

Survival probability of existing structures

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1. Introduction

Sections, bars and connections of structures belong to particular members for which the only possible failure mode exists. Structural members (beams, plates, columns, slabs) may be treated as separate systems representing their multicriteria failure mode. They consist of series, parallel or mixed connected stochastically dependent elements. For successful ordinary and scheduled maintenance of the structures it is expedient to know the revised values of residual survival probabilities of particular and structural members of the existing structures. This revision, including any correction of the partial safety factors, may be based on the principle of service-proven extreme actions.

Service loads and other actions of great intensity help to convince us not to allow rough human design and construction errors. Besides, these actions assist to some reductions of member resistance uncertainties and to the corrections of the survival reliability degrees of the structures. An additional new information about unfavourable actions cannot be used in the assessment of the capacity and serviceability of the structures, but it may be successfully used in the reliability prediction of members subjected to infrequent extraordinary gravity and lateral (horizontal) service or climate actions.

Unfavourable action effects of the members may be treated as an effective measure in the revised reliability predictions of the existing structures when they are confirmed by quality statistical information data on extraordinary service and climate actions [1]. That is why it is recommended to collect this information regularly and to fix it in observation logs. Updated statistical information may help to refine the probability density functions of member resistances if the new data has a small variance [2].

There are some limited attempts to transfer the approaches of deterministic limit state design for new structures to the existing ones. It was suggested to make the partial load factors more exact taking into account the

inspection data, system behaviour peculiarities and risk category extents of the existing structures [3]. This semi-probabilistic reliability checking format cannot be acknowledged as a universal method. Therefore, in design practice it is recommended to make use of the information on known service-proven loading situations using probabilistic approaches [2, 4, 5].

In spite of rather developed up-to-date concepts of reliability, hazard and risk theories, it is difficult to implant the probability-based methods in structural design practice due to the shortage of methodological approaches and applied mathematical models. The intention of the presented paper is to introduce structural to engineers and researchers the new practical probabilistic approaches in revised reliability predictions of the particular members of existing structures subjected not only to permanent and sustained variable, but also to intermittent rectangular pulse actions.

2. Analysis of time-dependent structural safety

2.1. Safety margin of particular members

Particular members of the structures are generally treated as the main design components in the probabilistic reliability analysis. In spite of a short period of recurrent intermittent pulse service and other actions, they belong to persistent design situations. The safety margin as time-dependent performance process of particular members in persistent design situations may be expressed as

$$M(t) = g[\theta, X(t)] \quad (1)$$

In the context of the analysis of the revised survival probabilities of members in design practice, the process (1) may be presented in more convenient form

$$M(t) = \theta_R R - \theta_{g_1} S_{g_1} - \theta_{g_2} S_{g_2} - \theta_{q_s} S_{q_s} - \theta_{q_1} S_{q_1}(t) - \theta_{q_2} S_{q_2}(t) \quad (2)$$

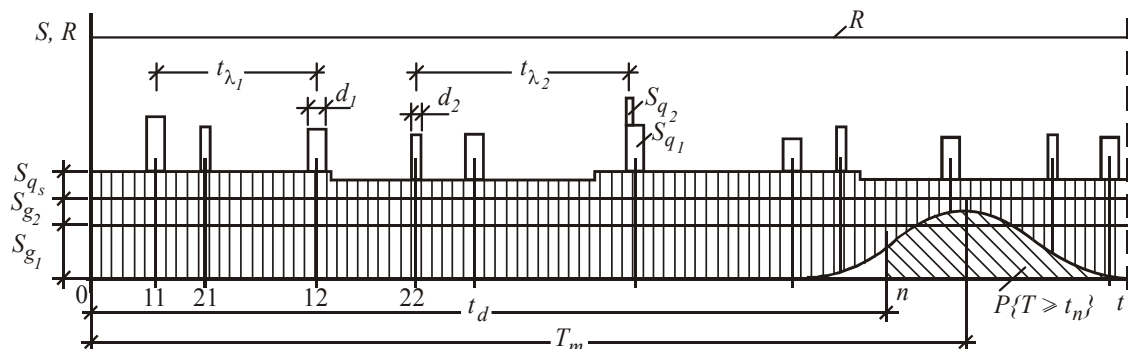


Fig. 1 Model for time-dependent reliability analysis of particular members

here S_{g_1} , S_{g_2} and S_{q_s} are the action effects caused by execution (pre-use) g_1 and service (use) permanent g_2 and sustained variable q_s loads; $S_{q_1}(t)$ and $S_{q_2}(t)$ are the action effects caused by extraordinary gravity q_1 and lateral (horizontal) q_2 actions (Fig. 1); θ_i is additional random variable representing uncertainties of models which give the values of member resistance and action effects. The additional random variables may be expressed by their probability density functions or simply as their means and standard deviations.

The design codes (ASCE 7 1995, EN 1990 2002) ignore the presence of two different by nature action effects S_{g_1} and S_{g_2} caused by the mass of structures and other permanent units depending on execution work peculiarities. At the same time, a nice verification of the limit states of structures by the partial factor method can lose its meaning due to erroneous values of permanent action effects.

According to ISO 2394 [6] and EN 1990 [7] recommendations, a Gaussian distribution law is to be used for permanent actions. Lognormal, Weibull and gamma distributions may be convenient for sustained variable actions. For simplicity, a Gaussian distribution may also be used for these actions [6]. Intermittent extraordinary service and seismic actions may be assumed to be distributed by exponential and Gumbel distributions [8]. Climate actions may be modelled by a Type 1 extreme value distribution [2].

The maximum sum value of sustained and extraordinary variable actions may be modelled by Gumbel distribution [9]. However, for the sake of simplified but rather exact probabilistic analysis, it is more expedient to present Eq. (2) in the form

$$M(t) = R_c - S(t) \quad (3)$$

Here

$$R_c = \theta_R R - \theta_{g_1} S_{g_1} - \theta_{g_2} S_{g_2} - \theta_{q_s} S_{q_s} \quad (4)$$

is the conventional resistance of members which may be modelled by Gaussian distribution

$$S(t) = \theta_{q_1} S_{q_1}(t) + \theta_{q_2} S_{q_2}(t) \quad (5)$$

is the joint action effect caused by intermittent rectangular pulse processes of transient extraordinary actions.

2.2. Survival probability of particular members

When structures are subjected to intermittent extraordinary gravity or lateral actions, the random safety margin process in design practice may be treated as a random sequence. In the case of the only extraordinary action effect, Eq. (3) may be presented as

$$M_k = R_c - S_k, \quad k = 1, 2, \dots, n \quad (6)$$

Here $n = t_d \lambda$ is the recurrence number of the extraordinary action effect during the design working life of structures t_d , where λ is the action renewal rate. The co-

efficient of autocorrelation of random sequence cuts is:

$$\rho_{kl} = \text{Cov}(M_k, M_l) / (\sigma M_k \times \sigma M_l) \quad (7)$$

where $\text{Cov}(M_k, M_l) = \sigma^2 R_c$ and σM_k , σM_l are their autocovariance and standard deviations.

When the action effect (5) is caused by two stochastically independent actions, three stochastically dependent random sequences of safety margins should be considered as follows

$$M_{1k} = R_c - \theta_{q_1} S_{q_1k}, \quad k = 1, 2, \dots, n_1 \quad (8)$$

$$M_{2k} = R_c - \theta_{q_2} S_{q_2k}, \quad k = 1, 2, \dots, n_2 \quad (9)$$

$$M_{3k} = R_c - \theta_{q_1} S_{q_1k} - \theta_{q_2} S_{q_2k}, \quad k = 1, 2, \dots, n_3 \quad (10)$$

Here the reiteration number of the coincident actions q_1 and q_2 during the design working life t_d may be calculated by the formula

$$n_3 = t_d (d_1 + d_2) \lambda_1 \lambda_2 \quad (11)$$

where d_1 and d_2 are durations of extraordinary actions (Fig. 1). The coefficient of cross-correlation of the random sequences is

$$\rho_{ij} \approx \sigma^2 R_c / \left[(\sigma^2 R_c + \sigma^2 S_i)^{1/2} (\sigma^2 R_c + \sigma^2 S_j)^{1/2} \right] \quad (12)$$

The integrated time-dependent survival probability of particular members may be expressed as

$$P_s = P\{T \geq t\} = P\{M_1(t) > 0 \cap M_2(t) > 0 \cap \cap M_3(t) > 0\} \approx P\{M_3(t) > 0\} \quad (13)$$

The value of P_s may be calculated by rather sophisticated and practical method of transformed conditional probabilities. When action effects of the members are caused by one and two extraordinary actions, the long-term survival probabilities are calculated, respectively, by the equations

$$P_s = P\{M_1(t) > 0\} = \rho_{kl} P_k + (1 - \rho_{kl}) \times \exp[-n_1 (1 - P_k)] \quad (14)$$

$$P_s = P\{\cap M_i(t) > 0\} \approx P_{s1} P_{s2} P_{s3} \times \left[1 + \rho_{23} \left(\frac{1}{P_2} - 1 \right) \right] \left[1 + \rho_{123} \left(\frac{1}{P_1} - 1 \right) \right] \quad (15)$$

Here the coefficients of correlation ρ_{kl} by Eq. (7) and ρ_{23} by Equation (12); $\rho_{123} = 0.5(\rho_{12} + \rho_{13})$ is the transformed coefficient of correlation

$$P_k = \int_0^{\infty} f_{R_c}(x) F_S(x) dx \quad (16)$$

is instantaneous survival probability of the members, where f_{R_c} is density function of the conventional member resistance by Equation (4) and $F_S(x)$ is distribution function of the extraordinary action effects or their combinations.

3. Revised survival probability of the members

3.1. Account of truncated distribution

Service actions may be treated as use-proven proof loads [10]. According to these suggestions (Fig. 2, a), the probability density function of revised resistance of particular members at the time t may be presented in the form

$$f_{R_r}(x, t) = \frac{f_R(x) F_{S_c}(x, t)}{\int_{-\infty}^{\infty} f_R(x) F_{S_c}(x, t) dx} \quad (17)$$

Here $f_R(x)$ is the primary density function; $F_{S_c}(x)$ is the cumulative distribution function of the joint action effect as

$$S_c = \theta_g S_g + \theta_q (S_{q_s} + S_{q_e}) \quad (18)$$

The dominator of Equation (17) is a normalizing factor of revised density functions.

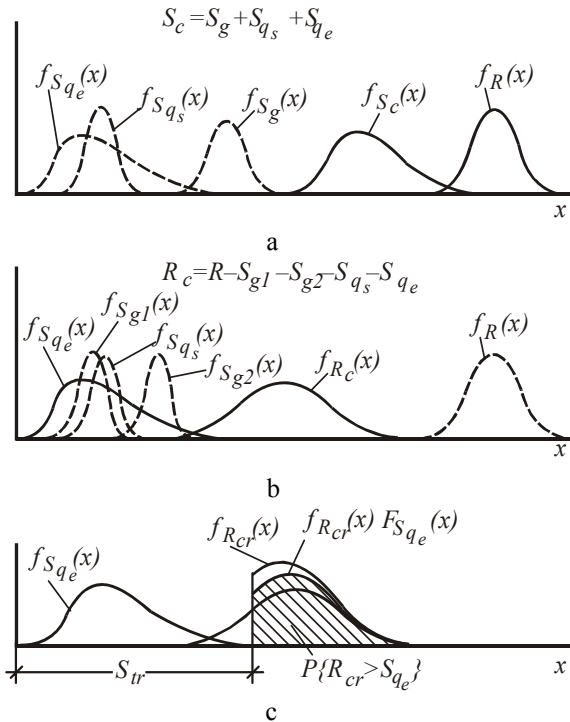


Fig. 2 Models for instantaneous safety analysis of particular members by Hall (a) and Kudzys (b, c) using their conventional action effect S_c and conventional resistance R_c , respectively

The investigations [11, 12] showed that it is not simple to adopt the presented approach in engineering practice due to complicated probability distributions of joint action effects. It would be better to use the revised conventional resistance of members (Fig. 2, b) the density function of which may be presented as follows

$$f_{R_{cr}}(x, t) = \frac{f_{R_c}(x) F_S(x, t)}{\int_{-\infty}^{\infty} f_{R_c}(x) F_S(x, t) dx} \quad (19)$$

where $F_S(x, t)$ is the distribution function of the extraordinary action effect $S = \theta_s S_{q_e}(t)$.

When the extraordinary action effect can be admitted as the deterministic value S_{tr} , the revised density function of conventional resistance of particular members may be treated as a truncated one presented as

$$f_{R_{c,tr}}(x, t) = f_{R_c}(x) / [1 - F_{R_{c,tr}}(x, t)] \quad (20)$$

The mean and variance of the revised resistance $R_{c,tr}$ may be calculated by the formulae

$$R_{c,tr,m} = R_{cm} + \lambda \sigma R_c$$

$$\sigma^2 R_{c,tr} = \sigma^2 R_c \left[1 + \lambda \left(1 + \frac{S_{tr} - R_{cm}}{\sigma R_c} \right) \right]^2 \quad (21)$$

Here the conversion factor of statistical moments is

$$\lambda = \varphi \left(\frac{S_{tr} - R_{cm}}{\sigma R_c} \right) / \left[1 - \Phi \left(\frac{S_{tr} - R_{cm}}{\sigma R_c} \right) \right] \quad (22)$$

where $\varphi(\bullet)$ and $\Phi(\bullet)$ are density and distribution functions of standard normal distributions.

When t_d and t_{ex} are the design and existing working lives, the revised survival probability of the members during the residual service life $t_{res} = t_d - t_{ex}$ is

$$P_{sr} = P\{T \geq t_{res}\} = \int_{-\infty}^{\infty} f_{R_{cr}}(x, t_{ex}) F_S(x, t_{res}) dx$$

$$= \rho_{kl} P_{kr} + (1 - \rho_{kl}) \exp[-n(1 - P_{kr})] \quad (23)$$

Here P_{kr} is the revised instantaneous survival probability calculated by Eq. (16); n is the recurrence number of the extraordinary action effect during the period of time t_{res} ; ρ_{kl} is the coefficient of correlation of cuts of the random sequence presented by Eq. (7). Standard deviation of recurrent in due course extraordinary actions may be considered as constant value. Therefore, this coefficient may be calculated by the formula

$$\rho_{kl} = 1 / \left(1 + \sigma^2 S / \sigma^2 R_{c,tr} \right) \quad (24)$$

where $\sigma^2 S$ and $\sigma^2 R_{c,tr}$ by Equation (21) are the variances of the action effect $S = \theta_s S_{q_e}(t)$ and member resistance.

When the deterministic joint action effect S_{tr} caused by permanent, sustained and extraordinary loads is less than the design value of member resistance $R_d \approx R_c - 3.1 \sigma R_c$, the revised survival probability, practically, is the same as primary one and the account on truncated distributions is aimless.

3.2. Account of Bayes theorem

When additional information is gathered about the existing particular members, it might be applied to improve the prior data on structural reliability of the members using Bayesian statistics. According to the recommendation presented in [4, 13], the updated probability of failure can be expressed as follows:

$$P_{fr} = P[g(\boldsymbol{\theta}, \mathbf{X}) < 0 | H] = \frac{P\{g(\boldsymbol{\theta}, \mathbf{X}) < 0 \cap H > 0\}}{P\{H > 0\}} \quad (25)$$

where $g(\boldsymbol{\theta}, \mathbf{X}) = M$ is random safety margin; $H > 0$ is the event of inspection or current maintenance results. Practically, the event $H > 0$ shows a successful withstanding of particular members to service-proven extreme actions when their resistances at the time t_{ex} are treated as deterministic values.

The analysis of Eq. (25) disclosed that it is difficult to get updated quantitative reliability parameters of existing members and to present relevant partial safety factors for deterministic design methods due to some conditionality of the event $H > 0$ and its correlation with the event $g(\boldsymbol{\theta}, \mathbf{X}) < 0$ [2, 5].

When an extraordinary action effect may be treated as deterministic proof force S_{tr} , two safety margins of particular members at the time t_{ex} should be considered as follows

$$M = \theta_R R - \theta_g (S_{g_1} + S_{g_2}) - \theta_q S_{q_s} - \theta_q S_{q_e} \quad (26)$$

$$H = R_k - \theta_g (S_{g_1} + S_{g_2}) - \theta_q S_{q_s} - S_{tr} \quad (27)$$

where R_k is characteristic resistance of the members. When an indispensable condition $H > 0$ is in force, two instantaneous survival probabilities of considered members may be calculated as follows

$$P_k = P\{M > 0\} = \int_0^{\infty} f_{R_c}(x) F_{S_{q_e}}(x) dx \quad (28)$$

$$P_{tr} = P\{H > 0\} = \Phi(H_m / \sigma H) \quad (29)$$

Here the conventional resistance R_c is calculated by Equation (4), and standard deviation of the function H by Eq. (27) may be presented in the following form

$$\sigma H = \left[\sigma^2 (\theta_g S_{g_1}) + \sigma^2 (\theta_g S_{g_2}) + \sigma^2 (\theta_q S_{q_s}) \right]^{1/2} \quad (30)$$

The revised instantaneous survival probability of existing members may be presented as

$$\begin{aligned} P_{kr} &= P\{M > 0 | H\} = \frac{P\{M > 0 \cap H > 0\}}{P\{H > 0\}} = \\ &= \frac{P\{M > 0\} P\{H > 0 | M > 0\}}{P\{H > 0\}} \end{aligned} \quad (31)$$

According to the method of transformed conditional probabilities, Eq. (31) may be rewritten as follows

$$P_{kr} \approx P_k \left[1 + \rho \left(\frac{1 - P_k}{P_{tr}} \right) \right] \quad (32)$$

where P_k is the survival probability by Eq. (28)

$$\rho = \rho(M, H) = \sigma H / \sigma M \quad (33)$$

is the coefficient of correlation between the safety margins (26) and (27).

The Eq. (32) is in force, when the probability $P_k > P_{tr}$. When the probability P_{tr} by Eq. (28) exceeds the value P_k by Eq. (29), the revised survival probability of existing members is

$$P_{kr} \approx P_k \left[1 + \rho \left(\frac{1}{P_{tr}} - 1 \right) \right] \quad (34)$$

The revised survival probability of the members during their residual service life is calculated by Eq. (23), where P_{kr} is their instantaneous survival probability by Eqs. (32) or (34).

The analysis of revised survival probabilities by the truncated distribution and Bayes theorem approaches leads approximately to the same results.

4. Conclusions

It is rather difficult to predict or give an objective and quantitative survival probability of load-carrying existing structures. When unfavourable service-proven extreme service loads or climate and seismic actions were fixed during the existing period of load-carrying structures, their structural safety may be assessed and predicted rather exactly by engineering probability-based approaches. The revised values of survival probability of particular members of the structures during their residual service life may be analysed by the new conceptions based on the truncated probability distribution and Bayes theorem approaches.

Generally, the extreme values of action effects of particular members of the structures are caused by intermittent rectangular pulse processes of gravity and lateral actions. Therefore, the performance processes or safety margins of the members in design practice may be treated as random sequences the cuts of which correspond to extreme service events and situations.

It is recommended to use the principle of conventional resistance of particular members and the method of transformed conditional probabilities in the revised analysis of safety margin process and residual instantaneous and long-term survival or failure probabilities of load-carrying structures.

The revised survival probability of particular members of existing structures leads to the correction of their technical service lives and allows us to avoid both unexpected failures and unfounded premature repair costs.

References

1. **Mori, V. & Ellingwood, B.** Reliability based service-life assessment of aging concrete structures.-J. of Structural Engineering, ASCE, 1993, v.119 (No5), p.1600-1621.
2. **Melchers, R.E.** Structural reliability analysis and prediction.-Chichester: John Wiley & Sons, 1999.
3. **Allen, D.E.** Limit state criteria for structural evaluation of existing buildings.-Canadian Journal of Civil Engineering, 1991, No18, p.995-1004.
4. **Madsen, H.** Model updating in reliability theory.-Proc. ICASP5 Vancouver, BC, 1987, p.564-577.
5. **Ellingwood, B.R.** Reliability-based condition assessment and LRFD for existing structures.-Structural Safety, 1996, v.18 (No2/3), p.67-80.
6. ISO 2394. General principles on reliability for structures. Switzerland, 1998.
7. EN 1990:2002 E. Eurocode-basic of structural design. CEN, Brussels, 2002.
8. **Vrouwenvelder, A.C.V.M.** Developments towards full probabilistic design codes.-Structural Safety, 2002, v.24 (No2-4), p.417-432.
9. **Ellingwood, B.R. & Tekie, P.B.** Wind load statistics for probability-based structural design.-J. of Structural Engineering, ASCE, 1999, v.125 (No4), p.453-463.
10. **Hall, W.B.** Reliability of service-proven structures.-J. of Structural Engineering, ASCE, 1998, v.114 (No3), p.608-624.
11. **Stewart, M.G.** Proof loads, construction area and the reliability of service proven structures. Probabilistic Mechanics and Structural Reliability, ASCE, Eds. D.M.Frangopol & M.D.Grigoriu, 1996, p.226-229.
12. **Stewart, M.G.** Effect of construction and service loads on reliability of existing RC structures.-J. of Structural Engineering, ASCE, 2001, v.127 (No10), p.1232-1235.
13. **Tang, W.H.** Probabilistic updating in reliability analysis.-J. of Testing and Education, ASTM, 1973, v.1 (No6), p.459-467.

A. Kudzys

ESAMŲ KONSTRUKCIJŲ TVERMĖS TIKIMYBĖ

Reziumė

Aptariama sudaromųjų (pjūvių, strypų, jungių) ir struktūrinių (sijų, plokščių, kolonų) elementų patikslinta ištvermės tikimybė kaip patikslintas esamų konstrukcijų patikimumo parametras. Analizuojami sudaromųjų elementų sutartiniai atspariai, saugos ribos ir tvermės tikimybės. Nagrinėjama dviejų trūkiųjų stačiakampių pulsinių poveikių įtaka ilgalaikei elementų tvermės tikimybei.

Rekomenduojama šią tikimybę vertinti transformuotų sąlyginių tikimybių metodu. Patikslintos elementų tvermės per likusią eksploatacijos laikotarpį tikimybės prognozavimas grindžiamas nupjautinio skirstinio ir Bajeso teoremos metodais.

A. Kudzys

SURVIVAL PROBABILITY OF EXISTING STRUCTURES

S u m m a r y

Revised survival probability of particular (sections, bars, connections) and structural (beams, plates, columns, slabs) members as a residual reliability parameter of existing structures is discussed. The conventional resistances, safety margins and time-dependent survival probabilities of particular members are analysed. The effect of two intermittent rectangular pulse action processes on a long-term survival probability of the members is considered. It is recommended to assess this probability by the method of transformed conditional probabilities. The revised survival probability prediction of the members during their residual service life is based on the truncated distribution and Bayes theorem approaches.

A. Кудзис

ВЕРОЯТНОСТЬ БЕЗОТКАЗНОСТИ СУЩЕСТВУЮЩИХ КОНСТРУКЦИЙ

Р е з ю м е

Обсуждается уточненная вероятность безотказности составных (сечений, стержней, соединителей) и структурных (балок, плит, колонн) элементов как уточненный параметр надежности существующих конструкций. Анализируются приведенные сопротивления, границы безопасности и вероятности длительной безотказности элементов. Рассматривается влияние двух прерывистых прямоугольных пульсирующих воздействий на вероятность длительной безотказности элементов. Рекомендуется оценить данную вероятность методом трансформированных условных вероятностей. Прогнозирование уточненной вероятности безотказности элементов в течение остального периода их эксплуатации основано на подходе усеченного распределения и теоремы Байеса.

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