

# Convergence of lift force calculation of a tapered wing using non-linear section data

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## Nomenclature

$AR$  - wing aspect ratio;  $b_0$  - wing root chord;  $b_t$  - wing tip chord;  $C_l$  - section lift coefficient;  $C_d$  - section drag coefficient;  $C_m$  - section moment coefficient;  $C_L$  - wing lift coefficient;  $C_D$  - wing drag coefficient;  $\alpha$  - wing angle of attack;  $\alpha_{ekv}$  - section angle of attack;  $\Delta\alpha$  - change of plate's angle of attack during iteration process;  $\phi$  - angle, which defines the speed of result approaching;  $\Delta$  - percentage difference between calculated sectional lift coefficient and that one, from airfoil data.

### Subscripts

$max$  - maximum;  $min$  - minimum;  $pl$  - defines a plate (element of wing model);  $i$  - wing section number.

### Superscripts

\* - defines airfoil data;  $k$  - iteration number.

## 1. Introduction

Prandtl's lifting line theory is the base of the most methods used for calculations of lifting force change of flat, low sweep angle, moderate and high aspect ratio wings. In 1934 Tani [1] has developed the first successful technique for handling nonlinear section lift-curve slopes in Prandtl's lifting line theory formulation. NACA report of Sivells and Neely [2] in 1947 provides a detailed description of Tani's method for unswept wings with arbitrary planform and airfoil lift-curve slopes. They apply this method for the analysis of wings up to stall, i.e. until angle of attack of the wing at any section has  $C_l$  equal to  $C_{lmax}$ .

Numerical solutions of the Prandtl's lifting line theory were also developed and still in use. Mostly known researches, which use these methods, are the researches of McCormick [3] and Anderson et al. [4]. Also the method is reflected in a resent research of Phillips and Snyder [5].

The researches of Mutteperl [6] and Weisinger [7] made a base for so called Finite-Step method or Vortex Step method, which also was developed from Prandtl's lifting line theory. Later Campbell [8] and Blackwell [9] simplified their method. These methods presented the first attempts to couple sectional (two-dimensional) viscous results with inviscid wing (three-dimensional) theory. Most resent research, which reflects their work, was made by Barnes [10]. He presents Semi-Empirical Vortex Step Method, which includes empirical adjustments in lifting line position and shape.

Piszkin and Levinsky [11] developed a nonlinear lifting-line method based in part on the iterative method originally conceived by Tani [1]. Their method differs

from Prandtl's classical LLT (Lifting Line Theory) in the implementation of the boundary condition. Tseng and Lan [12] developed entirely different approach to the use of nonlinear section data. In their method, the reduction at any given wing section is determined by the difference between potential flow solution and the viscous  $C_l$  from the nonlinear section of  $C_l$ - $\alpha$  curve.

In all methods, which are using nonlinear section data, the main objective is that for the final solution of three-dimensional flow,  $\Gamma$  distribution across the span would be consistent with the distribution of effective  $\alpha$  for each section and the  $C_l$  and  $C_m$  for each section would be consistent with the effective  $\alpha$  for that section and the section  $C_l$ - $\alpha$  and  $C_m$ - $\alpha$  data. Mukherjee, Gopalarathnam, and Kim [13] achieved that condition by finding the effective reduction in the camber distribution for each section along the span.

Another possibility to take into account non-linear section data in wing calculation is presented in the research of Jacob [14]. His method combines inviscid 3D-lifting surface theory with 2D-airfoil theory, which includes boundary layer calculations and a displacement model for rear separation.

In the research presented, lifting-line solution method by Horseshoe vortex elements was used, which differs from the methods mentioned above, by the nonlinear section data implementation technique. Here an idea of Lasauskas [15] was developed into separate method for wing lift and drag force calculation using nonlinear section characteristics.

## 2. Some features of calculation procedure

Calculation method used here is described in more details in reference [16]. The method combines numerical solution of lifting-line theory and a special approach to evaluate nonlinear section lift data. A lattice of Horseshoe vortex elements models the wing. The reduction of section lift in viscous flow in comparison with the potential flow is estimated through the change of zero lift angle of attack. This change is determined by an iterative procedure from a known sectional non-linear data. In general, calculated sectional data can be used as well as an experimental data.

The iterative procedure allows predicting wing lift at critical and post critical angles of attack for the wings with moderate sweep and high aspect ratios.

As the research showed, the range of possible angles of attack and Reynolds numbers only depends on

available airfoil data.

During the iteration procedure, calculated sectional lift coefficients are compared with available airfoil lift coefficients, which could be also calculated from the airfoil theory (X-FOIL [17] used for the presented case) or obtained by an experiment

$$\Delta = \left| \frac{C_{li}^{*k} - C_{li}^k}{C_{li}^{*k}} \right| \cdot 100 \quad (1)$$

$\Delta$  decreases to zero through the iteration process, which assures the solution convergence. Maximum number of iterations was limited to 50 and the limit of difference  $\Delta$  (Eq. (1)) was set to 0.03%.

Another feature of calculation procedure is that it allows getting the results directly after each iteration, which ensures to follow up the changes until the optimal result is obtained.

Special research was made in order to evaluate the condition of result approach. The value of  $\tan\phi$  was introduced

$$\tan\phi = \frac{2\pi C_{li}^k}{2\pi\alpha_{pli}^k - C_{li}^k} \quad (2)$$

Eq. (2) was derived with the assumption, that  $\Delta\alpha_i^k$  is small comparing it to  $\alpha_i^k$  between two iterations. Talking about  $\tan\phi$ , two cases are important.

1.  $\phi=0^\circ$  situation is not defined. It means that the method is not suitable to calculate such angles of attack at which zero lift coefficients is obtained at any section.

2.  $\phi=90^\circ$  the result is obtained directly, without iteration process. It is only theoretical case, indicating that slope of  $C_l=f(\alpha)$  at appropriate section has reached  $2\pi$ .

With an assumption that nonlinear section characteristic could be approximated by a straight line between two iterations due to small step of  $\Delta\alpha^k$ , a meaning of result approach speed could be given to the value of  $\phi$ , as its magnitude indicates how fast the result will be reached for the appropriate slope of nonlinear lift curve of the airfoil.

As it could be found from Eq. (2), sign change of  $\tan\phi$  from positive to negative indicates the event when the slope of  $C_l=f(\alpha)$ , calculated from linear equation, exceeds that one of an infinite plate. That kind of event ends in a non-converged situation. Overrun of sectional angle of attack (more than calculated angle of attack) confirms that case. Also it should not be confused with an overrun of critical angle of attack of appropriate airfoil, as the overrun of critical angle of attack of airfoil during the iteration process does not denote the non-converged event.

### 3. Results for a tapered wing

All figures present calculation results for a tapered wing without sweep:  $b_0/b_t=2$ ,  $AR=6$ . The wing was constructed of a variable section, starting with NACA 0018 at the root and ending with NACA 0009 at the tip.

Fig. 1 presents lift and drag distribution of the calculated wing. Calculations were made using sectional characteristics obtained by means of X-FOIL [17] at Reynolds number  $3.09 \times 10^6$ .

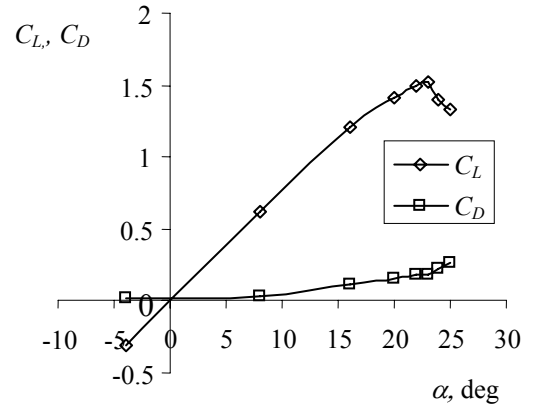


Fig. 1 Lift and drag distribution of calculated wing

Effect of iterative process is presented in Figs. 2 and 3. Figs. 2 and 3 show only that part of curves, where the iterative process has the mostly significant effect. It is evident from the figures, that the iterative process has the highest influence on the angle of attack near and above stall. Here the section characteristics is pure non-linear and the highest difference from the linear part appears.

Fig. 4 presents the dependence of calculation accuracy on the number of iterations. Proves the fact, that iteration procedure has the highest influence at the angles of attack near and beyond stall. It should be said, that a required accuracy was reached within 10 iterations for the angles of attack up to  $22^\circ$ , which is almost the critical angle of attack for the calculated wing. More iterations are needed for the higher angles.

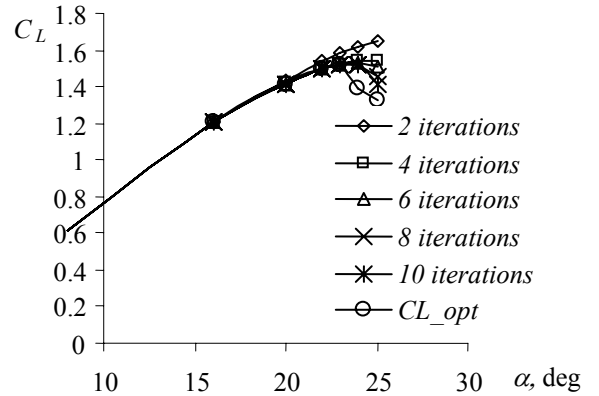


Fig. 2 Effect of iterative process on wing  $C_L$  calculation.  $CL_{opt}$  presents the final solution of wing lift

Average values of  $\tan\phi$  for the calculated wing could be found in Fig. 5. This figure and successive figures present results for a half span wing. It was noticed that  $\tan\phi$  almost does not depend on the iteration number (except for a non-converged cases), but is different for each wing section. The highest value for the calculated case was reached at the section Nr. 4. The lowest value noticed at the wing tip. Also  $\tan\phi$  depends on the angle of attack, what is presented in Fig. 5.

Fig. 8 presents iteration procedure results for  $\alpha=24^\circ$ . Here the curves of  $\tan\phi$  show, that after 18 iterations non-converged situation was reached at section 4

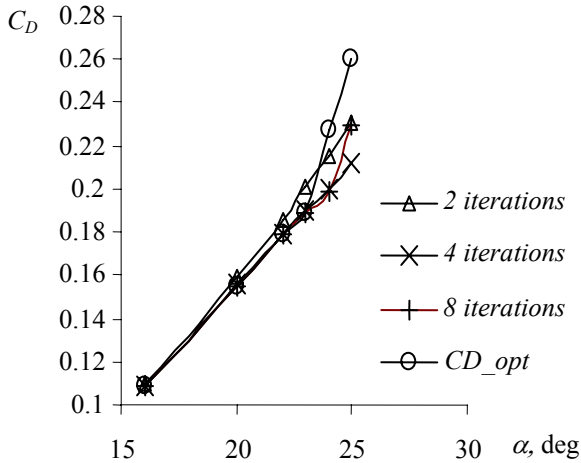


Fig. 3 Effect of iterative process on  $C_D$  calculation.  $CD_{opt}$  presents the final solution of wing drag

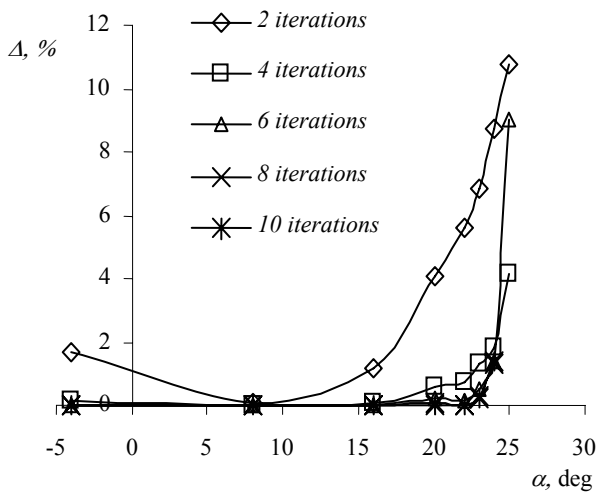


Fig. 4 Dependence of accuracy on number of iterations

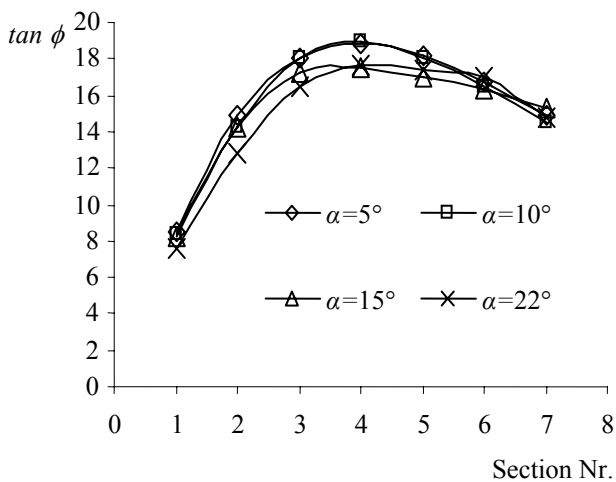


Fig. 5  $Tan \phi$  dependence on section position

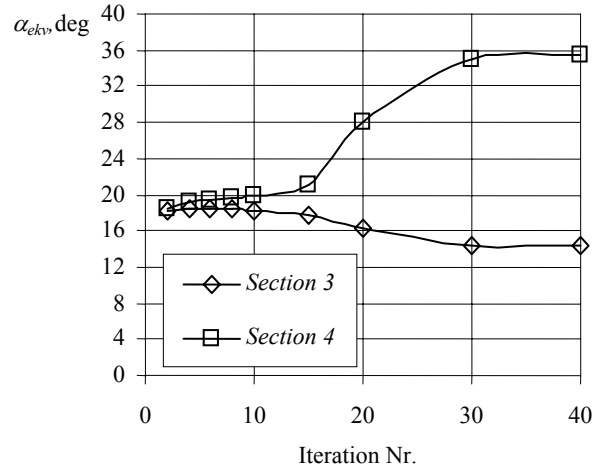


Fig. 6 Change of section angle of attack through iteration process at  $\alpha=24^\circ$

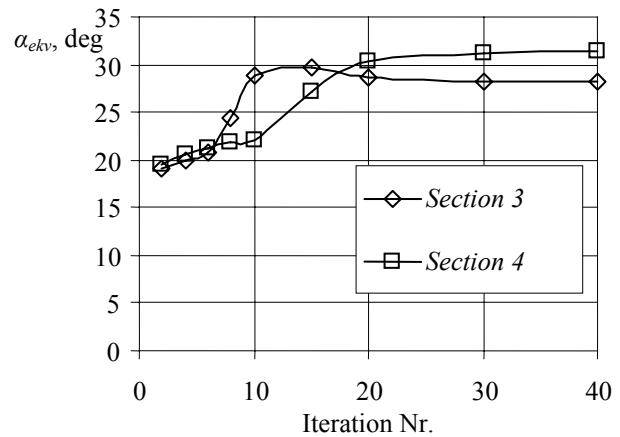


Fig. 7 Change of section angle of attack through iteration process at  $\alpha=25^\circ$

(it is indicated by  $\tan \phi$  sign change). At  $\alpha=25^\circ$  additionally non-converged state was reached at section 3, which is presented in Fig. 9. Also it should be noticed, that at the higher angle non-converged state was reached more quickly: after 8 iterations at section 3 and after 13 at section 4. These cases are explained by an overrun of sectional angle of attack more than the calculated angle of attack of complete wing. As shown in figures 6 and 7, that angle was overrun at iterations, coincident to those predicted by  $\tan \phi$ .

It should be mentioned, that before the non-converged state is reached, some sections of wing reaches the set accuracy. Despite the fact that calculated lift coefficients of the sections not going to converge also are in small difference from the airfoil data (e. g. 1.4% for  $24^\circ$  angle of attack), these results should be used with extreme caution as the lift coefficient of each wing section has the influence on the other sections.

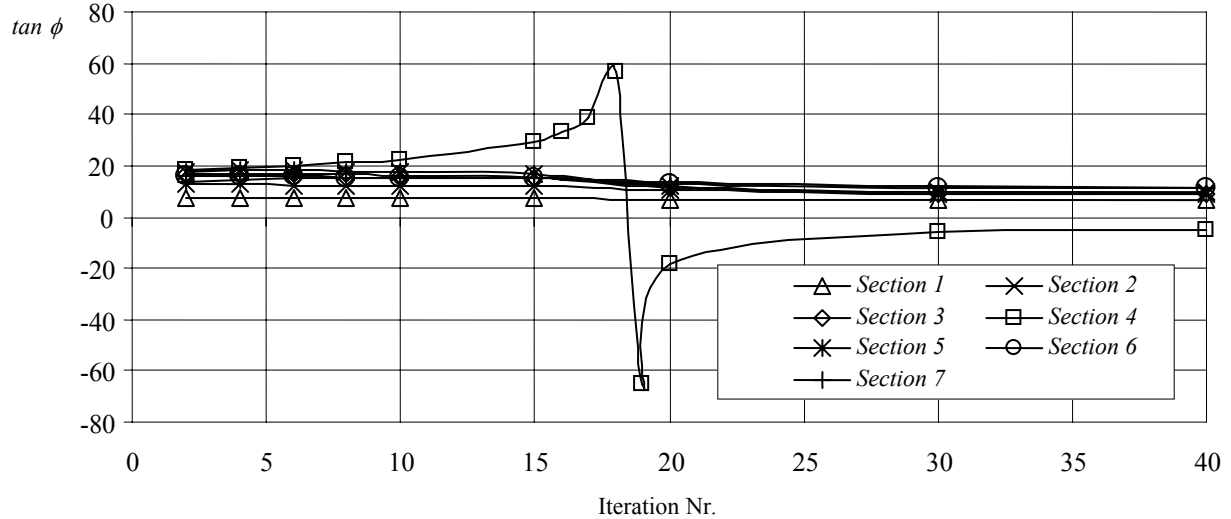


Fig. 8  $\tan \phi$  dependence on iteration number at  $\alpha=24^\circ$

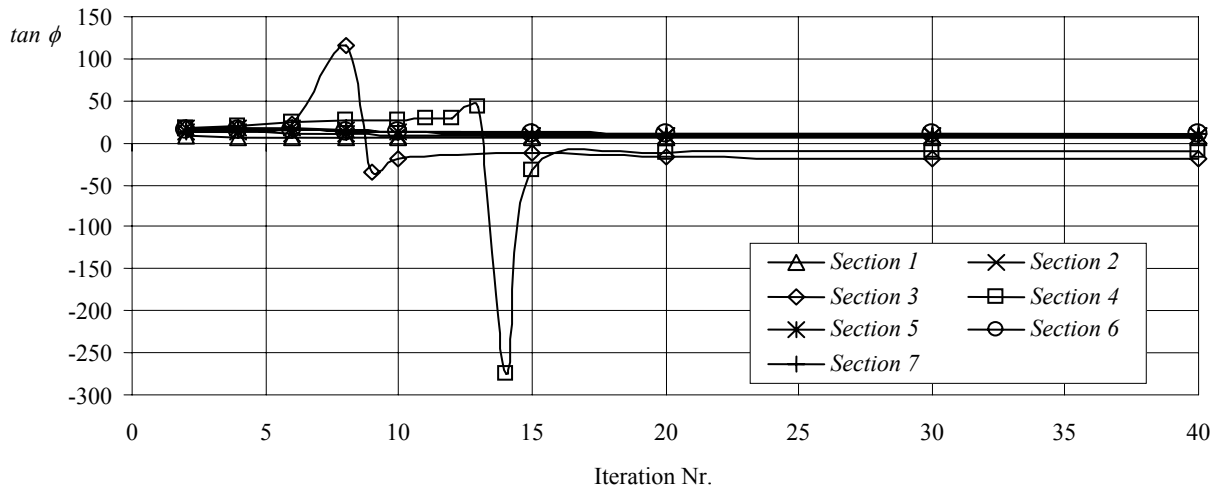


Fig. 9  $\tan \phi$  dependence on iteration number at  $\alpha=25^\circ$

#### 4. Conclusions

Same kind of research was made for another plan forms of wing (rectangular, swept), which proved the same features of the iteration procedure. So the results, presented here, could be rated as general for the calculation method used. As the results show, difference  $\Delta$  decrease together with the number of iterations. Here it should be noted that after a number of iterations, the calculated sectional lift coefficient comes equal to the non-linear lift coefficient of the infinite wing. After this situation is reached further calculation has no significance, i.e. further increase of the iteration number has no influence on the result. For the set limit of the difference, that number of iteration depends on wing geometry, section number and calculated angle of attack.

The process of iteration itself has a higher influence on the lift coefficient difference at the angles of attack near critical. For the tapered wing, this difference tends to decrease in the span wise direction from the wing tip to the root.

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TRAPECINIO SPARNO KELIAMOSIOS JĖGOS  
SKAIČIAVIMO, NAUDOJANT NETIESINES PJŪVIO  
CHARAKTERISTIKAS, KONVERGENCIJA

Re z i u m ė

Straipsnyje pateikiamos sparno keliamosios jėgos skaičiavimo metodo, naudojant netiesines pjūvio charakte-

ristikas, ypatybės. Pagrindinis tikslas - nustatyti iteracinio proceso įtaką sprendinio tikslumui. Šis darbas yra pradėto tyrimo metodo galimybėms ir pritaikymo riboms nustatyti dalis. Daugiau dėmesio skiriama skaičiavimo tikslumui. Nustatyta iteracinio proceso įtaka trapecinio sparno skaičiavimo rezultatams.

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CONVERGENCE OF LIFT FORCE CALCULATION OF  
A TAPERED WING USING NON-LINEAR SECTION  
DATA

S u m m a r y

The article presents some peculiarities of lift force calculation method using nonlinear section data. The main task is to determine an influence of iteration procedure on solution accuracy. This evaluation is a part of initiated research on potentials and application limits of the method. Set of accuracy is described in more details. Influence results of iteration procedure are presented for lift calculation of a tapered wing.

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КОНВЕРГЕНЦИЯ ПОДЪЕМНОЙ СИЛЫ, ПРИ  
РАСЧЕТЕ ТРАПЕЦЕИДАЛЬНОГО КРЫЛА  
ИСПОЛЬЗУЯ НЕЛИНЕЙНЫЕ ХАРАКТЕРИСТИКИ  
СЕЧЕНИЙ

Р е з ю м е

В работе представлены особенности расчета подъемной силы крыла, используя нелинейные характеристики сечений. Поставлена цель определить влияние итерационного процесса на точность результата. Эта работа представляет часть исследования с целью определения возможностей и границы применения данного метода. Особое внимание уделено точности расчета. Представлены результаты влияния итерационного процесса при расчете подъемной силы трапецидального крыла.

Received October 11, 2004