

## Optimization of linear precise movements

R. Bentkus\*, A. Kasparaitis\*\*, A. Šukys\*\*\*

\*State Tax Inspectorate of Lithuania, Vasario 16-th str. 15, LT-01514 Vilnius, Lithuania, E-mail: rbentkus@vmi.lt

\*\*Vilnius Gediminas Technical University, Basanavičiaus str. 28, LT-03224 Vilnius, Lithuania, E-mail:

a.kasparaitis@bsp.lt

\*\*\*Vilnius Gediminas Technical University, Basanavičiaus str. 28, LT-03224 Vilnius, Lithuania, E-mail: a.sukys@bsp.lt

### 1. Introduction

Elimination of the impact of linear movement errors to the total error of measuring device is one of the most important subjects in the development of precise measuring equipment, e.g. high accuracy coordinate measuring machines, length measuring devices, comparators for the calibration of standard block gauges and scales [1, 2]. It may be achieved by producing linear movement assemblies and single components of high accuracy, or by compensating errors of movement [2, 3]. The latter method is more effective and advanced. Both ways are used to achieve extra high accuracy, e.g. in the development of linear standards.

Various methods for error compensation are used, but their effectiveness depends on the character of errors of the movement. The compensation of high frequency errors is more complicated and results in a residual error. Random errors are kept uncompensated when methods of calculating compensation are used.

Thus, it is important to obtain the objective information about accuracy characteristics of different constructions of linear movement assemblies to be used for the creation of mentioned precise measuring equipment. The stability of linear movement trajectory, its limited values and other parameters are also very important. The information about the cooling effect of the air jet outgoing from the air bearing (support) on thermal deformation of guide-way and, accordingly, on the accuracy of linear movement is significant as well.

### 2. Evaluation criteria

The analysis of functional characteristics was made and the following criteria were evaluated for the optimal selection of bearings.

- Straightness, stability and character of the trajectory of the measuring unit movement. It is important to estimate what frequency components dominate in the evaluation of movement character.

- Parameters of rigidity.
- Stability of accuracy parameters.

Values of friction forces, their dependence on the movement velocity, the complexity of assembling and adjusting, required conditions of maintenance, the influence on the environment and other units of measuring equipment, durability, production cost were evaluated as subsidiary criteria.

Different kinds of bearings-rolling, aerostatic, hydrostatic, hydrodynamic, boundary lubrication and mixed lubrication sliding bearings – are being used for precise

equipments.

Hydrostatic bearings were not included in this research because a special equipment to maintain bearings by compressed oil is need, draining of the bearing oil leakage is complicated, variation of temperature balance may be undefined.

An unacceptable floating up of the unit on lubricant film at the beginning of motion in the precise equipment, the dependence of friction force on the velocity are specified for the hydrodynamic bearings.

Boundary and mixed lubrication bearings were not investigated there also, because of the absence of experience to design it and provide the uniformity of movement with the prevention of stick-slip phenomenon.

Thus, only rolling and aerostatic bearings (supports) were chosen for investigations.

The exactitudes of bearings were evaluated by the following statistical characteristics of movement errors.

- Standard deviation.
- Spectral density.
- Covariate function.

Errors are determined by the dispersion of the trajectory points in the perpendicular directions to the unit movement axis.

The standard deviation specifies the dependence of a random component of the error of linear motion on the unit position, i. e. this error is defined in various positions of movable measuring unit. The analysis of standard deviation diagrams enables to evaluate the stability of movement, to find positions where maximal instability of moving unit happens, to identify instability causes, to look for means to eliminate them [4, 5].

The results of measurements are:  $\delta_{ij}$ ,  $i=1, 2, \dots, M$  (values of displacements),  $j=1, 2, \dots, N$  (realizations).

The standard mean deviation is defined by this equation

$$S_i = \left\{ \frac{1}{N} \sum_{j=1}^N (\delta_{ij} - \bar{\delta}_i)^2 \right\}^{\frac{1}{2}} \quad (1)$$

here  $\bar{\delta}_i = \frac{1}{N} \sum_{j=1}^N \delta_{ij}$ ,  $i=1, 2, \dots, M$ .

The character of errors and reasons of their occurrence are determined, the possibilities to evaluate and eliminate them are achieved during the analysis of spectral density. The evaluation is performed by period-diagram  $I(v)$  of errors [6, 7]

$$I(\nu) = \frac{1}{2\pi M} \left| \sum_{k=1}^M (\bar{\delta}_k - \mu) e^{-ik2\pi\nu} \right|^2 \quad (2)$$

here  $0 \leq \nu \leq \frac{1}{2}$ ,  $\mu = \frac{1}{M} \sum_{k=1}^M \bar{\delta}_k$ ,  $k=1, 2, \dots, M$  is an average by the displacement,  $\bar{\delta}_k$  are results of experiment,  $\nu$  is a frequency.

The integrated period-diagram  $F(\nu)$  is the proper characteristic to show obviously the relative influence of different components of the error harmonics on general error

$$F(\nu) = \int_0^{\nu} I(\nu) d\nu, \quad 0 \leq \nu \leq \frac{1}{2} \quad (3)$$

Covariant function  $R_k$  is used to determine whether the relation between error values appears in different positions of unit

$$R_k = \frac{1}{M} \sum_{i=1}^{M-k} \bar{\delta}_i \bar{\delta}_{i+k} \quad (4)$$

here  $i=1, 2, \dots, M$ ,  $k=1, 2, \dots, M-1$

### 3. Subject of the research

Experiments were performed with the different units of linear movement. Fig. 1 shows the unit to investigate parameters of movement on the air bearings (supports) consisting of three triangularly displaced bearings in the horizontal plane 1 and two bearings in the vertical plane 2. Analogous bearings 3 and 4 from the opposite sides of guideway were mounted on the elastic elements and displaced by one on each plane to preload the assembly.

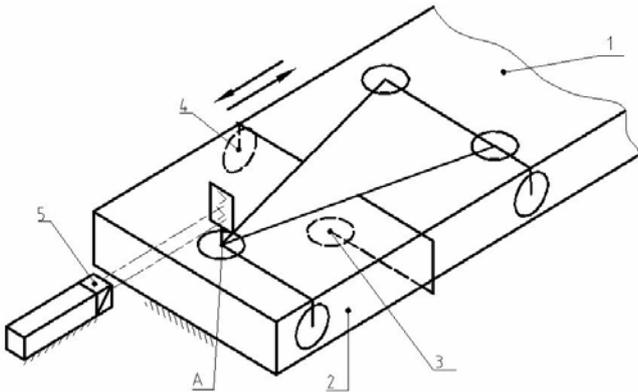


Fig. 1 Unit of linear movement with aerostatic bearings

Aerostatic bearings (supports) of different constructions were also investigated. These bearings comprise individual feed elements consisting of round pockets, microgrooves and the inner volume specially formed.

The scheme of unit for investigation of rolling bearings (supports) is similar to the scheme shown in the Fig. 1. Special rolling bearings containing a heavy outer ring were produced to prevent high frequency movement errors, which can appear as a result of elastic deformations during rolling. This is characteristic feature of the bearing

which outer ring thickness is normal. The run-out of outer ring of this bearing during rotation does not exceed  $1 \mu\text{m}$ .

Based on the current experience, the hardened, grinded and finish lapped steel guideway was produced to explore the investigation unit comprising mentioned rolling bearings. The deviation of straightness of working surfaces along the all length was less than  $2 \mu\text{m}$ , the surface roughness  $R_z=0.32 \mu\text{m}$ , the hardness - 55 HRC.

A lapped guideway from hard gabbro-diabase granite was used for the investigation of the unit comprising aerostatic bearings. Deviations of its working surface were close to those of the steel guideway.

The trajectory of unit motion was evaluated using the trajectory of one point *A* near one bearing and straightness of motion was evaluated from the projection of this trajectory to the vertical plane. So, linear oscillations in the vertical plane were accepted as the value of the motion straightness.

The straightness of movement was measured by laser interferometer 5 with the set of optical elements.

### 4. Results of experiment

After performing experiments with rolling bearings, it appears that the deviations of movement straightness are much more larger than deviations of straightness of the working surface of the guideway. High frequency components from the run-out of the bearing outer ring and alternating contact deformations are included as well. Sources of these errors are form deviations of bearing elements, lack of surface entirety, defects and contaminations of surfaces. Straightness deviation of the movement is up to  $4 \mu\text{m}$  for the mentioned errors of the bearing and the guideway, i.e. for the run-out  $1 \mu\text{m}$  of the bearing outer ring and  $2 \mu\text{m}$  deviation of the guideway straightness. Also it appeared that during the experiment hysteresis takes place when the motion direction changes. Redistribution of forces in the bearing during the motion reversing was the reason of the hysteresis. It increases considerably because under real conditions it is impossible to adjust exactly the plane of rotation of each bearing to the direction of unit movement, regardless the fact that special means for the bearings orientation were used. Angular displacements of carriage up to  $0.3$  angular sec. were measured additionally during reversal.

Aerostatic bearing does not have a direct contact with the working surface of the guideway, because of the film of compressed air. Therefore high frequency components of form errors and local defects on the guideway working surface are averaged during the movement of the aerostatic bearing on the guideway. Investigations with different bearing constructions show that the degree of mentioned averaging depends on the relation of longitudinal periods of the guideway form errors and the periods of inequality of the air film rigidity. The averaging increases by increasing dimensions of bearings and uniformity of the air film rigidity into its and the bearing pocket whole volume.

Low frequency form errors dominate in the lapped precise guideways. Therefore, the projection of the measured point movement trajectory of assembly with air bearings into the vertical plane is near to the contour obtained by perpendicular sectioning of the working surface. This trajectory has a monotonic changing character without

components of high frequency. The uniformity of air film rigidity depends on the parameters of air supply elements, their number, positions and uniformity.

The comparison of characteristics of motion straightness errors between rolling and aerostatic bearings illustrates the mentioned conclusions.

Diagrams of Fig. 2 present the dependence of standard deviation of motion in vertical plane on the displacement of mentioned two assemblies with different bearings. They show clearly that deviations are up to 5 times bigger in the case of rolling bearings. Deviations differ significantly during the displacement. Frequency components of rolling bearings deviations is higher as well.

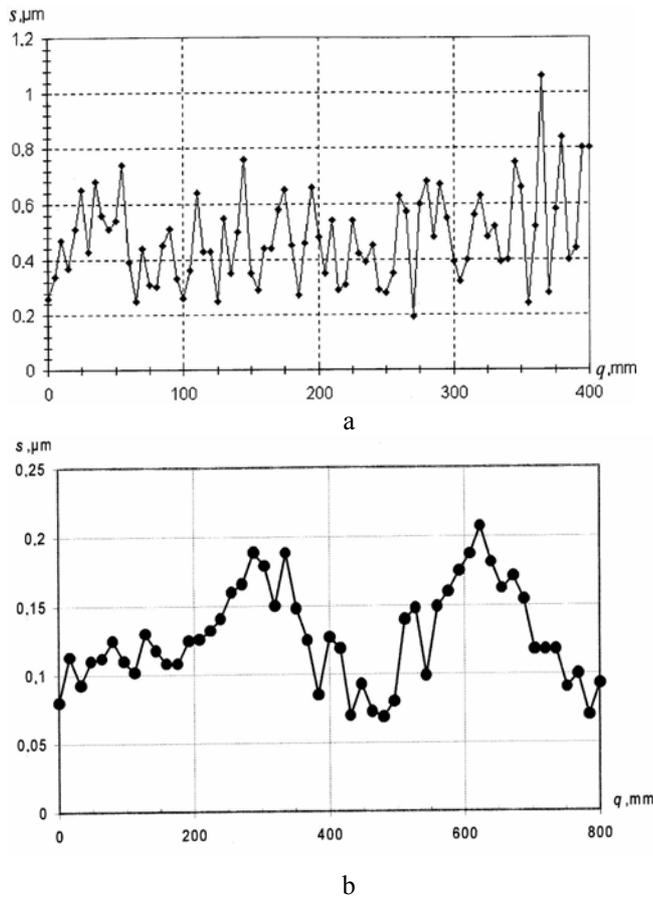


Fig. 2 Dependence of standard deviation in vertical plane on the displacement  $q$  for assemblies with rolling (a) and aerostatic (b) bearings

Fig. 3 shows the period-diagrams of straightness errors of the mentioned assemblies.

The period-diagram (a) shows maximal values of errors for the assembly with rolling bearings when frequencies are  $\nu_1=0.012$ ,  $\nu_2=0.025$ ,  $\nu_3=0.07$ ,  $\nu_4=0.3$ . The first two peaks of the period-diagram appear for low frequency components that are due to form errors of the guideway. The third peak coincides with the frequency of two turns of the outer bearing ring (on the rotational frequency of separator). The frequency of the fourth peak coincides with low frequency component of the outer bearing ring form errors.

The second case, i.e. the assembly with aerostatic bearings, gives the clearly expressed maximum on the period-diagram at the frequency  $\nu_1=0.008$ , which corresponds to the dominated component of the form error of the

guideway.

A graphical chart of integrated period-diagram for the assembly with rolling bearings is shown on the Fig. 4.

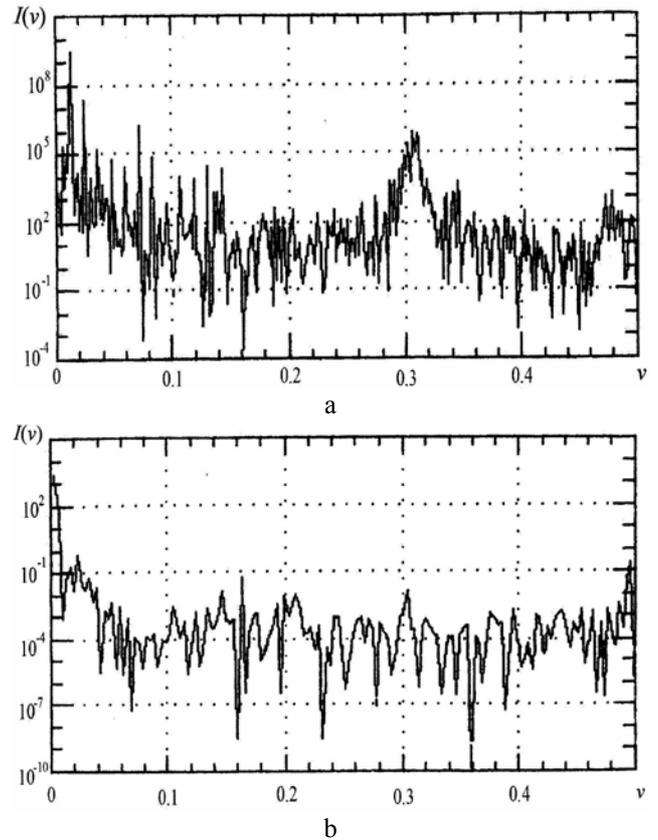


Fig. 3 Period-diagrams  $I(\nu)$  of motion errors for assemblies with rolling (a) and aerostatic (b) bearings

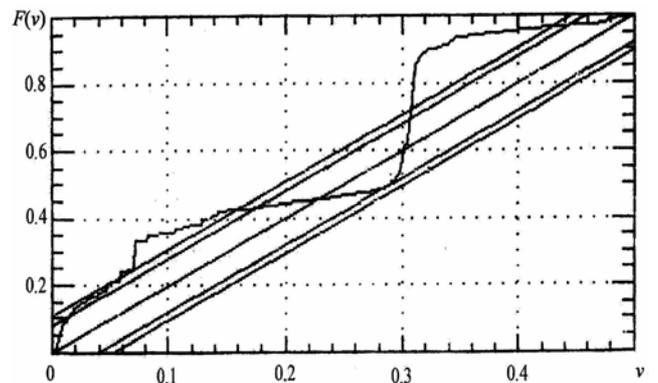


Fig. 4 Integrated period-diagram  $F(\nu)$  of motion errors for assembly with rolling bearings

This diagram illustrates a relative influence of components associated with bearing rotation on the motion error. This period-diagram is calculated after removing harmonics corresponded to frequencies  $\nu_1$  and  $\nu_2$ , i.e. after elimination of guideway form errors.

Correlation between form errors  $\Delta f$  of the guideway and motion errors  $\Delta$  of assembly was explored. This correlation is illustrated by an example of one realization.

Fig. 5 shows measured values of the total motion error, without the elimination of components correlated with form errors of the guideway, i.e. components with frequencies  $\nu_1$  and  $\nu_2$ , were included. This figure contains

the graph of motion errors approximated by algebraic spline and also a graph of measured form errors. Identity of the last two graphs is evident. The standard deviation of approximated function values from the values of form error is  $S_{PF}=0.06 \mu\text{m}$ .

It allows to make a conclusion that thermal influence of air bearing's air jet on the deformation of the guideway is unsubstantial.

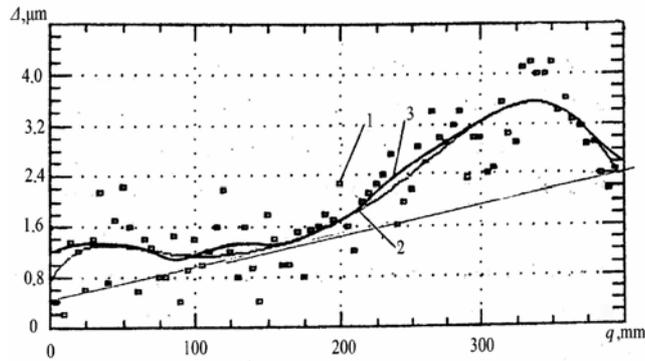


Fig. 5 Deviations of motion straightness (1), their approximation by the algebraic polynomial (2) and guideway form error (3) diagrams for the assembly with rolling bearings

The value of additional error which appears during rolling is up to  $2 \mu\text{m}$ . It is evident from Fig. 6 which shows this high frequency component of the motion error.

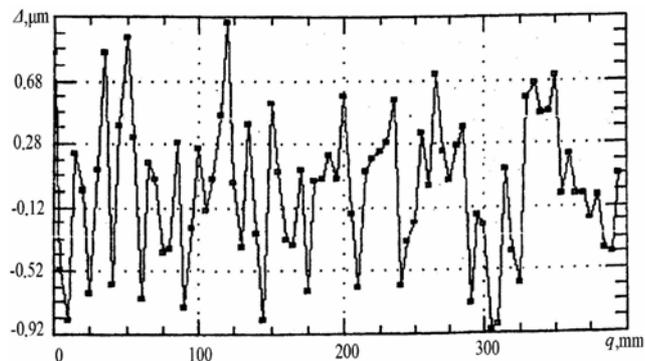


Fig. 6 Dependence of high frequency error of motion straightness on the displacement  $q$

Fig. 7 shows the period-diagram  $I(\nu)$  of the mentioned movement rolling straightness error the components of which correspond to the frequencies  $\nu_1$  and  $\nu_2$ , (guideway form errors) are eliminated. The graph shows the frequency component  $\nu_3=0.13$  which corresponds to the frequency of rotation of the outer ring of one bearing. The frequency  $\nu_6=0.47$  corresponds to the high frequency component of form error and to the frequency of deformations in the contact zone of the outer ring. Frequencies  $\nu_3=0.07$  and  $\nu_4=0.3$  are seen also and are described previously as form error components of balls and outer ring.

So, the main factors which influence on motion error of the investigated assembly are: form errors of the guideway (frequencies  $\nu_1=0.012$  and  $\nu_2=0.025$ ), low frequency form errors of the outer ring (frequency  $\nu_4=0.3$ ), variation of dimensions of balls (frequency  $\nu_3=0.07$ ). Others important factors are: high frequency form errors of the outer ring surface and deformations in the contact zone

(frequency  $\nu_6=0.47$ ), eccentricity of the outer surface of the outer ring in respect to its rotation axis (frequency  $\nu_5=0.13$ ).

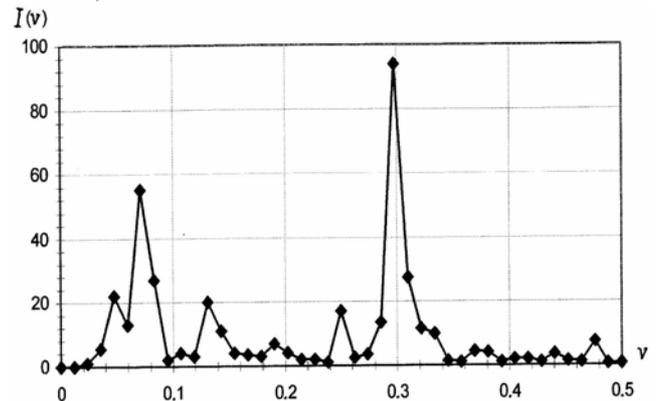


Fig. 7 Period-diagram  $I(\nu)$  of one realization of motion straightness errors for the assembly with rolling bearings

Fig. 8 shows that there is a small difference between the trajectory of an assembly containing aerostatic bearings and low frequency form (straightness) errors of the working surface of the guideway. These deviations do not exceed the value of  $0.15 \mu\text{m}$  on the whole measuring length and may be evaluated as errors of tests.

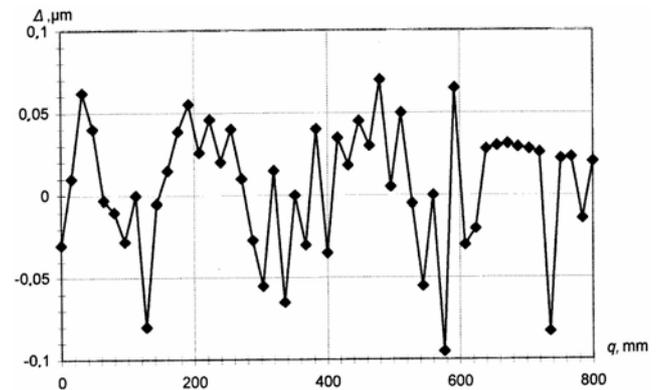


Fig. 8 Deviations of motion straightness errors from low frequency form errors of guideway on the displacement  $q$  for the assembly with aerostatic bearings

Variations of rigidity of air film and bearing clearance slightly influences on the total error as well. The reason of this is the variation of parameters of the compressed air (pressure, cleanliness, wetness etc.) supplied to the air bearing and micro-deviations of working surfaces of guideway (roughness, local defects).

Investigations of errors correlation confirm the described results.

#### 4. Conclusions

The proposed method provides an objective evaluation of the character of linear and rotational movement errors and the determination of their stability as a function of assembly displacement.

Linear movement errors in the assembly with aerostatic bearings are identical to low frequency form errors of the guideway surfaces. Assemblies with rolling bearings are impacted by high frequency errors the values

of which are near to the values of low frequency errors.

The main sources of high frequency errors in the investigated assemblies with rolling bearings are found to be: low and high frequency form errors and deformations in contact zones of the outer ring, inequality of bearing balls, eccentricity of outer surface of outer ring to its rotation axis.

## References

1. **Bentkus, R., Kasparaitis, A., Šukys, A.** Statistinė matavimo koordinatinėmis mašinomis kompensuojamųjų paklaidų analizė.-Matavimai.-Kaunas: Technologija, 2004, Nr.3(31), p.17-20.
2. **Abbe, M., Takamasu, K.** Modeling of spatial constraint in CMM error for uncertainty estimation.-Proc. of the 3th Euspen International Conference-Eindhoven, The Netherlands, May 2002, v.2, p.637-640.
3. **Weelers, W.G., Schellekens, P.H.J.** Compensation for dynamic errors of coordinate measuring machines. -Measurement, 1997, v.20, No3, p.197-209.
4. **Kay, Steven M.** Fundamentals of Statistical Signal Processing. Estimation Theory.-Prentice Hall PTR, 1993.-625p.
5. **Howard, Roy M.** Principles of Random Signal Analysis and Low Noise Design: the Power Spectral Density and its Applications.-Wiley-IEEE Press, 2002.-328p.
6. **Bendat, Julius S., Piersol, Allan G.** Random Data: Analysis & Measurement Procedures. -Wiley-Interscience, 2000.-594p.
7. **Oppenheim, Alan V., Schaffer, Ronald W., Buck, John R.** Discrete-Time Signal Processing. -Prentice Hall, 1999.-870p.

R. Bentkus, A. Kasparaitis, A. Šukys

## TIESIŲJŲ PRECIZINIŲ JUDESIŲ OPTIMIZAVIMAS

### Reziumė

Aprašytas precizinio judesio stabilumo eksperimentinio ir analitinio įvertinimo metodas. Vertinimo kriterijais pasirinkti judesio paklaidų vidutinis kvadratinis

nuokrypis, spektrinis tankis, kovariacinė funkcija. Taikant šį metodą ištirti precizinio tiesiojo judesio mazgai su aerostatiniais ir riedėjimo guoliais, nustatytas jų judesio paklaidų pobūdis, pagrindiniai jų atsiradimo šaltiniai. Atliktas įvairių paklaidų parametrų palyginimas.

R. Bentkus, A. Kasparaitis, A. Šukys

## OPTIMIZATION OF LINEAR PRECISE MOVEMENTS

### Summary

Here the method of experimental and analytic evaluation of precise movement stability is described. Standard deviation, spectral density, covariate function of motion errors are accepted for the criteria of evaluation. Assemblies of precise linear movement with aerostatic and rolling bearings have been investigated by using this method. The character of movement errors, main sources of their occurrence are also defined here. The article includes comparisons of different errors as well.

Р. Бенткус, А. Каспарайтис, А. Шукис

## ОПТИМИЗАЦИЯ ПРЕЦИЗИОННЫХ ЛИНЕЙНЫХ ПЕРЕМЕЩЕНИЙ

### Резюме

В статье представлен метод экспериментальной и аналитической оценки стабильности прецизионного движения. Критериями оценки приняты среднеквадратическое отклонение, спектральная плотность, ковариационная функция погрешностей движения. При помощи этого метода исследованы узлы перемещения прямолинейного движения, содержащие подшипники (опоры) как аэростатические, так и качения. Определен характер погрешностей движения и основные источники их возникновения. Сопоставлены параметры разных погрешностей.

Received October 12, 2004