



$$\sigma_y^{M*} = \sigma_x^{M*} + \frac{2}{\sqrt{3}} \sigma_i^{M*}(0) - D^M p C_p^* \xi^2 \quad (8)$$

$$p = \int_0^1 \sigma_y d\xi \quad (9)$$

$$p = \frac{2}{\sqrt{3}} \sigma_i^{M*}(0) + \frac{1}{3} p C_p^* [f_2(\alpha) - D^M] \quad (10)$$

$$p C_p^* = \frac{2\sqrt{3}(\gamma_N - 1) \sigma_i^{M*}(0)}{f_2(\delta^H) + f_2(\alpha) - (C_1^M D^M - D^H)} \quad (11)$$

$$\sigma_i^{H*}(0) = \frac{\sqrt{3}}{2} \left\{ p + \frac{1}{3} [f_2(\eta^H) p C_p^* + |\tau_{xy}^H(1)| D^M] \right\} \quad (12)$$

$$\sigma_i^{H*}(1) = \sqrt{\left[ \sigma_i^{H*}(0) - \frac{\sqrt{3}}{2} D^H p C_p^* \right]^2 + 3(p C_p^*)^2} \quad (13)$$

$$\sigma_i^{M*}(1) = \sqrt{\left[ \sigma_i^{M*}(0) - \frac{\sqrt{3}}{2} D^M p C_p^* \right]^2 + 3(p C_p^*)^2} \quad (14)$$

where  $f_2(\eta)$ ,  $f_2(\eta^H)$  and  $\Phi(\eta^H)$  are functions of stresses distribution [1].

For stress strain state determination strain intensity at the centre of contact plane  $e_i^*(0)$  must be chosen and the factor of mechanical heterogeneity  $\gamma_N = \sigma_i^{H*}(0)/\sigma_i^{M*}(0)$  determined.

Parameter  $D^M$  is determined from equality of strains at external contact plane point, i.e.  $e_y^{H*}(1) = e_y^{M*}(1)$ .

Parameter  $D^H = 0$  when  $\sigma_i^{H*}(1) \leq \sigma_e^H$ . When material H at the external point of contact plane  $\zeta^* = 1$  is deformed elastic-plastic ( $\sigma_i^{H*}(1) > \sigma_e^H$ ) stress  $\sigma_i^{H*}(1)$  is calculated from presumption, that potential energy of material H at this point under elastic and elastic-plastic loading is the same [2]. When tensile curve of material H at elastic-plastic zone is approximated by power function

$$\sigma_i^{H*}(1) = \sigma_e^H \left[ \frac{\sigma_{if}^{H*}(1)}{\sigma_e^H} \right]^{\frac{2 m_0^H}{m_0^H + 1}} \quad (15)$$

The intensity of fictitious elastic stress  $\sigma_{if}^{H*}(1)$ , which corresponds to  $e_i^*(0)$  is calculated from Eq. (13) by substituting  $p C_{p_f}^*$  instead of  $p C_p^*$ . Parameter  $p C_{p_f}^*$  is calculated by Eq. (11) when  $D^H = 0$  and  $\gamma_N = \gamma_{Nf}$ . Fictitious factor of mechanical heterogeneity  $\gamma_{Nf} = \bar{e}_i^{M*}(0)$ . Parameter  $D^H$  when  $\sigma_i^{H*}(1) > \sigma_e^H$  is calculated from the equation

$$D^H = \frac{2}{\sqrt{3}} \frac{\sigma_i^{H*}(0) - \sqrt{[\sigma_i^{H*}(1)]^2 - 3(p C_p^*)^2}}{p C_p^*} \quad (16)$$

Designations (0), (0.5) and (1) denotes values at longitudinal sections of  $\zeta = 0$ ,  $\zeta = 0.5$  and  $\zeta = 1$  respectively.

In any section equilibrium condition  $p = \int_0^{1+\bar{h}_{fc}} \sigma_y d\xi$

must be used instead of Eq. (9). Determination of this integral is very complicated. Therefore at first approaching integral equilibrium condition

$$\frac{p}{1 + \bar{h}_{fc}} = \int_0^1 \sigma_y d\xi \quad (17)$$

was accepted.

Stress intensity at the centre of square weld with reinforcement, which relative height  $\bar{h}_{fc}$ , is equal

$$\sigma_{ic}^M(0) = \frac{\sqrt{3}}{2} \left[ \frac{p}{1 + \bar{h}_{fc}} - \frac{1}{3} p C_p^* f_2(0) \right] \quad (18)$$

*Determination of strains.* From Eq. (18) follows that strains at the central cross-section of real square weld is smaller than in the simplified plane weld. Longitudinal strain at this zone

$$e_{yc}^M = \frac{\sqrt{3} \sigma_{ic}^M(0)}{2 E_c'^M} \quad (19)$$

where  $E_c'^M = \sigma_{ic}^M / e_{ic}^M$  is secant modulus of mild material loading curve in this zone. Because this solution is approximate the mean longitudinal strain at deformation base  $\bar{L}$  is determined in longitudinal section  $\zeta = 0.5$  from condition that the distribution of strains in direction of  $\eta$ -axis corresponds to the law of square parabola. This condition showed a good agreement with experimental data in welded joint with the simplified weld. Then mean value of longitudinal strains of mild metal  $e_{ym}^M = e_y^*(0.5) + 2/3 [e_{yc}^M - e_y^*(0.5)]$ , when  $e_{yc}^M > e_y^*(0.5)$  and  $e_{ym}^M = e_{yc}^M + 1/3 [e_y^*(0.5) - e_{yc}^M]$ , when  $e_{yc}^M < e_y^*(0.5)$ . Mean longitudinal strain in section  $\zeta = 0.5$  at deformation base  $\bar{L}$  of the welded joint with the reinforcement, when  $\bar{L} > \alpha + 2$ , is

$$e_{ym} = \frac{1}{\bar{L}} \left[ \left( \bar{L} - \bar{l}_l \right) e_{yl}^H + \frac{\bar{l}_l}{3} \left( e_y^*(0.5) + 2 e_{yl}^H \right) + \alpha e_{ym}^M \right] \quad (20)$$

where  $\bar{l}_l = 2$  is base metal (in this case hard metal) relative distance from the contact plane in which stress  $\sigma_x = 0$ ;  $e_{yl}^H = 3 p / (4 E)$  is hard metal strain in cross-section where stress  $\sigma_x^H = 0$ . At elastic-plastic loading of hard metal ( $\sigma_i^H > \sigma_e^H$ ) secant modulus  $E_l'^H$  must be used instead of  $E$ .

When  $e_{ym}$  is the same longitudinal strain value at the central cross-section of the weld with reinforcement is

lower than this one in simplified weld (see Fig. 2).

*Determination of strain intensity at the external contact plane point of welded joint with the real mild square weld.* Longitudinal strains at the central cross-section of real weld decreases with increasing of the reinforcement height  $\bar{h}_{rfc}$  (Fig. 2). In this case stress intensity at external point of the contact plane ( $\zeta^* = 1$ ) increases due to stress concentration of reinforcement. But this solution does not evaluate strain increasing at this point. Therefore strain intensity at contact plane point  $\zeta^* = 1$  was determined by the equation

$$e_{i \max}^* = e_{in} K_e \quad (21)$$

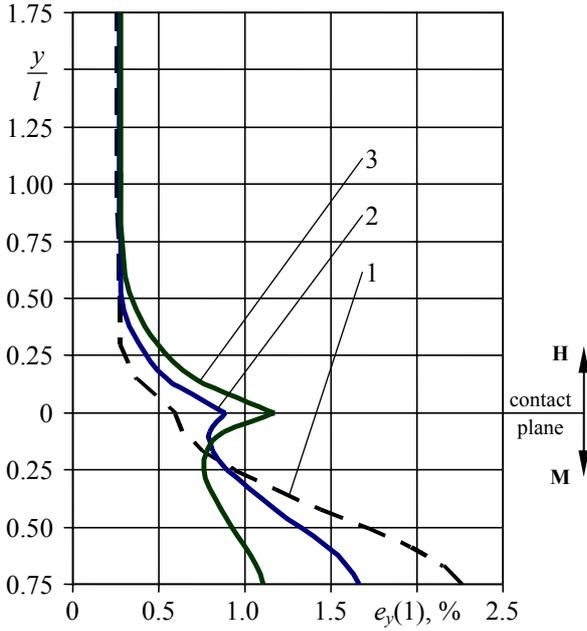


Fig. 2 Comparison of longitudinal nominal strains distribution in welded joint with simplified (---) and real mild weld (—) when  $\alpha = 0.75$ ,  $\bar{L} = 5$ ,  $e_{y,m} = 0.5\%$ ,  $2l = 10$  mm,  $r = 1$  mm,  $m_0^H = 0.102$ ,  $m_0^M = 0.069$ ,  $\gamma_e = \sigma_e^H / \sigma_e^M = 1.35$ : 1 -  $\bar{h}_{rfc} = 0$ ; 2 -  $\bar{h}_{rfc} = 0.05$ ; 3 -  $\bar{h}_{rfc} = 0.1$

where  $K_e$  is strain concentration coefficient in stress concentration zone at elastic-plastic loading;  $e_{in}$  is intensity of nominal strains. In this work it was assumed that  $e_{in} = e_i^*(1)$ , which is determined from Eqs. (13) - (15) and tensile curve of M or H materials for simplified welded joint.

In work [3] it is determined that strain concentration coefficient proposed by N. Machutov [4]

$$K_e = \frac{\alpha_\sigma^{\frac{2}{1+m_0}} \bar{\sigma}_{in}^{\frac{1-m_0}{1+m_0}}}{(\alpha_\sigma \bar{\sigma}_{in})^n \frac{1-m_0}{1+m_0} [1 - (\bar{\sigma}_{in} - 1/\alpha_\sigma)]} \quad \text{when } \bar{\sigma}_{in} \leq 1 \quad (22)$$

and

$$K_e = \frac{\alpha_\sigma^{\frac{2}{1+m_0}}}{(\alpha_\sigma \bar{\sigma}_{in})^n \frac{1-m_0}{1+m_0} [1 - (\bar{\sigma}_{in} - 1/\alpha_\sigma)]} \quad \text{when } \bar{\sigma}_{in} > 1 \quad (23)$$

showed good agreement with experimental data. In Eqs. (22) and (23)  $\alpha_\sigma$  is stress concentration coefficient at elastic loading;  $\bar{\sigma}_{in} = \sigma_{in} / \sigma_e$ ;  $m_0$  is the power index of material tensile curve which at elastic-plastic zone is approximated by power function;  $n$  is material coefficient (for steels  $n \approx 0.5$ ).

By G. Neuber [5] it was shown that when the radius of concentrator rounding is small real value of stress concentration coefficient for metals is smaller than  $\alpha_\sigma$ . Because metals have a granular structure the radius of concentration rounding can not be smaller than one half size of grain  $r_g$ . In work [5] is shown that for steel the value of  $\alpha_\sigma$  determined experimentally and theoretically is the same, when fictitious value of one half size of grain  $r_{fg} \approx 0.48$  mm is assumed. Therefore when  $r \leq r_{fg}$  the radius of concentrator rounding  $r = 0.48$  mm must be accepted.

Stress concentration coefficient  $\alpha_\sigma = \sigma_{i \max} / \sigma_{in}$  increases with increasing of  $\bar{h}_{rfc}$  and decreasing of  $\bar{b}_{rf}$ .

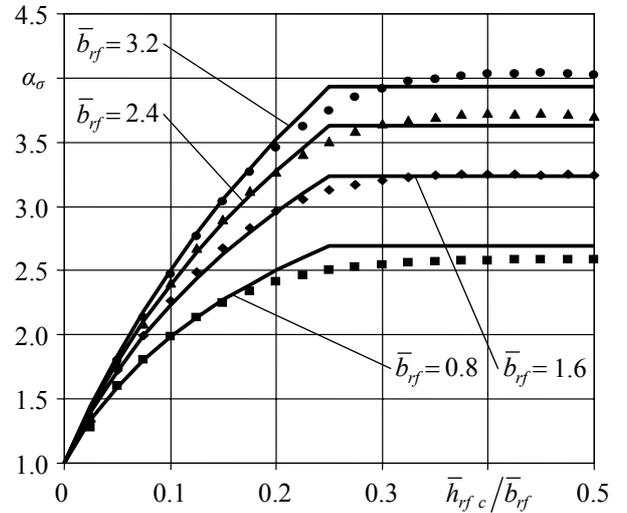


Fig. 3 Dependence  $\alpha_\sigma$  on  $\bar{h}_{rfc}$  and  $\bar{b}_{rf}$  calculated analytically (—) and determined by FEM (points), when  $2l = 20$  mm;  $r = 0.48$  mm; the length of finite element edge at the concentration zone  $l_e \approx 0.005l$ :  $\blacksquare$  -  $\bar{b}_{rf} = 0.8$  ( $\alpha = 0.4$ );  $\blacklozenge$  -  $\bar{b}_{rf} = 1.6$  ( $\alpha = 0.8$ );  $\blacktriangle$  -  $\bar{b}_{rf} = 2.4$  ( $\alpha = 1.2$ );  $\bullet$  -  $\bar{b}_{rf} = 3.2$  ( $\alpha = 1.6$ )

Dependence  $\alpha_\sigma$  on  $\bar{h}_{rfc}$  and  $\bar{b}_{rf}$  was determined for mechanically homogeneous welded joint by FEM, when  $\sigma_{i \max} < \sigma_e$  and the rounding radius of reinforcement beginning  $r = 0.48$  mm, is shown in Fig. 3 (points).

From Fig. 3 follows that dependence  $\alpha_\sigma$  on  $\bar{h}_{rfc}$  and  $\bar{b}_{rf}$  when  $\bar{h}_{rfc} / \bar{b}_{rf} \leq 0.25$  for welded joints may be expressed by the equation

$$\alpha_\sigma = 1 + C_l C_r (1 + 2 \bar{b}_{rf}) \lg \left[ 1 + \frac{\bar{h}_{rfc}}{\bar{b}_{rf}} \left( \frac{25}{1 + \bar{b}_{rf}} \right) \right] \quad (24)$$

and when  $\bar{h}_{rfc} / \bar{b}_{rf} > 0.25$ ,  $\alpha_\sigma$  is determined from Eq. (24) by substituting 0.25 instead of  $\bar{h}_{rfc} / \bar{b}_{rf}$ . Where

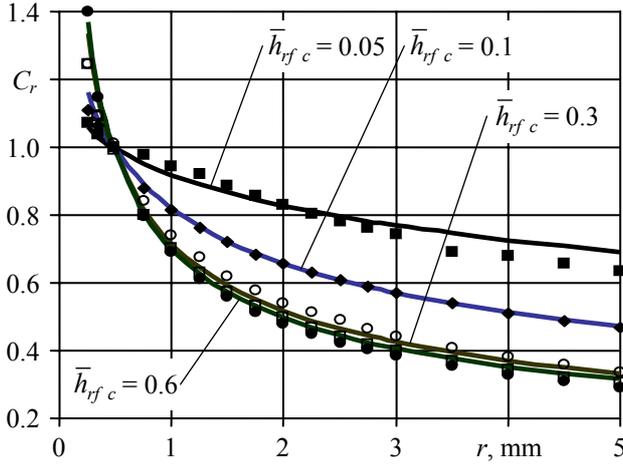


Fig. 4 Dependence of  $C_r$  on  $r$  and  $\bar{h}_{rfc}$  calculated analytically (—) and determined by FEM (points), when  $2l = 20$  mm; the length of finite element edge at the concentration zone  $l_e \approx 0.005 l$ :  $\blacksquare$  -  $\bar{h}_{rfc} = 0.05$  and  $\alpha = 0.8$ ;  $\blacklozenge$  -  $\bar{h}_{rfc} = 0.1$  and  $\alpha = 0.8$ ;  $\bullet$  -  $\bar{h}_{rfc} = 0.6$  and  $\alpha = 0.8$ ;  $\square$  -  $\bar{h}_{rfc} = 0.3$  and  $\alpha = 0.4$ ;  $\circ$  -  $\bar{h}_{rfc} = 0.3$  and  $\alpha = 1.2$

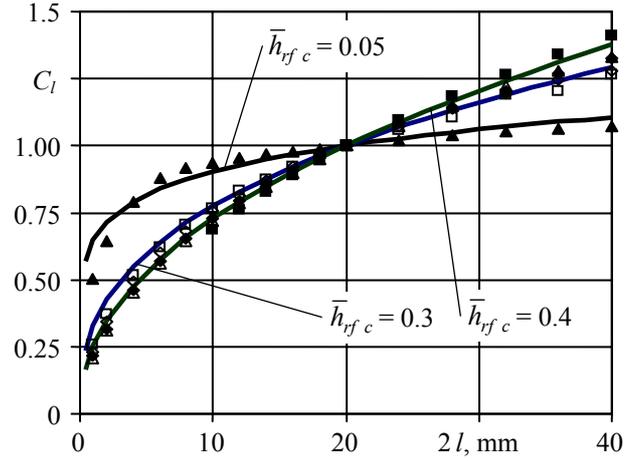


Fig. 5 Dependence of  $C_l$  on  $2l$  calculated analytically (—) and determined by FEM (points), when the length of finite element edge at the concentration zone  $l_e \approx 0.005 l$ :  $\blacktriangle$  -  $\bar{h}_{rfc} = 0.05$ ,  $r = 0.48$  mm and  $\alpha = 0.8$ ;  $\triangle$  -  $\bar{h}_{rfc} = 0.4$ ,  $r = 0.48$  mm and  $\alpha = 0.8$ ;  $\blacksquare$  -  $\bar{h}_{rfc} = 0.3$ ,  $r = 3$  mm and  $\alpha = 0.8$ ;  $\square$  -  $\bar{h}_{rfc} = 0.3$ ,  $r = 0.25$  mm and  $\alpha = 0.8$ ;  $\blacklozenge$  -  $\bar{h}_{rfc} = 0.3$ ,  $r = 0.48$  mm and  $\alpha = 0.4$ ;  $\diamond$  -  $\bar{h}_{rfc} = 0.3$ ,  $r = 0.48$  mm and  $\alpha = 1.2$

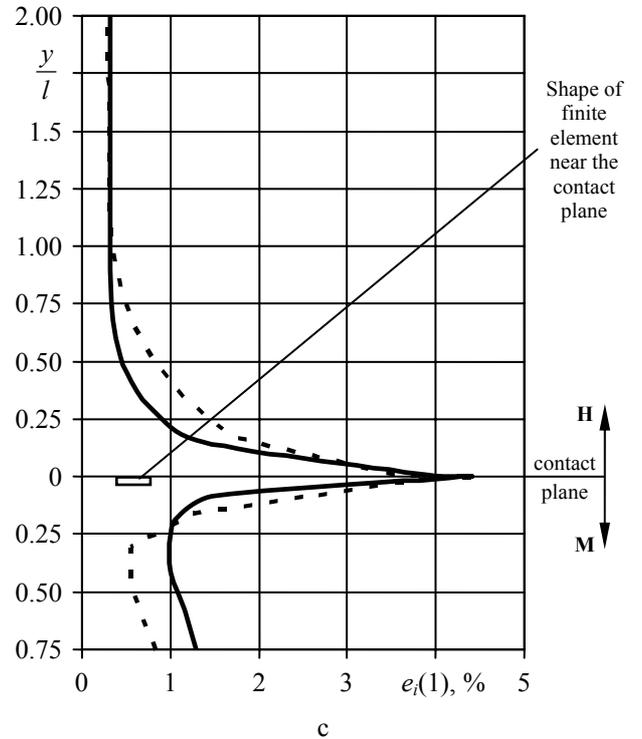
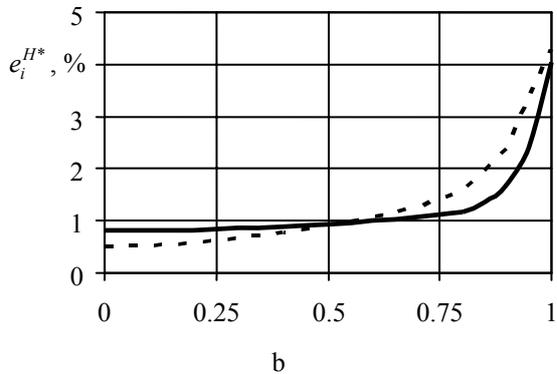
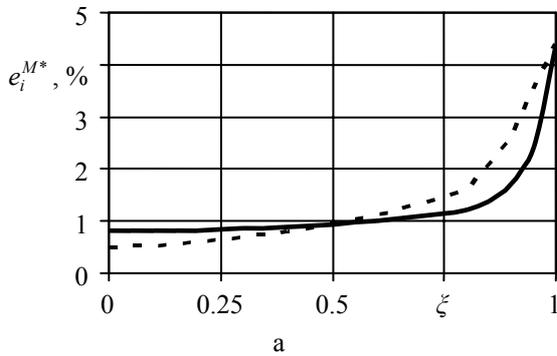


Fig. 6 Comparison of strain intensity distribution in a welded joint with the real mild weld determined analytically (—) and by FEM (---) when the length of finite element edges at the contact plane zone  $l_e \approx (0.1 \times 0.01) l$ ,  $\alpha = 0.75$ ,  $\bar{L} = 5$ ,  $e_{ym} = 0.5\%$ ,  $2l = 20$  mm,  $r = 0.48$  mm,  $\gamma_e = \sigma_e^H / \sigma_e^M = 1.35$ ,  $m_0^H = 0.102$ ,  $m_0^M = 0.069$ : a and b - on the contact plane; c - on the surface of welded joint

$C_r = \frac{1 + (10 \bar{h}_{rfc})^2}{1 + (10 \bar{h}_{rfc})^2 \sqrt{r/0.48}}$  is the parameter which evalu-

ates rounding radius of weld reinforcement beginning;  $C_l = (2l/20)^{(0.9 \bar{h}_{rfc} + 0.1)}$  is the parameter which evaluates thickness of welded sheets. Radius  $r$  and thickness of sheets  $2l$  must be expressed in millimetres. Dependencies of parameter  $C_r$  on  $r$ ,  $\bar{h}_{rfc}$  and parameter  $C_l$  on  $2l$  are shown in Fig. 4 and Fig. 5 respectively.

Strain concentration coefficient of mild metal  $K_e$  at contact plane (point  $\zeta^* = 1$ ) is determined from Eq. (23) by substituting  $m_0^M$  and  $\bar{\sigma}_i^{M*}(1) = \sigma_i^{M*}(1)/\sigma_e^M$  instead of  $m_0$  and  $\bar{\sigma}_{in}$ . At contact plane point  $\zeta^* = 1$  strain concentration coefficient of hard metal  $K_e$  is determined from Eq. (22) when  $\bar{\sigma}_i^{H*}(1) \leq 1$  and Eq. (23) when  $\bar{\sigma}_i^{H*}(1) > 1$  by estimating that  $m_0 = m_0^H$  and  $\bar{\sigma}_{in} = \bar{\sigma}_i^{H*}(1)$ .

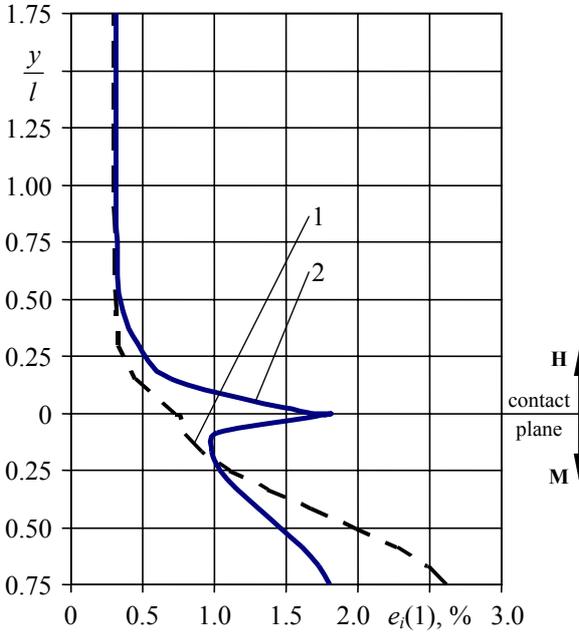


Fig. 7 Distribution of strain intensity on the surface of welded joint with real mild weld when  $\alpha = 0.75$ ,  $\bar{L} = 5$ ,  $e_{ym} = 0.5\%$ ,  $2l = 10$  mm,  $r = 1$  mm,  $\gamma_e = \sigma_e^H / \sigma_e^M = 1.35$ ,  $m_0^H = 0.102$ ,  $m_0^M = 0.069$ : 1 -  $\bar{h}_{rfc} = 0$  (simplified weld); 2 -  $\bar{h}_{rfc_{opt}} = 0.059$

Strain intensity on the central cross-section of real mild weld  $e_{ic}^M = \sqrt{3} e_{yc}^M / 2$ .

Strain intensity of H and M metals calculated analytically showed a good agreement with its values determined by FEM (Fig. 6).

*Determination of optimum relative value of mild weld reinforcement.* From Eqs. (18) and (19) follows that strain intensity in central cross-section of the mild real weld decreases with increasing  $\bar{h}_{rfc}$ . But in this case stress intensity, as follows from Eq. (21), increases in stress concentration zone at the external contact plane point  $\zeta^* = 1$ . Therefore optimal relative value of reinforcement height  $\bar{h}_{rfc_{opt}}$  must be determined from condition  $e_{ic}^M = e_i^{M*}(1)$

(Fig. 7).

#### 4. Butt welded joints with the hard real weld

Stress strain state determination of simplified welded joint with hard interlayer is presented in works [1, 2]. Stress strain state in mild base metal and on the contact plane of welded joint with the real hard weld and simplified hard weld are the same. In this case stiffness of hard weld increases due to relative height of reinforcement  $\bar{h}_{rfc}$ . Stress intensity at the centre of real hard weld, which relative height is  $\bar{h}_{rfc}$ , can be determined by the equation

$$\sigma_{ic}^H(0) = \frac{\sqrt{3}}{2} \left[ \frac{p}{1 + \bar{h}_{rfc}} - \frac{1}{3} p C_p^* f_2(\eta^H = 0) \right] \quad (25)$$

Mean longitudinal strain is determined analogically as in simplified weld [1].

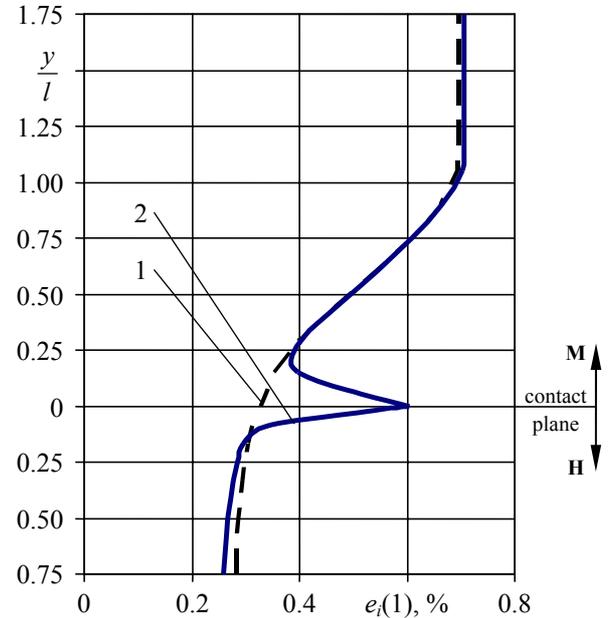


Fig. 8 Distribution of strain intensity on the surface of welded joint with the real hard weld when  $\alpha^H = 0.75$ ,  $\bar{L} = 5$ ,  $e_{ym} = 0.5\%$ ,  $2l = 10$  mm,  $r = 1$  mm,  $m_0^H = 0.102$ ,  $m_0^M = 0.069$ ,  $\gamma_e = \sigma_e^H / \sigma_e^M = 1.35$ : 1 -  $\bar{h}_{rfc} = 0$  (simplified weld); 2 -  $\bar{h}_{rfc} = 0.1$

Stiffness of hard weld and strain intensity in stress concentration zone  $e_i^{H*}(1)$  increase with increasing  $\bar{h}_{rfc}$ . Strains  $e_i^{H*}(1)$  and  $e_i^{M*}(1)$  are determined analogically as in welded joint with a real mild weld. In this case in Eqs. (2) - (11)  $\alpha^H$  and  $\delta = 1.2$  must be substituted instead of  $\delta^H = 1.2$  and  $\alpha$  respectively.

Coefficient  $\alpha_\sigma$  is determined from Eq. (24) by evaluating that in this case  $\bar{b}_{rfc} = 2 \alpha^H$ .

From Fig. 8 follows that strain intensity at the most heavily loaded zones of welded joint with the hard real weld is larger than in simplified welded joint. Therefore weld reinforcement is recommended to be take off by grinding.

## 5. Conclusions

1. Strain intensity on the central cross-section  $e_{ic}^M$  of real mild weld decreases when the relative height of reinforcement  $\bar{h}_{rfc}$  increases.

2. Strain intensity in stress concentration zone  $e_i^{M*}(1)$  at initial point of weld reinforcement increases with increasing of  $\bar{h}_{rfc}$ .

3. Optimum value of  $\bar{h}_{rfc}$  can be determined by the equality of strain intensities at zones pointed in 1st and 2nd conclusions.

4. Strain intensity in the most heavily loaded zones of welded joint with the real hard weld is larger than in the welded joint with simplified weld.

## References

1. **Bražėnas, A., Daunys, M.** The stress strain state and plasticity of mechanically heterogeneous welded joints with a flat interlayer subjected to tension (compression). ISSN 1392–1207.-Mechanika.-Kaunas: Technologija, 1995, Nr.1, p.5-13.
2. **Bražėnas, A.** Strength and Low Cycle Fatigue Life of Mechanically Heterogeneous Butt Welded Joints. Monograph.-Kaunas: Technologija, 2002.-284p.
3. **Daunys, M., Narvydas, E.** Investigation of the stress strain state at low cycle loading in stress concentration zones.-Proc. of 6th Int. Symposium on Creep and Coupled Processes.-Bialystok, 1998, p.163-172.
4. **Махутов Н.А.** Деформационные критерии разрушения и расчет элементов конструкций на прочность.-Москва: Машиностроение, 1981.-272с.
5. **Нейбер Г.** Концентрация напряжений. -Москва, 1947.-204с.

A. Bražėnas, D. Vaičiulis

DEFORMACIJŲ BŪVIS TEMPIAMOSE MECHANIS-  
KAI NEVIENALYTĖSE SUVIRINTOSE JUNGTYSE  
SU REALIA PLOKŠČIA SIŪLE ESANT  
PLOKŠTUMINEI DEFORMACIJAI

Резюме

Šiame darbe gautos priklausomybės apskaičiuoti deformacijų intensyvumui suvirintųjų jungčių su realia

minkšta ir kieta siūle perkrovimo zonose. Realios minkštos siūlės optimalus santykinis rumbelės aukštis apskaičiuotas iš deformacijų centriniame siūlės skerspjūvyje ir įtempimų koncentracijos zonoje (pradiniame rumbelės taške) intensyvumo lygybės. Kietos siūlės rumbelė padidina deformacijų intensyvumą suvirintosios jungties perkrovimo zonoje.

A. Bražėnas, D. Vaičiulis

THE STRAIN STATE OF MECHANICALLY  
HETEROGENEOUS WELDED JOINT WITH THE  
REAL SQUARE WELD SUBJECTED TO TENSION AT  
PLANE DEFORMATION

Summary

Dependencies for the calculation of strain intensity in the most heavily loaded zones of welded joint with the real mild and hard weld are obtained in this paper. The optimum relative height of mild weld reinforcement is obtained from equality of strain intensities in central cross-section and stress concentration zone (initial point of reinforcement). The hard weld reinforcement increase strain intensity in the most heavily loaded zones of welded joint.

А. Браженас, Д. Вайчюлис

ДЕФОРМИРОВАННОЕ СОСТОЯНИЕ РАСТЯГИВА-  
ЕМЫХ МЕХАНИЧЕСКИ НЕОДНОРОДНЫХ СВАР-  
НЫХ СОЕДИНЕНИЙ С РЕАЛЬНЫМ ПЛОСКИМ  
ШВОМ В УСЛОВИЯХ ПЛОСКОЙ ДЕФОРМАЦИИ

Резюме

В настоящей работе получены зависимости для определения интенсивности деформаций в зонах перегрузки сварного соединения с мягким и жестким швом. Оптимальная высота валика мягкого шва получена из равенства интенсивности деформаций в центральном его сечении и в зоне концентрации напряжений (в начальной точке валика). Валик твердого шва увеличивает интенсивность деформаций в зонах перегрузки сварного соединения.

Received November 18, 2004

DOI: 10.5755/j02.mech.12902