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Compressible fluid-structure interaction and modal representation

V. Kargaudas*, M. Žmuida**
**Kaunas University of Technology, Studentų 48, 51367 Kaunas, Lithuania, E-mail: vkargau@ktu.lt*
***Kaunas University of Technology, Studentų 48, 51367 Kaunas, Lithuania, E-mail: mykolas.zmuida@ktu.lt*
***Kaunas Technical College, Tvirtovės a. 35, 50155 Kaunas, Lithuania, E-mail: mykolas.zmuida@ktu.lt*

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1. Introduction

Vibrations of structures in civil engineering, aerospace, biomechanics are frequently connected with fluid influence. Fluid is a part of the mechanical system, and compressible gas or liquid. It is the significant component of the whole mechanical model. Four different dam - water reservoir models, the first rigid dam - incompressible water, the fourth flexible dam - compressible water, are presented by Tiliouine, Seghir [1]. Galerkin variational formulation is established for each model and earthquake response studies presented. A method to compute the vibration modes of an elastic shell or plate in contact with a compressible fluid is considered by Hernandez [2]. Presence of zero-frequency spurious modes with no physical meaning is indicated. Elastoacoustic vibration modes are investigated by Mellado and Rodriguez [3]. Interaction of compressible flow and deformable structures is solved by Gretarsson et al. [4]. Hydrodynamic pressure on underwater glide vehicle and surface stresses are investigated by Du et al. [5]. Vibrations in magnetorheological fluids are studied by Bansevičius et al. [6].

Forced vibrations of two plates in incompressible fluid are investigated in [7]. These two plates, not connected together, interact through an incompressible fluid. Interaction of the different eigenmodes of the same plate in vacuum is also presented.

2. Equations of plate motion

Deflections of a plate AB (Fig. 1), supported at opposite edges, can be approximated by the functions of distance y and time functions $q_s(t)$

$$u(y,t)=\sum_{s=1}^{n} q_s(t)\sigma_s(y) \tag{1}$$

where n is any integer. The base functions $\sigma_s(y)$ satisfy the boundary conditions of the plates when $y=y_1$, $y=y_2$. In Fig. 1 $y_1=0$, $y_2=h$ and $\sigma_s(y_j)=0$, $\sigma''_s(y_j)=d^2\sigma_s/dy^2=0$, $j=1,2$, but any other values of y_1 , y_2 and boundary conditions can be applied. Solution (1) is presented in n-dimensional vector space and is complete in the functional space $L_2[0,h]$ if $n\to\infty$. We can define the space of investigation when $n=const<\infty$.

A virtual deflection of the plate $\delta u_r=\delta q_r\,\sigma_r(y)$, $1\le r\le n$. The inertia forces are $-A\rho\sum_{s=1}^{n}\ddot{q}_s\sigma_s(y)$, so the

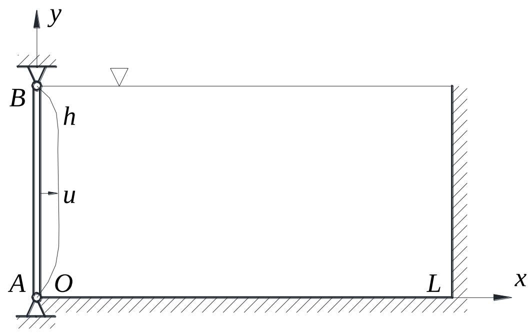

Fig. 1 Plate AB and rectangular fluid domain with free surface

virtual work $\delta W_i=-\left(\sum_{s=1}^{n} d_{rs}\ddot{q}_s\right)\delta q_r$, where $d_{rs}=A\rho\int_0^h\sigma_s\sigma_r dy$, A is the cross-section area, ρ - density of the plate. Potential energy of the deformation $\Pi=\frac{EI}{2}\int_0^l\left[\sum_{s=1}^{n}q_s\sigma''_s(y)\right]^2dy=\frac{1}{2}\sum_{s=1}^{n}\sum_{r=1}^{n}q_sq_rc_{sr}$, where $c_{sr}=EI\int_0^l\sigma''_s\sigma''_r dy$, therefore the virtual work of the plate deformation $-\frac{\partial\Pi}{\partial q_r}\delta q_r=-\left(\sum_{s=1}^{n}c_{sr}q_s\right)\delta q_r$. Modulus of elasticity $E=E^*/(1-\upsilon^2)$, where υ is Poison's ratio.

Sum of the virtual work for any $r=1,2,...,n$ is zero if influence of the fluid is neglected

$$\sum_{s=1}^{n}d_{rs}\ddot{q}_s+\sum_{s=1}^{n}c_{rs}q_s=0 \tag{2}$$

or

$$D\ddot{\bar{q}}+C\bar{q}=0 \tag{2*}$$

where $D=\|d_{rs}\|$, $C=\|c_{rs}\|$ are n-by-n matrices, $\bar{q}^T=[q_1,q_2,...,q_n]$. If $\sigma_s(y)$ are orthogonal, the matrix D and may be the matrix C is diagonal.

When the fluid is compressible and inviscid, the classical Helmholtz equation $\Delta\phi=\frac{1}{c_o^2}\frac{\partial^2\phi}{\partial t^2}$ for the potential function $\varphi(x,y,t)$ holds true, where c_o is the sound speed in the fluid. By using the separation of variables method the velocity potential can be expressed

$$\phi_j = A_j\left(sinh\psi_j x - \frac{cosh\psi_j x}{tanh\psi_j L}\right)cos\,\chi_j y\,\dot{q}_s(t) \qquad (3)$$

where $\dot{q}_s = sin\,\omega t$, $\chi_j = (j-0.5)\pi/h$,

$$h\psi_j = \frac{\pi}{2}\sqrt{(2j-1)^2-\theta_o^2}\,,\quad \theta_o = \frac{2\omega h}{\pi c_o} = \frac{4fh}{c_o} \qquad (4)$$

Particular solution (3) depends on the frequency $\omega = 2\pi f$, and this changes the whole solution of the fluid-structure interaction problem. If the sound speed $c_o \to \infty$, then $\theta_o \to 0$ and $\psi_j \to \chi_j$, solution coincides with [7].

The boundary condition on the line $x = 0$ is $\frac{\partial\varphi}{\partial x} = \frac{\partial u_s}{\partial t}$. If $u_s = q_s(t)\sigma_s(y)$, then from Eq. (3) $\phi_s = \dot{q}_s\sum A_{js}\left(sinh\psi_j x - \frac{cosh\psi_j x}{tanh\psi_j L}\right)cos\,\chi_j y$ and

$$A_{js}\psi_j\frac{h}{2} = \int_0^h \sigma_s(y)cos\,\chi_j y dy = -hD_{js}^*.$$

If $\sigma_s(y) = sin\frac{\pi s(y-y_1)}{y_2-y_1}$, then

$$D_{js}^* = \frac{2s\vartheta_1}{(j-0.5)^2-(s\vartheta_1)^2}\frac{D_{js}}{\pi} \quad \text{if } s\vartheta_1 \neq j-0.5;$$

$$D_{js}^* = \frac{sin\,\chi_j y_1}{2\vartheta_1} - \frac{D_{js}}{2\pi(2j-1)} \quad \text{if } s\vartheta_1 = j-0.5,$$

where $D_{js} = cos\,\chi_j y_1 - cos\,\pi s\,cos\,\chi_j y_2$, $\vartheta_1 = h/l$. On the plate surface $x = 0$, applying relation $p = \rho_o\dot{\phi}$, ρ_o - fluid density, pressure is expressed $p_s = \rho_o h\ddot{q}_s\sum_{j=1}^{\infty}\frac{2D_{js}^*}{h\psi_j\,tanh\psi_j L}cos\,\chi_j y$. The virtual work of the fluid pressure, when virtual deflection is $\delta u_r = \sigma_r(y)\delta q_r$, can be expressed $\delta W = -\rho_o h^2\alpha_{rs}\ddot{q}_s\delta q_r$,

$$\alpha_{rs} = \sum_{j=1}^{\infty}\frac{2D_{js}^*D_{jr}^*}{h\psi_j\,tanh\psi_j L} \qquad (5)$$

When the virtual work of the fluid pressure is added to the virtual work of plate deformation and inertia forces, the linear system of equations $\sum_{s=1}^{n}(d_{rs}+\rho_o h^2 d\alpha_{rs})\ddot{q}_s + \sum_{s=1}^{n}c_{rs}q_s = 0$ follows. In matrix notation

$$(D+\rho_o h^2 dH)\ddot{\bar{q}} + C\bar{q} = 0 \qquad (6)$$

where $H = \|\alpha_{rs}\|$, and d is width of the plate, parallel to the axis z, perpendicular to the x, y plane.

3. Eigen frequencies and modal representation

When vibrations are harmonic $\bar{q} = \bar{g}e^{i\omega t}$, then from Eq. (2) $(D-\lambda C)\bar{g} = 0$, where $\lambda = \omega^{-2}$ and $\bar{g}$ does not depend on time. If $\bar{r} = C^{1/2}\bar{g}$, then the matrix equation is $(B-\lambda I)\bar{r} = 0$, where $B = C^{-1/2}DC^{-1/2}$ is symmetric matrix and I is the unit matrix. If the base functions $\sigma_s(y)$ are orthogonal, then the matrix D is diagonal. When $y_1 = 0$, $y_2 = h$ (Fig. 1) $d_{ss} = m/2$, $c_{ss} = s^4/\Gamma^2$, where $\Gamma^2 = 2h^3/\pi^4\,EI$, m is mass of the plate.

If D is replaced by $D_H = D + \rho_o h^2 dH = m(\tilde{D}+\varepsilon H) = m\tilde{D}_H$, then vibrations Eq. (6) of the plate in fluid are presented

$$(I-\omega^2 m\tilde{B}_H)\bar{r} = 0 \qquad (7)$$

where $\tilde{B}_H = C^{-1/2}\tilde{D}_H C^{-1/2}$ is symmetric matrix, $\tilde{D}_H = \tilde{D}+\varepsilon H$, $\varepsilon = \rho_o h^2 d/m$. Really, from (5) the equality $\alpha_{rs} = \alpha_{sr}$ follows, and the matrix H is symmetric, but every entry of the matrix $\alpha_{sr} = \alpha_{sr}(\omega)$ depends on the vibration frequency ω. So, the entries $\tilde{b}_{sr}$ of the matrix $\tilde{B}_H(\omega)$ depend on the vibration frequency, the eigenvalues $\bar{\lambda}_s$ and eigenvectors $r_s = r_s(\omega)$ also depend on ω. The eigenvectors of the matrix $\tilde{B}_H$ do not represent all vibration modes of the plate in compressible fluid. Alternatively, as the matrix $\tilde{B}_H$ is symmetric, the real eigenvectors and eigenmodes can be determined by iterations.

For the steel plate ρ = 7.8 kg/dm^3, $E = 2.1\times10^7$ N/cm^2, $\delta = 2.5$ cm, $l = h = 1$ m, the first free frequency in vacuum $f_1 = 6.54$ Hz. If fluid is assumed incompressible water $\rho_o = 1$ kg/dm^3, dimensionless parameter $\varepsilon = 15.4$, the first frequency is f_{o1} = 0.634 Hz. The main parameter for compressible fluid is θ_o (4), and it depends on the distance h and sound speed c_o = 1470 m/s for the water. The parameter θ_o does not depend on fluid density, and this is important when influences of water and air are compared.

The first approximation of the first eigenfrequency $f_{o1} = 0.634$ Hz is applied and the new values of α_{sr} from (5) give new matrix H. After that the first eigenvalue $\tilde{\lambda}_1$ of the matrix is deduced from (7) and eigenfrequency $f_1 = \sqrt{\tilde{\lambda}_1/m}/2\pi$ practically coincides with the value f_{o1}. The set of eigenvectors $\bar{r}_s(f_1)$, $s = 1,2,\ldots,7$ of the matrix $\tilde{B}_H(f_1)$ is complete and orthogonal in the n-dimensional vector space, but only the first eigenvalue and the first eigenvector have physical meaning. In the second line $s = 1$, $\theta_o = 0.0052$ of the Table 1 are presented all eigenvalues of the matrix $\tilde{B}_H(f_1)$.

Calculations of the matrix $H(\omega_s)$ and the matrix $\tilde{B}_H(f_s)$, $s = 2,3,\ldots,7$ were performed, the eigenvalues in the lower lines $s = 2,3,\ldots,7$ of the Table 1. Every eigenvalue f_s of the corresponding matrix $\tilde{B}_H(f_s)$ is almost the same as in the line with $\theta_o = 0$ of the Table 1. But the first eigenvalue of the matrix $\tilde{B}_H(f_7)$ $f_1(f_7) = 0.486 < 0.634 = f_1(f_1)$ of the matrix $\tilde{B}_H(f_1)$.

Table 1

Vibrations of the steel plate

s	θ_o	f_1	f_2	f_3	f_7
-	0	6.54	26.1	58.8	320
1	0.0052	<u>0.634</u>	4.08	11.47	92.3
2	0.0333	0.634	<u>4.08</u>	11.47	92.3
3	0.0936	0.632	4.07	<u>11.46</u>	92.3
4	0.192	0.627	4.07	11.46	92.3
5	0.333	0.613	4.06	11.45	92.3
6	0.519	0.578	4.03	11.42	92.2
7	0.753	0.486	3.96	11.36	<u>92.2</u>

There are the set of eigenvectors $\bar{r}_s(f_j)$, $s = 1, 2, \ldots$, for every frequency f_j, $s = 1, 2, \ldots$, but only the eigenvectors $\bar{r}_s(f_s)$ have the physical meanings of the eigenmodes of the plate. All the vectors $\bar{r}_s(f_j)$, $j = const$, $s = 1, 2, \ldots$, are orthogonal and complete in the vector space of investigation. Only the vector $s = j$ have physical sense. The eigenmodes $\bar{r}_s(f_s)$, $s = 1, 2, \ldots, n$ are not orthogonal and may be not complete in the vector space of investigation.

Table 2

Vibrations of the wood plate

s	θ_o	f_1	f_2	f_3	f_4	f_5
-	0	6.072	30.93	75.58	140.1	224.6
1	0.0714	<u>6.067</u>	30.93	75.58	140.1	224.6
2	0.363	5.914	<u>30.82</u>	75.49	140.0	224.5
3	0.879	4.142	29.94	<u>74.77</u>	139.4	224.0
4	1.65	7.47	31.1	75.7	<u>140.3</u>	224.8
5	2.71	10.26	35.9	81.0	145.8	<u>230.4</u>

Another example presents vibrations of wood plate in air, when $h = 1$ m, plate thickness $\delta = 0.4$ cm, density $\rho = 0.4$ kg/dm^3, $E = 12 \times 10^5$ N/cm^2. Density of air $\rho_o = 1.2$ g/dm^3, therefore $\varepsilon = 0.75$. Density of the air is much less then the density of water, and diminution of frequency in the fluid is not so significant (Table 2). The speed of sound in air $c_o = 340$ m/s, and therefore parameter θ_o is higher and exceeds critical value $\theta_o = 1$ when eigenvibration number $s > 3$. If $\theta_o > 1$, then $h\psi_j$ in (4) has an imaginary value and some terms in α_{rs} (5) are negative with product $h\psi_j^* \tan L\psi_j^*$ in denominator, where $h\psi_j^* = \frac{\pi}{2}\sqrt{\theta_o^2 - (2j-1)^2}$.

The matrix of hydrodynamic interaction H does not depend on fluid density ρ_o, but depends on compressibility. If $c_o \to \infty$ then the matrix $H(f)$ coincides with the matrix $H(0)$ in compressible fluid when $f = 0$

$$10H(0) = \begin{Vmatrix} 11.15 & 2.62 & 1.82 \\ 2.62 & 4.72 & 1.09 \\ 1.82 & 1.09 & 2.91 \end{Vmatrix},$$

$$10H(6.07) = \begin{Vmatrix} 11.18 & 2.64 & 1.83 \\ 2.64 & 4.73 & 1.10 \\ 1.83 & 1.10 & 2.92 \end{Vmatrix},$$

$$10H(230) = \begin{Vmatrix} -0.705 & -4.94 & -2.54 \\ -4.94 & 5.92 & 0.78 \\ -2.54 & 0.78 & 2.80 \end{Vmatrix}.$$

The matrices $H(6.07)$, $H(230)$ are presented for the eigenfrequencies $s = 1$, $s = 3$, $s = 5$ (Table 2). The latter matrix corresponds to the parameter $\theta_o = 2.71 > 1$ and some entries are negative. Nevertheless, the matrix $H(230)$ and the matrix

$$\tilde{B}_{H(230)} = \begin{Vmatrix} 1.43 & -0.30 & -0.07 \\ -0.30 & 0.19 & 0.05 \\ -0.07 & 0.05 & 0.03 \end{Vmatrix}$$

are symmetric, therefore all eigenvalues and eigenvectors are real and can be defined positive (Table 2)

$$T_{H(6.07)} = \begin{Vmatrix} 0.999 & -0.039 & -0.010 \\ 0.038 & 0.998 & -0.041 \\ 0.011 & 0.040 & 0.998 \end{Vmatrix}.$$

Only the first column of the matrix $T_{H(6.07)}$ is the true first mode of the plate in compressible fluid. All other columns are the eigenvectors of the plate, and all these eigenvectors, with the first mode included, make a set of orthogonal vectors, complete in the functional space of investigation. This is true with the set of eigenvectors

$$T_{H(230)} = \begin{Vmatrix} 0.974 & 0.214 & 0.064 & 0.027 & 0.014 \\ -0.220 & 0.971 & 0.086 & 0.032 & 0.016 \\ -0.046 & -0.100 & 0.992 & 0.057 & 0.024 \\ -0.018 & -0.032 & -0.062 & 0.996 & 0.042 \\ -0.009 & -0.015 & -0.023 & -0.044 & 0.999 \end{Vmatrix}.$$

The last column is not only the eigenvector, but also can be assumed as eigenmode number 5. The product of the matrices $T_{H(f_s)} T'_{H(f_s)} = I$, f_s are the eigenfrequencies. But if the set of eigenmodes forms the matrix

$$T_{MODE} = \begin{Vmatrix} 0.999 & -0.042 & -0.017 & -0.001 & 0.014 \\ 0.038 & 0.998 & -0.057 & -0.011 & 0.016 \\ 0.011 & 0.041 & 0.997 & -0.030 & 0.024 \\ 0.005 & 0.016 & 0.045 & 0.999 & 0.042 \\ 0.001 & 0.008 & 0.019 & 0.027 & 0.999 \end{Vmatrix}$$

then $T_{MODE}\,T'_{MODE} \neq I$. Notice that absolute values of entries in diagonal of T_{MODE} are much larger then all other absolute values of the same matrix, even though the matrix $T_{H(230)}$ has the diagonal values less then in T_{MODE}.

4. Discussion

When vibrations are forced by harmonic force $F = F_z \, sin\omega_z t$, $f_z = \omega_z / 2\pi$, and the frequency f_z coincides or is near the eigenfrequency f_j of the plate in compressible fluid (underlined values in Tables 1, 2), the mode of vibration can be assumed equal to the eigenvector – the *j*-th column in the matrix $T_{H(f_j)}$. If the mode of the forced vibrations should be more precise, the other eigenvectors of the matrix $T_H(f_s)$ can be applied. If all *n* eigenvectors are necessary, any set of eigenvectors $T_H(f_s)$, $s = 1, 2, ..., n$ is acceptable. The set of eigenmodes can be unsuitable as base functions because the set can be not complete in the vector space of investigation. Moreover, the eigenmodes are not orthogonal. It may be indicated, that added masses are useful only when rigid bodies are in fluid. In some sense the coefficients $\alpha_{rs} = \alpha_{sr}$ can be presented as substitute to the added mass.

The eigenmodes are important when resonant vibrations are induced and one or two eigenvectors of the corresponding matrix are required to present the forced vibration. Real fluid always is compressible, so any investigation of the fluid and structure raise the problem – what is the practical and general theoretic significance of the fluid compressibility.

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V. Kargaudas, M. Žmuida

SPŪDAUS SKYSČIO IR KONSTRUKCIJŲ SĄVEIKA BEI SAVŲJŲ VIRPESIŲ FORMŲ SAVYBĖS

R e z i u m ė

Tiriami tamprios plokštės ir spūdaus neklampaus skysčio virpesiai. Skysčio spūdumas keičia plokštės savąsias virpesių formas ir tik šiek tiek savuosius virpesių dažnius, o skysčio tankis savuosius virpesių dažnius keičia žymiai daugiau, negu formas. Aprašomi plieninės plokštės virpesiai vandenyje ir medinės plokštės virpesiai ore, aptariamas vandens ir oro spūdumo poveikis . Plokštės savųjų virpesių formų vakuume sąveika teikiama hidrodinamine matrica, kurios elementai priklauso nuo garso bangų skystyje greičio. Plokštės savosios virpesių formos vakuume ir savosios virpesių formos nespūdžiame skystyje gali būti tyrimų bazinėmis funkcijomis, bet plokštės savųjų virpesių formos spūdžiame skystyje nesudaro pilnos ortogonalių bazinių funkcijų aibės.

V. Kargaudas, M. Žmuida

COMPRESSIBLE FLUID-STRUCTURE INTERACTION AND MODAL REPRESENTATION

S u m m a r y

Vibrations of elastic plate and compressible fluid are investigated. Compressibility of the fluid influences eigenmodes and to some degree eigenfrequencies also, while density of the fluid eigenfrequencies changes more than eigenmodes. Calculations of steel plate vibrations in water and wood plate vibrations in air are presented, influence of the water and the air compressibility discussed. Interaction of plate eigenmodes in vacuum is represented by hydrodynamic matrix, entries of which depend on sound wave speed in fluid. Eigenmodes of the plate in vacuum and eigenmodes in incompressible fluid can be applied as base functions of the investigation, but the set of eigenmodes of the plate in compressible fluid does not form a set of complete orthogonal base functions.

Keywords: vibrations, compressible fluid, deformable plate, eigenmodes, interaction.

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