation or computerization. Such computerisation is used for flexibility in manipulating the numbers and due to the number of repetitiveness of steps involved. Computer codes are provided in Fortran Language. The results of the simulation are as shown in Figs. 1-3. The horizontal axis is the parameter change in % of acceleration due to gravity while the vertical axis is the measured quantity before the end condition of simulation is satisfied. This procedure was repeated smoothly along the horizontal axis with the results in each round being kept in an output file. The Excel-Fortran interaction is involved for importation of the simulated data in a named file into the Excel environment for graphing purposes. Graphs are produced in a scattered form. The smoother the transverse on the horizontal axis the more detail of the graph.

In particular, four different programmes have been written. The first computer programme concerns 'lift ball bounce distance diagram'. The programme first specifies the initial distance of the lift and its acceleration. Then, the ball is specified while accelerating under gravity. The next step is to specify the distance of the lift and ball respectively from rest. Incremental variation of lift acceleration is then considered. This procedure is the same for all the programmes with modifications in the coding elements in order to achieve the set goals.

Although the x-axis of all the three Figs. 1-3 are labelled as lift acceleration, which is measured in percentage of gravity, the label on the y-axis for each of the figures are different, and are shown as (i) (1/3) of drop height, (ii) number of bounces, and (iii) time of bounce (normalised), for Figs. 1, 2, and 3, respectively. Fig. 1 displays the scattered diagram of the relationship between the drop height and the lift acceleration. The shape obtained is bell-like and is skewed to the left. The mean of this distribution seems to be at 28 units with the start and end positions along the x-axis taken from 0 to about 83. For the y-axis, the range of values is from 0 to about 0.065 units. The gradient towards the left side of the graph is about 45.27° while it is less for other part of the graph, which is to the right. This informs the reader that with small increments in lift acceleration, greater value of drop height usually results.

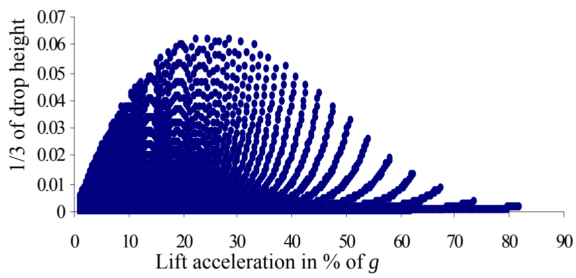

Fig. 1 Scattered diagram of (1/3) of drop height

Fig. 2 is ogive curve of the number of bounces plotted against lift acceleration. It is observed that the number of bounces on the y-axis has a maximum value of about 1070 units while it is asymptotic towards the x-axis with a terminal value of about 98 units. The number of bounces decreases as the acceleration increases steadily until the earlier reaches 75 units. Consequently, the lift acceleration moves asymptotically towards zero. From the

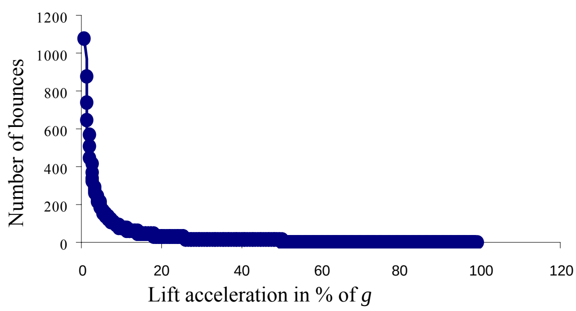

Fig. 2 Number of bounces before 10.000 metre covered

graph, the gradient of the curve is 3.38°.

Fig. 3 shows the portion the bell shape of the time of bounce (normalised) against lift acceleration. It seems to be skewed toward the right with scattered diagram spread in from 0 to about 96 units. The gradient of time of bounce plotted against lift acceleration is 45.5°. This shows that increase in lift acceleration also brings almost an equal increase in time bounce of the ball.

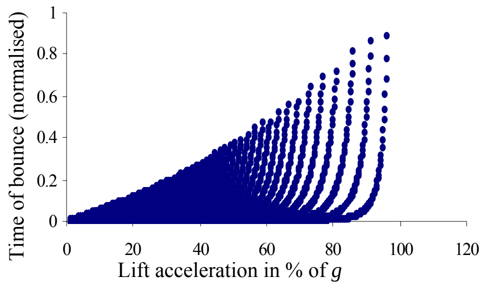

Fig. 3 Lift ball next time of bounce diagram

4. Conclusion

The importance of understanding the theory behind bouncing ball falling consecutively on an accelerating lift tabletop has been emphasized. Consequently, this study investigated the dynamic interaction of accelerating lift tabletop from the viewpoint of fractal analysis. The mathematical model developed was tested with simulation results. From the results obtained, the ball bounce-off height has a normal distribution shape with fractal details. It is concluded that fractal representation of the problem brings a new perspective to its solution; and should be explored in order to take advantage of this knowledge in its application to orange juice production and other applications.

Notably, understanding the dynamics and kinetics of a bouncing ball has both theoretical and practical significance to the researchers and managers in industries who are interested in optimal management of energy consumed during manufacturing processes. Particularly, for food industries that utilize ball-shaped objects as in-process materials in manufacturing the concept proposed here is of importance. Consequently, the motion of the bouncing ball on the table-top of an accelerating lift is modelled as fractals. This is motivated by the shared-characteristics of the bouncing ball and fractals. The model formulated is then tested with simulated data in order to evaluate its practical dimensions. The results are then plotted as graphs describing the relationship between the set of two important parameters indicated.

Having developed the model, there is a need for future outlook of research in the area. Since fractal is an already established area, there is a wide array of opportunities in applying some advanced fractal techniques and theory in the promotion of research in this area. Another aspect that readily attracts the development by its integration to existing fractal-bouncing ball structure is the application of soft computing tools such as fuzzy theory, genetic algorithm, simulated annealing, artificial neural network, and neurofuzzy systems. Efforts could initially be focused on the motion of the ball during bounces.

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VERTIKALIAI GALINČIO ATŠOKTI KAMUOLIUKO, ESANČIO ANT KELTUVO PLOKŠTĖS, KURI GREITĖDAMA JUDA TA PAČIA KRYPTIMI, FRAKTALINĖ ANALIZĖ

Rezi u m ė

Įrodyta, kad kamuoliuko atšokimo problema, kylanti vibruojančiuose transporteriuose ar virpančiose tiekimo sistemose yra svarbus reiškinys sprendžiant tam tikrų techninių sprendimų įgyvendinimą. Šiame straipsnyje nagrinėjama kamuoliukų, be pertraukos krintančių ant greitėjančios keltuvo plokštės, atšokimo dinamika. Modeliuojant nustatyta, kad dinaminė sąveika tarp viena kryptimi didėjančiu greičiu judančios keltuvo plokštės ir ant jos krintančio kamuoliuko atšokimo yra fraktalinio pobūdžio. Keltuvo plokštės paviršiaus pagreitis kito tolygiai procentine priklausomybe (daugiau nei vieno tūkstančio žingsnelių 10.000 m atstumu) nuo pagreičio, atsirandančio dėl sunkio jėgos, galinčių atšokti kamuoliukams atsitrenkus į keltuvo plokštės paviršių. Tai užrašyta grafiškai. Taip pat grafiškai užfiksuotas keltuvo plokštės paviršiaus pagreitis trečdaliu užpildžius jį kroviniu. Kamuoliukų atšokimas gerokai sumažėja ir artėja prie nulio keltuvo plokštę 40% užpildžius kroviniu. Kamuoliukų atšokimo sumažėjimo grafikas kinta pagal normalinį pasiskirstymo dėsnį ir yra fraktalinio pobūdžio. Šis tyrimas parodo, kad du objektai, pradžioje esantys skirtinguose aukščiuose, veikiami gravitacijos jėgų, pasiekia ir atsitrenkia į paviršių skirtingame aukštyje. Lygtis, aprašanti atšokančio kamuoliuko dinamiką ir keltuvo plokštės kilimą, yra kvadratinio pobūdžio, tačiau kamuoliuko atšokimo išnykimas, kintant krovinio aukščiui yra fraktalinio pobūdžio.

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FRACTAL DYNAMICS OF A BOUNCING BALL ON ACCELERATING LIFT TABLETOP BOOTH CONSTRAINED TO VERTICAL MOTION

S u m m a r y

The bouncing ball problem has proved to be an important phenomenon in engineering applications involving vibro-transportation and vibratory feeder systems. In this paper, the dynamics of a bouncing ball falling consecutively on an accelerating lift tabletop is studied. Using simulation, it is established that the dynamic interaction of accelerating lift tabletop constrained to one-dimensional motion on which the ball is bouncing is fractal. The acceleration of the lift table top was varied gradually as a percentage of acceleration due to gravity over one thousand steps while the number of bounces-off made by the bouncing ball before the lift table top covered a fall distance of 10.000 m was recorded graphically. Similarly, every lift tabletop acceleration has the set of bounce-off height of the bouncing ball recorded graphically, and taken to be one third of height of fall. The number of bounce off drastically dropped to about zero when the acceleration of the lift tabletop was 40% of acceleration due to gravity. The graphical presentation of the ball bounce off height has normal distribution shape with fractal detail. This study showed that two objects, initially at different heights, falling under gravity, maintain separating heights for the period of their fall. The equation governing the dynamics of the bouncing ball and the lift tabletop are of quadratic type but the ball bounce off height graphical results contain fractal details.

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ФРАКТАЛЬНЫЙ АНАЛИЗ ШАРИКА ВЕРТИКАЛЬНО ОТСКАКИВАЮЩЕГО ОТ ПОВЕРХНОСТИ ПЛАТЫ ПОДЪЕМНИКА С УСКОРЕНИЕМ ДВИЖУЩЕГОСЯ В ТОМ ЖЕ НАПРАВЛЕНИИ

Р е з ю м е

Установлено, что проблема отскока шарика от поверхности платы, возникающая, например, в вибротранспортерах и вибрирующих системах снабжения, является важной для решения определенных технических задач. В статье изучается динамика отскока шариков постоянно падающих на подъемную плату с ускорением движущегося подъемника. При моделировании установлено, что динамическое взаимодействие между поверхностью платы с ускорением движущегося подъемника и на него падающего с отскоком шарика носит фрактальный характер. Ускорение платы подъемника изменялось постоянно, с процентальной зависимостью (больше чем одна тысяча шагов на дистанцию 10.000 м) от ускорения возникшего от силы тяжести шариков при их ударе на поверхность платы подъемника. Все это записано графически. Графически установлено ускорение платы подъемника при ее заполнении грузом на треть. Отскок шариков значительно уменьшается и приближается к нулю при заполнении грузом платы подъемника на 40%. График отскока шариков меняется по нормальному закону распределения и носит фрактальный характер. Исследования показали, что два объекта в начале находящиеся на разных уровнях при воздействии сил гравитации ударяются на поверхность на разной высоте. Уравнение динамики воздействия платы подъемника с отскакивающим от нее шариком является квадратной, но исчезновение явления отскока шарика при изменении высоты груза носит фрактальный характер.

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