

Interacting heat transfer processes in water droplets

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Nomenclature

a – thermal diffusivity, m^2/s ; B – Spalding transfer number; c_p – mass specific heat, $J/(kg\ K)$; D – mass diffusivity, m^2/s ; Fo – Fourier number; I – intensity of radiation, $W/(m\ ster)$; k_{σ} – spectral index of absorption; L – latent heat of evaporation, J/kg ; m – mass flux density, $kg/(s\ m^2)$; n – number of the term in infinite sum; n_{σ} – spectral index of refraction; Nu – Nusselt number; q – heat flux density, W/m^2 ; p – pressure, Pa ; R – radius of a droplet, m ; r – coordinate, m ; s – freely selected direction; T – temperature, K ; w – velocity, m/s ; β – Brewster's angle; γ – angle evaluating sphere's geometry; λ – thermal conductivity, $W/(m\ K)$; χ_{σ} – spectral radiant absorption, m^{-1} ; θ – subsidiary function; ρ – density, kg/m^3 ; τ – time, s ; ω – wave number, m^{-1} .

Subscripts: c – convective; g – gas; e – equilibrium evaporation; f – phase transformations; k – conductive; l – liquid; n – normalized; r – radiation; R – droplet surface; v – vapour; Σ – total; 0 – initial state; ∞ – far from a droplet.

Superscripts: $+$ – external side of a droplet; $-$ – internal side of a droplet.

1. Introduction

Two-phase flows, containing gas and liquid droplets, are common in various energetic technologies. Such two-phase flows are present in heat generating devices, where water is being sprayed in order to reduce pollution of atmosphere, in gas turbines, in contact type heat exchangers and other devices. Efficiency of the above-mentioned devices significantly depends on the intensity of transfer processes in two-phase flows. Optimal control of the technological processes is possible only when the regularities of transfer processes are familiar. Research of heat and mass transfer in two-phase gas droplet flows is generally called a droplet problem.

Liquid droplets intensively warm and evaporate in high temperature gas. Equilibrium and unsteady modes of droplet evaporation are possible. During equilibrium evaporation mode all energy supplied to the droplet, is used to evaporate it. It is enough to solve the external droplet problem to determine the intensities of droplet heating and evaporation. During unsteady evaporation mode only a part of energy supplied to the droplet from outside is used for its evaporation. Other part heats the droplet

$q_{\Sigma}^{\pm} \equiv q_f + q_h$. Droplet heating intensity is indicated by a total heat flux density, taken inside the droplet $q_h \equiv q_{\Sigma}^-$. It is important to determine temperature of an evaporating droplet. The peculiarities of transfer process interaction depend on this temperature. Often premises are made,

which allow neglecting the internal droplet problem. The premise of liquid temperature constancy is crude; it is not valid in the case of a droplet unsteady evaporation mode, when relatively cold droplets get into high temperature medium and therefore heat intensively. In the case of a droplet equilibrium evaporation mode the premise of liquid temperature constancy is more valid, though it should not be accepted freely, because temperature of an evaporating droplet surface is defined by the interaction of energy flows on it. Instant value of the droplet surface temperature can be calculated according to the condition of the balance of energy flows on the droplet surface

$$\bar{q}_{\Sigma}^+(\tau) + \bar{q}_{\Sigma}^-(\tau) + \bar{q}_f^+(\tau) = 0 \quad (1)$$

So, a droplet surface temperature always changes during unsteady evaporation mode. During equilibrium evaporation the change of the droplet surface temperature is determined by the boundary conditions [1]. The character of this temperature change depends on the boundary conditions of heat and mass transfer. A droplet surface temperature, at which all energy supplied to the droplet, is used for its evaporation, is called equilibrium evaporation temperature $T_{R,e}$. The main objective of the droplet problem is to calculate the intensities of an evaporating droplet heating and phase transformations.

The main objective of this study is to review shortly the methods of heat transfer in a droplet; to evaluate achievements and shortcomings; to construct a universal method of analytical - numerical research of heat transfer process' interaction in evaporating semitransparent liquid droplets and to examine the dependence of state change of water droplets on their heating way.

2. Research of heat transfer in droplets

Researches of the droplet problem have been carried out more than a hundred years. These researches cover such wide spectrum of heat and mass transfer problems that it is impossible to overview all research methods in this study, even very briefly. It can be generally mentioned that steady heat transfer in low temperature media was examined in early studies. Such research did not require the evaluation of internal droplet problem. The results of such research are generalized in [2]. Rapid technological development required deeper and more detailed cognition of combined heat and mass transfer. Unsteadiness and intensive interaction under the influence of Stefan hydrodynamic flow and selective absorption of radiant flow are common features of the transfer processes, taking place in two-phase flows, containing droplets and gas. Detail review of convective transfer researches is presented in [3]. The results of this study indicate the achievements of the droplet problem in the case when it is possible to neglect

radiant flow influence. Though the latter premise limits the application of the research [3] in evaluation of transfer interaction in radiant two-phase flows. Hence, the methods of research of heat and mass transfer in gas-droplets flow can be classified according to their attitude towards the necessity of radiant flow evaluation.

The D^2 model of a droplet heating and evaporation can be considered as a classical one. The model is constructed under the condition that the square of an evaporating droplet diameter decreases in time linearly; droplet temperature remains constant [4]. The importance of the internal droplet problem firstly was shown in “rapid mixing limit” and “infinite conductivity” models [5], in which the change of an evaporating droplet temperature was predicted. Though practical application of those models is limited by the condition of a droplet isothermality, which is being assured by stating that, the intensity of heat transfer processes is sufficient to maintain constant temperature in the entire warming droplet volume. “Conduction limit” model states that heat inside a droplet is transferred only by conduction and temperature in the entire droplet surface is constant [6]. Under conditions of usual thermal processes Archimedean forces do not evoke circulation in non-isothermal droplets [7], though drag forces, which occur due to different droplet and gas velocities, evoke liquid circulation, as liquid Peclet number $Pe_l > 5$ [8]. A “vortex” model was developed for the evaluation of this effect [9,10], as liquid circulation in a droplet is presented as Hill vortex, stating that Peclet number significantly decides coincidence of isotherms with stream lines in a droplet. The aforementioned models in the study [8] were specified, evaluating liquid circulation by an effective thermal conductivity $\lambda_e = \lambda_l \cdot k_c$. The coefficient, which evaluates liquid circulation, is the function of Peclet number. This method of the internal droplet problem solution allows evaluating the influence of heat transfer with circulating liquid on a droplet energetic balance. Though it is not possible to evaluate the status of every droplet cross-section.

Radiation effects are significant in high temperature two-phase flows. Evaluation of radiation is one of the most complicated problems of two-phase fluid mechanics. The possibilities of its numerical investigation were expanded by a rapid progress of computational technique. Therefore, the evaluation of peculiarities of spectral radiation in semitransparent liquids [11-19] and interaction of combined transfer process interaction in droplets [1,20-23] is positive aspect of a modern approach to the droplet problem. Volumetric radiation absorption in droplets using geometric optics approach was examined in the study [11]. It is stated that absolutely black body radiation acts on liquid from its surroundings. Significantly nonlinear radiation absorption was verified. Radiation absorption in water, n-heptane and n-decane droplets was calculated using electromagnetic theory method in [12]. It is shown that geometric optics approach can result in a sensible error of radiation absorption distribution. In the study [13] the influence of radiation absorption on a droplet heating was examined under the condition that a droplet is an isotropic homogeneous sphere. The results of the numeric research affirm radiation influence on droplet heating and evaporation processes. Local radiant flux in a liquid was examined using light ray tracing method in [14]. In this study radia-

tion absorption was modeled in semitransparent liquid sphere, slab and cylinder, as they are uniformly acted by isotropic radiant flux. The influence of a volume shape on radiation absorption process was indicated. External radiation absorption in a dispersed system of low temperature spherical volumes is modeled in [15]. Asymmetric illumination of separate particles is approximately equated to a uniform illumination from two hemispherical volumes, oriented according to the direction of spectral radiation flow. Calculations were carried out using Mie radiation theory. The results of numeric research of radiation absorption in water and diesel droplets [15] confirmed the importance of liquid spectral optical properties. The integral equation, which calculates local radiant flux in a semitransparent sphere, is constructed in the study [16]. For that spectral optical density along freely chosen direction was replaced by optical density along the direction of a semitransparent sphere dimension. The advantage of this method is that liquid radiation emission and dependence of optical spectral properties on liquid temperature is evaluated. The influence of liquid temperature on energy transfer in water droplets is verified in [17]. Spectral absorption and reflection coefficients, as well as total internal reflection angles in wide range of boundary conditions were calculated in this study. The research results showed that the change of optical properties influences local radiation flux in warming water droplets.

Heating and evaporation of stagnant water droplets under the influence of monochromatic radiation was thoroughly examined in [18,19]. Possible fast heating and slow heating modes of droplets were determined. As the intensity of external radiation is high, large droplet warms rapidly without any sensible influence of conduction. In such case temperature profiles repeat radiant flux profiles in a droplet. Energetic equilibrium on the droplet surface has not been stabilized yet, though non-intensive phase transformations quickly shift to an explosive mode of droplet evaporation. External energy source dominates in this case, and it is possible to neglect droplet evaporation. During slow heating mode the influence of thermal conduction and evaporation processes is sensible. In this case a widely used condition of quasi-steady state of a droplet is valid. The existence of energy flux equilibrium on a droplet surface is based on this condition. Under conditions of asymmetric radiation temperature fields inside a droplet are close to spherically symmetric, which are characteristic to a uniform source case. Maximal temperature of a droplet center is higher than liquid saturation temperature only in a few degrees, i.e., explosive evaporation mode is not reached.

Radiation absorption can significantly distort steady temperature field in a semitransparent liquid droplet and even change the direction of conductive component of the total heat flux. That is shown in [20]. The influence of unsteady radiant-conductive transfer process interaction in initially isothermal droplet is thoroughly examined in [1,21,22]. These studies present the conception of a complex droplet problem. Very close interrelation between internal and external droplet problems is indicated. It is suggested to classify the change of a droplet state into the initial, transient and final periods according to the place of temperature field maximum. Transfer process interaction is peculiar in those periods. Though the influence of liquid circulation is neglected in these studies. Besides, a numeric

scheme requires iterative calculations, which are CPU time consuming. A very important step towards liquid circulation evaluation was made in the studies [23,24]. They contain a developed method of unsteady radiant-conductive heat transfer problem solution (presented in [21]) and indicate a possibility to use time - intermittent in numeric research scheme.

Application of the results of fundamental two-phase flow research in various CFD (Computational fluid dynamics) codes is limited by strict requirements for a droplet problem solution time. Therefore some problems of the droplet internal problem simplification and the reliability of obtained results might arise. A heat transfer model of droplets, which are influenced by combined heating is presented in the study [25]. In this case radiation absorption is calculated using Mie theory. Finite conduction of droplets and internal liquid circulation are evaluated by an effective conductivity k_e . Analytical solutions of energy equation were obtained under conditions of heat transfer coefficient α variation. In the simplest case, as α is set to be constant, a defined equation of temperature field inside a droplet is derived. As α is variable, differential equations are transformed into Volterra type second kind integral equation. Calculation results were adapted to heating and evaporation of standard diesel fuel droplets. It was determined that droplet temperature increases due to radiation influence, especially in bigger droplets. Heating and evaporation of fuel droplets using a parabolic temperature profile model was analyzed in [26]. Advanced D^2 law model evaluates heating of a droplet. This form of the D^2 law is called ellipse law and is based on elliptical presentation of the droplet diameter variation in time. Actually it is one more approach, suitable to be used in CFD codes. This approach does not require a lot of CPU time, though it does not allow evaluating the interaction of transfer processes.

Above - discussed droplet problem solution methods do not avoid limitations of their practical application: in some cases radiation is neglected, in other cases – convection. Very often the interaction of unsteady transfer processes is not taken into account. The influence of droplet liquid initial temperature and its heating way on the droplet unsteady evaporation process is not evaluated sufficiently. In this paper the main attention will be paid to those problems.

2. Research methodology

In the case of interacting radiant - conductive heat transfer in a semitransparent liquid droplet an internal droplet problem is defined by a system of integral and differential equations

$$\rho c_p \frac{\partial T(r, \tau)}{\partial \tau} = \frac{1}{r^2} \frac{\partial}{\partial r} \left\{ r^2 \left[\lambda \frac{\partial T(r, \tau)}{\partial r} + q_r(r, \tau) \right] \right\} \quad (2)$$

$$\left. \frac{\partial T(r, \tau)}{\partial r} \right|_{r=0} = 0 \quad (3)$$

$$q_r(s) = \int_0^\infty \int_0^{2\pi} \int_0^\pi I_\omega(s) \sin \varphi \cos \varphi d\varphi d\psi d\omega \quad (4)$$

$$\frac{1}{c} \frac{\partial I_\omega(s)}{\partial \tau} + \frac{\partial I_\omega(s)}{\partial s} = \chi_\omega(s) [n_\omega^2(s) I_{\omega,0}(s) - I_\omega(s)] \quad (5)$$

$$\chi_\omega = 4\pi \frac{k_\omega \omega}{n_\omega} \quad (6)$$

$$\bar{n}_\omega = n_\omega - ik_\omega \quad (7)$$

Energy Eq. (2) defines the change of unsteady temperature field of a droplet in time and space. This change is evoked by the variation of conductive and radiant local heat fluxes. The condition (3) selects the case of symmetric heat transfer. Integral Eq. (4) defines local radiant heat flux density in a droplet, which depends on spectral radiant intensity, described by the expression (5). The latter depends on the intensity of light wave absorption, which is determined by a liquid spectral index of absorption, defined by the expression (6). The real part of complex spectral index of light refraction \bar{n}_ω corresponds to the spectral index of reflection n_ω ; the imaginary part corresponds to the spectral index of absorption k_ω . Complex spectral index of light refraction \bar{n}_ω describes optical properties of semi-transparent liquid. Physics of interacting heat transfer processes and their effects on each other is highly complicated. Interacting heat transfer processes deform droplet temperature field and change its gradients. That evokes direct change of conductive local heat flux. Though the influence of the transfer process interaction on radiant heat flux is indirect. It evidences as the change of liquid optical spectral properties due to liquid temperature change. In this case the intensity of spectral radiation absorption in a droplet is changed; the conditions of light reflection on the droplet surface and light ray refraction between gas and liquid is changed as well.

Evaluating the Eqs. (3)-(7), which define radiant flux density, the energy Eq. (2) can be defined as a non-linear second-degree parabolic type differential – integral equation. In general case its analytical solution is not possible. Note that a direct numeric solution of the Eqs. (2)-(7) using the method of finite differences is also highly complicated – difficulties arise due to calculation of radiant flux density divergence in the Eq. (2), indeterminacy of the selection of time and space coordinate step and selectivity of spectral radiation. Therefore a method of combined analytic – numeric research of the system (2)-(7) is selected. This method is based on experience, gained in [1,23,24] studies. The essence of the method of combined analytic – numeric research is the following: at the beginning the system of Eqs. (2) –(7) is being transformed into a system of integral equations and the scheme of iterative numeric calculations for a new system is constructed. The convergence of such system would be easy to control.

Transfer processes in a droplet and its surroundings are interrelated, hence the solution of the internal droplet problem is unambiguous for the investigative case of droplet heating, as boundary conditions will be formulated evaluating the external droplet problem. In its turn, the external droplet problem solution will be unambiguous only when the solution of internal droplet problem is known. Therefore the solution of a “complex” droplet problem is possible only using iterative calculations, when the droplet surface temperature, which is important parameter for internal and external problems, is calculated according to the condition (1), requiring energy flux balance of the droplet surface. Hence, the Eq. (1) is a bound-

ary condition of the droplet internal and external problems. It corresponds to the third degree boundary conditions, which are actually defined by the interaction of transfer processes on the droplet surface. Therefore the boundary conditions for the system of Eqs. (2) – (7) can be formed at the beginning of the research in regards to yet unknown values: droplet diameter, droplet surface temperature and spectral radiation intensity in the droplet

$$r(R, \tau) = R(\tau), T(R, \tau) = T_R(\tau), I_\omega(R, \tau) = I_{\omega, R}(\tau) \quad (8)$$

In general case the initial conditions are the following

$$T(r, 0) = T_{R,0}(r), R(0) = R_0 \quad (9)$$

The premise of the future iterative numeric research scheme allows solving the energy Eq. (2) and radiant flux density Eq. (3) independently: when solving Eq. (2), radiant flux density is set to be a known parameter; when solving the Eq. (3) – temperature field is set to be defined.

It is possible to state that the first member of the Eq. (5), which defines the change of spectral radiant intensity in time, is infinitesimal, because light propagation speed is far and away higher than thermal conduction propagation speed. Then the change of light intensity along freely chosen direction in a droplet s is easily linked to its change along the droplet coordinate r [20]

$$\frac{dI_\omega}{ds} = \pm \frac{\sqrt{r^2 - R^2} \sin \beta}{r} \frac{dI_\omega}{dr} \quad (10)$$

By introduction of symbolic denotation of optical thickness

$$\tau_{r_{j-1}}^{r_j} \equiv \int_{r_{j-1}}^{r_j} d\tau_{r_{j-1}}^{r_j} = \frac{\chi_\omega dr'}{\sqrt{1 - (r/r')^2} \sin^2 \gamma} \quad (11)$$

the Eq. (3) is transformed to the expression, convenient for construction of a numerical scheme [1]

$$\begin{aligned} q_r(r) = & 2\pi \int_0^{\pi/2} \int_0^{\infty} \cos \gamma \sin \gamma \left[I_{\omega, R} \exp(\tau_r^r) \right. \\ & + \int_r^R n_\omega^2 I_{0\omega} \exp(\tau_r^r) d\tau_r^{r'} - I_{\omega, R} \exp(\tau_R^{r \sin \gamma} + \tau_r^{r \sin \gamma}) \\ & - \int_{r \sin \gamma}^r n_\omega^2 I_{0\omega} \exp(\tau_r^{r \sin \gamma} + \tau_r^{r \sin \gamma}) d\tau_r^{r'} \\ & \left. - \int_{r \sin \gamma}^r n_\omega^2 I_{0\omega} \exp(\tau_r^{r'}) d\tau_r^{r'} \right] d\gamma d\omega \quad (12) \end{aligned}$$

here $R \sin \beta = r \sin \gamma$ and $\gamma = \pi - \varphi$.

The use of a subsidiary function:

$$\Theta(r, \tau) = r \left[T(r, \tau) - T_R(\tau) \frac{T_0(r)}{T_{R,0}} \right] \quad (13)$$

allows transforming the Eq. (2) to the form of a transfer problem, known as the Dirichlet problem

$$\frac{\partial \Theta(r, \tau)}{\partial \tau} = a_1^2 \frac{\partial^2 \Theta(r, \tau)}{\partial r^2} + f(r, \tau) \quad (14)$$

here $a_1^2 \equiv \frac{\lambda}{c_p \rho}$; the source function is unambiguously

defined by local radiant flux density in a droplet

$$f(r, \tau) = \frac{1}{r^2 c_p \rho} \frac{\partial}{\partial r} [r^2 q_r(r, \tau)] \quad (15)$$

The peculiarities of the function (13) ensure uniform conditions of unambiguity of the Eq. (14)

$$\Theta(0, \tau) = 0, \Theta(R, \tau) = 0 \text{ and } \Theta(r, 0) = 0 \quad (16)$$

Taking into account that the source function (15) in iterative calculations can be carried out according to the temperature field, determined in earlier iteration, the Eq. (2) is transformed to integral form [24]

$$\begin{aligned} T(r, \tau) = & T_R(\tau) \frac{T_0(r)}{T_{R0}} + \frac{2R}{r} \sum_{n=1}^{\infty} \sin \frac{n\pi r}{R} \\ & \times \int_0^\tau \left\{ \frac{(-1)^n T_0(r)}{n\pi T_{R0}} \frac{dT_R(\tau')}{d\tau'} - \int_0^R \left(\frac{n\pi r'}{R} \cos \frac{n\pi r'}{R} \right. \right. \\ & \left. \left. - \sin \frac{n\pi r'}{R} \right) \left[\frac{a}{R^2} \frac{dT_0(r')}{dr'} \frac{T_R(\tau')}{T_{R0}} + \frac{q_r(r', \tau')}{R^2 \rho c_p} \right] \right\} d\tau' \\ & \times \exp \left[-a \left(\frac{n\pi}{R} \right)^2 (\tau - \tau') \right] d\tau' \quad (17) \end{aligned}$$

The solution algorithm of the integral Eqs. (12) and (17) is formed by predicting control time τ_I and by dividing it into $I-1$ number of finite size time intervals; a droplet radius is divided into $J-1$ number of finite size intervals of the droplet coordinate

$$\tau_I = \sum_{i=2}^I \Delta \tau_i = \sum_{i=2}^I (\tau_i - \tau_{i-1}) \quad (18)$$

$$R_J = \sum_{j=2}^J \Delta r_j = \sum_{j=2}^J (r_j - r_{j-1}) \quad (19)$$

Time integral in the Eq. (17) and coordinate integrals in the Eqs. (12) and (17) are changed by the sum of integrals in those equations and solved analytically. Iterative scheme of numeric research of combined transfer process interaction in a droplet is constructed according the solutions (12) and (13) of the internal droplet problem by concretizing the Eq. (1) and by evaluating the peculiarities of heat and mass transfer between an evaporating droplet and its surrounding gas using the method [1]. Convective heat flux density into an evaporating droplet

$$q_c^+ = f_B^{-0.7} \frac{\lambda_{\text{vg}} (2 + 0.57 Re^{1/2} Pr^{1/3})}{2R} (T_g - T_R) \quad (20)$$

$$f_B = 1 + \frac{c_{p,g}(T_g - T_R)}{L} \left[1 + \frac{\lambda_l \left. \frac{\partial T(r, \tau)}{\partial r} \right|_{r=R^-}}{q_c^+} \right] \quad (21)$$

If no liquid circulation occurs in an evaporating droplet, in the expression (21) liquid thermal conductivity is replaced by effective thermal conductivity $\lambda_e = k_c \lambda_l$. The parameter k_c , which evaluates convective heat transfer, is the Peclet number function [8]. Heat flux density of phase change indicates energy expenditure for droplet evaporation

$$q_f^+ = Lm = L \frac{D}{RT_R R_g} p \ln \frac{p - p_{g,R}}{p - p_{g,\infty}} \quad (22)$$

Motion dynamics of an evaporating droplet

$$\frac{dw_l}{d\tau} = \frac{3 C_D}{8 R} \frac{\rho_g}{\rho_l} \frac{|w_l - w_g|}{(w_l - w_g)^{-1}} \quad (23)$$

All members of the Eq. (1) are the function of evaporating droplet surface temperature T_R , the latter is calculated using the method of the fastest descent and minimizing the Eq. (1) according to temperature $T_{R,i}$ and searching for its value to satisfy the condition

$$\left(1 - \frac{q_\Sigma^+}{q_\Sigma^- + q_f^+} \right) \cdot 100\% < 0.01 \quad (24)$$

Time intervals $\Delta\tau_i$ are selected in such a way that the influence of an evaporating droplet coordinate change on transfer process interaction can be neglected during those intervals. Then, as temperature of evaporating droplet surface $T_{R,i}$, satisfying the condition (24), is determined, all parameters of combined transfer at time instant τ_I are easily calculated. Then according to the equation

$$\rho \frac{dR}{d\tau} = - \frac{D}{RT_R R_g} p \ln \frac{p - p_{g,R}}{p - p_{g,\infty}} \quad (25)$$

a droplet R_I is calculated. In this case the solution of the Eq. (2) is obtained for the droplet of non-isothermal initial state, therefore a consequential time change scheme can be applied. Then the initial conditions (9) are corrected according to the calculated values of the droplet radius R_I and temperature field $T(r, \tau_I)$. Calculations are repeated until the droplet completely evaporates out.

3. Research results

When partial pressure of liquid in gas is higher than saturated vapor pressure near droplet surface, the droplet evaporates. In opposite case liquid vapor, which is present in gas, condenses on the droplet surface. Condensation continues until droplet surface warms up to dew point temperature. As phase changes occur, the Knudsen layer is formed around the droplet, which is proportional to

free path of molecules. Combined heat transfer in a large size droplet is modeled, as $K_n \ll 0,01$. Then temperature of gas mixture near a droplet surface is very close to the droplet surface temperature.

The intensity of evaporating droplet state change depends on gas parameters, liquid nature and on the peculiarities of heat and mass transfer. In order to emphasize the influence of heat and mass transfer peculiarities, the premise of gas parameter constancy is made. This premise is valid, as the ratio of gas and liquid initial flow rates is close to zero. The initial velocity of droplet slip in gas $\Delta w_0 = w_{l,0} - w_{g,0}$ and radiant absorption intensity determine droplet heating mode. In the most cases droplets are injected into a flow in higher velocity, comparing to the gas flow velocity. Therefore a combined heating of droplets is present. To evaluate the peculiarities of combined heating, it is necessary to know the change of droplet state in the case of the simplest boundary conditions.

3.1. Conductive heating of evaporating droplets

Cognition of droplets state change during their conductive heating forms a basis for further evaluation of combined heating. Conductive heating case is thoroughly examined, by modeling heating and evaporation of water droplets in various boundary conditions, as velocity of droplet slip in air $\Delta w_l = 0$. It is determined that in the case of conductive heating without reference to the initial water temperature and size of droplets, during unsteady evaporation droplets reach a new isothermal state (Fig. 1), which can be determined by a droplet surface temperature $T_{R,e,k}$.

A droplet warms up to this temperature at the moment τ_e of equilibrium evaporation beginning. By generalization of research results from various cases it is possible to state that during equilibrium evaporation droplet temperature remains constant and the droplet remains isothermal till its extinction moment only in the case of its conductive heating, if the parameters of gas flow do not change. The condition $T_{R,e,k} = const$ is the peculiarity of the droplet conductive heating. This temperature is the function of gas temperature and liquid partial pressure in gas (Fig. 1, a). A water droplet is non-isothermal during its unsteady evaporation mode; its temperature field gradient is positive (Fig. 2, b). The direction of conductive heat flux density vector remains constant; its modulus corresponds to the part of external conductive heat flux density, which heats the droplet (Fig. 2, a). Therefore the intensity of a droplet unsteady evaporation is defined by the difference of conductive heat fluxes outside and inside the droplet. Though heating dynamics of droplets that differ in size and have different initial liquid temperature is distinctive (Fig. 3, b), they reach equilibrium evaporation mode being at the same energetic level (Fig. 3, a). Hence, as a droplet is in its equilibrium evaporation mode and is heated by conduction, its surface temperature value does not depend on the droplet size and initial state. The value of $T_{R,e,k}$ depends on gas temperature (Fig. 4) and is sensible to the pressure ratio $p_{g,\infty} / p$ (Fig. 5).

Droplet lifetime covers the initial and final heating periods. In the initial period an unsteady droplet evaporation occurs; in the final period – equilibrium evaporation.

Temperature field has no extreme points neither in the initial, nor in the final periods; its gradient gradually decreases during unsteady evaporation mode and is equal to zero during equilibrium one. The dependence of temperature of water droplets, which evaporate in equilibrium mode, on boundary conditions of heat and mass transfer is verified by experimental results [27,28] et al. Experimental results, obtained by different authors, are widely scattered (Fig. 6, points).

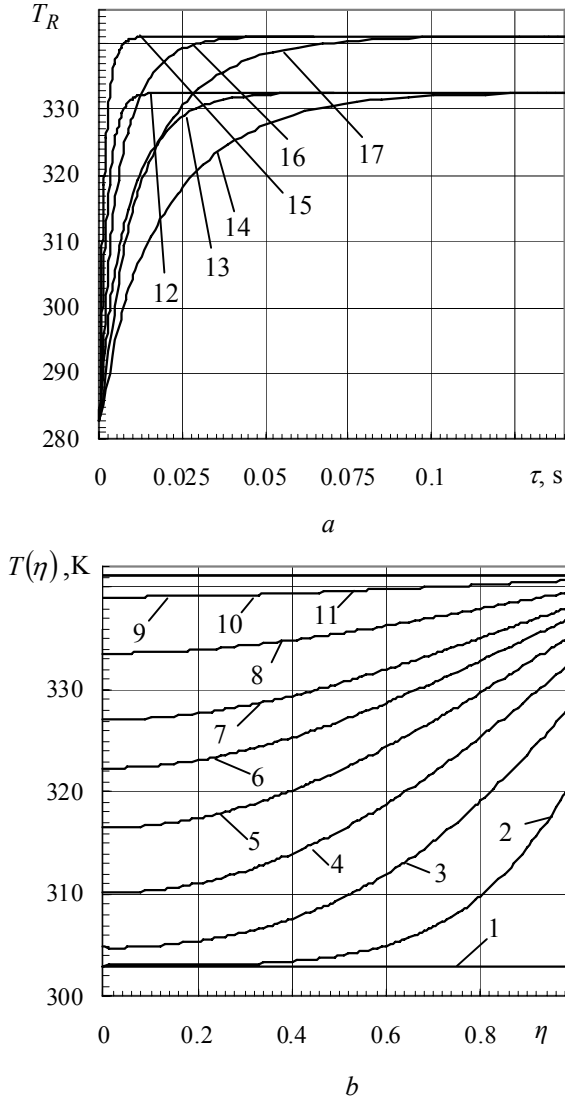


Fig. 1 Change of water droplet temperature in its conductive heating case. *a* – dynamics of surface temperature; *b*– temperature field. τ , s: 1 – 0; 2 – 0.0028; 3 – 0.0055; 4 – 0.0083, 5 – 0.0111; 6 – 0.0138; 7 – 0.0166; 8 – 0.022, 9 – 0.033; 10 – 0.072; 11 – 0.27. R_0 , m: 1-11,13,16 – 0.0001; 12,15 – 0.00005; 14,17 – 0.00015. $p_{g,\infty}/p$: 1-14 – 0.1; 15-17 – 0. $T_d=1273$ K

It is possible to state that sufficient attention was not paid to air humidity, limiting only to air temperature control. Numerical research results of water droplets that evaporate in equilibrium mode, during their conductive heating, correlate with experimental results (Fig. 6, lines). Analysis of the obtained results of numerical research allows noticing that water surface temperature dynamics, which reflects the peculiarities of water droplet state

change in the case of conductive heating can be generalized according to the universal time Fourier number. Dynamics of variation of water droplet surface temperature

$$T_{R,n}(\tau) = \frac{T_R(\tau) - T_{R,0}}{T_{R,e,k} - T_{R,0}} \quad (26)$$

is individual in universal time coordinate.

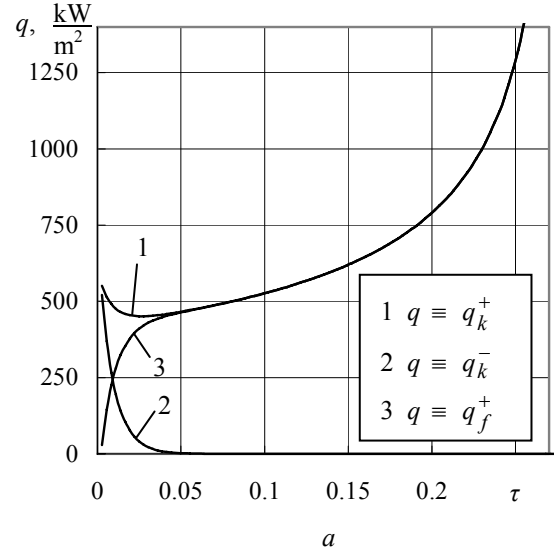


Fig. 2 Change of heat flux densities (*b*) and temperature field gradient (*a*) in an evaporating water droplet. Boundary conditions the same as in Fig. 1, when $p_{g,\infty}/p=0.1$

Dynamics of a water droplet surface temperature in regards to the droplet size is different for different initial liquid temperatures (Fig. 3) and different air humidity (Fig. 1), though in universal time coordinate dynamics of evaporating droplet surface temperature does not depend on initial droplet size (Figs. 7, 8). This conclusion is verified by the fact that diagrams of the functions that define different size droplet surface temperatures coincide $T_{R,n}(R_0, Fo)$, as the droplets with the same initial temperature $T_{R,0}$ evaporate in a gas flow with constant parameters T_d and $p_{g,\infty}/p$.

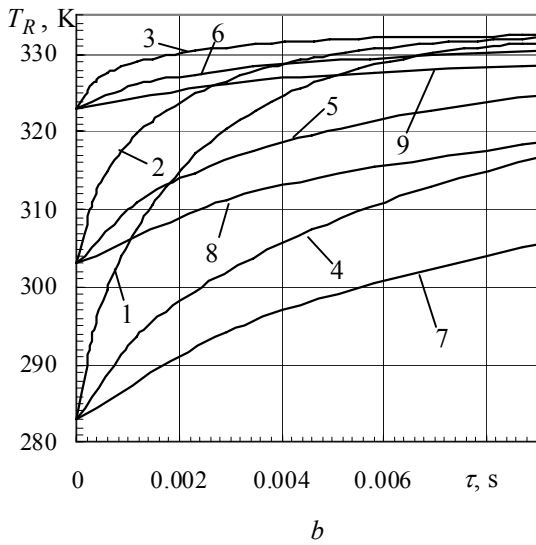
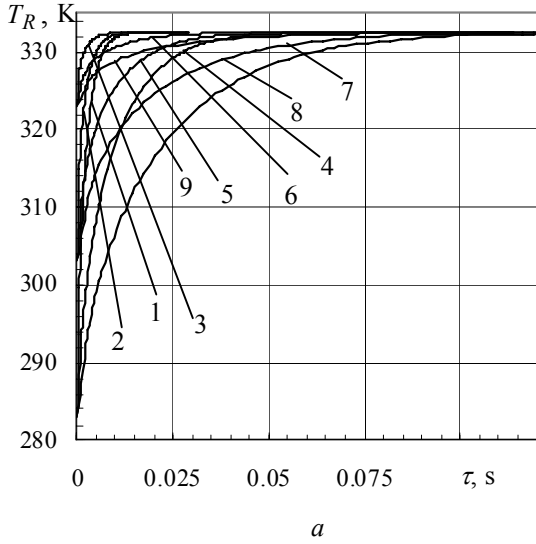


Fig. 3 Change of temperature of different initial state water droplets in the case of conductive heating. R_0 , m: 1 – 0.00005; 4–6 – 0.0001; 7–9 – 0.00015. $p_{g,\infty}/p=0$. $T_d=1073$ K

Though as sprayed liquid temperature (Fig. 7) or vapor partial pressure in gas (Fig. 8) change, the charts of the function $T_{R,n}(R_0, Fo)$ alter. The generalization of liquid droplet surface temperature variations, normalized according to the expression (26), under different boundary conditions into an empiric equation was not successful. More detailed analysis of water droplet evaporation in the conditions of conductive heating showed that the product of Fourier number and the function $T_{R,n}(R_0, Fo)$, defined by the expression (26)

$$\bar{T}(R_0, Fo) = Fo T_{r,n}(R_0, Fo) \quad (27)$$

in nondimensional coordinate not only is independent of the droplet size, but also is not sensible to the change of the parameters $T_{R,0}$, T_d and $p_{g,\infty}/p$ (Fig. 8), when equilibrium evaporation is reached under the condition $Fo_e < 1$. The droplet surface temperature, normalized according to the Eq. (27), is presented in the Fig. 9.

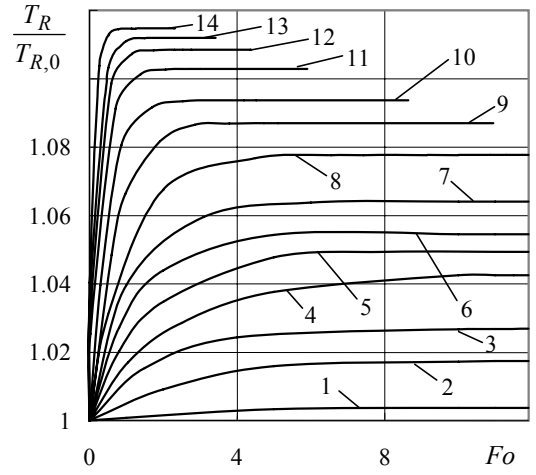


Fig. 4 Dependence of evaporating water droplet surface temperature on dry air temperature T_d , K: 1 – 373; 2 – 400; 3 – 423; 4 – 473; 5 – 500; 6 – 523; 7 – 573; 8 – 673; 9 – 773; 10 – 873; 11 – 1073; 12 – 1273; 13 – 1473; 14 – 1873. $\Delta w_l = 0$. $T_n \equiv T_R / T_{R,0}$. $T_{R,0} = 300$ K. $R_0 = 0.00005$ m

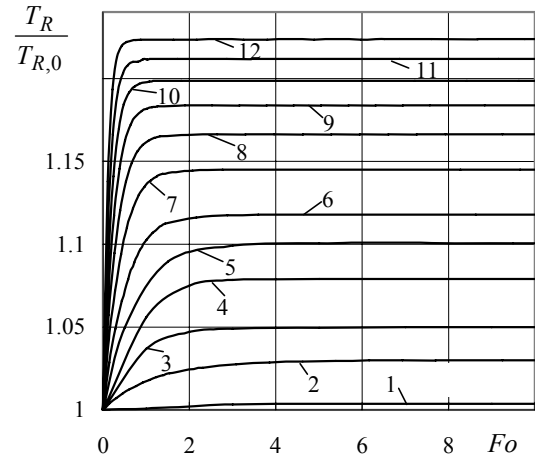


Fig. 5 Dependence of evaporating water droplet surface temperature on liquid vapor in air $p_{g,\infty}/p$: 1 – 0; 2 – 0.025; 3 – 0.05; 4 – 0.1; 5 – 0.15; 6 – 0.2; 7 – 0.3; 8 – 0.4; 9 – 0.5; 10 – 0.6; 11 – 0.7; 12 – 0.8. $\Delta w_l = 0$. $T_n \equiv T_R / T_{R,0}$. $T_d = 373$ K

Determined peculiarities of droplet state change in the case of conductive heating are important when creating the method of engineering calculation of unsteady droplet evaporation.

3.2. Combined heating of evaporating droplets

The intensity of heat supplied to a droplet is determined by conductive, convective and radiant components of the total external heat flux density. When $q_r^+ = 0$, q_c^+ evidences only by quantitative changes of the evaporating droplet state, if compared to conductive heating. Therefore during convective heating, similarly to the conductive heating case, a droplet reaches its equilibrium

evaporation mode being isothermal, though at higher energetic level. The increment of the droplet surface temperature, if compared to the droplet surface temperature in the case of conductive heating, is determined by the ratio of $q_{c,e}^+$ and $q_{k,e}^+$

$$\Delta T_{e,c} = T_{R,e,c} - T_{R,e,k} = f(q_{c,e}^+ / q_{k,e}^+) \quad (28)$$

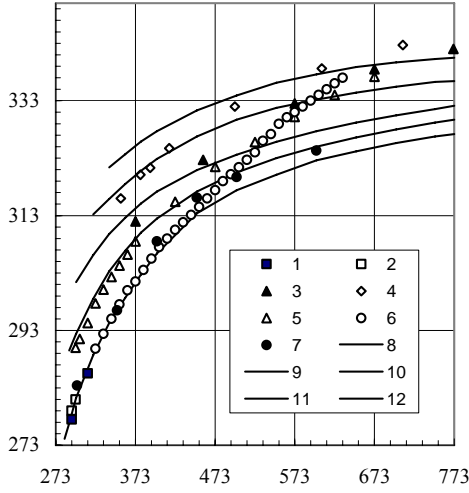


Fig. 6 Temperature of evaporating water droplet. Points – experiment [26,27]: 1 – G. Langstrofh; 2 – W. Ranz, W. Marschall; 3 – A. Kobajjisi; 4 – N. Nishiaki; 5 – V. Fedoseev, D. Poliscuk; 6 – M. Apasev, R. Malov; 7 – C. Downing. Lines – numerical research, as $\Delta w_l = 0$, $p_{d,\infty} / p$: 8 – 0; 9 – 0.025; 10 – 0.05; 11 – 0.1; 12 – 0.15

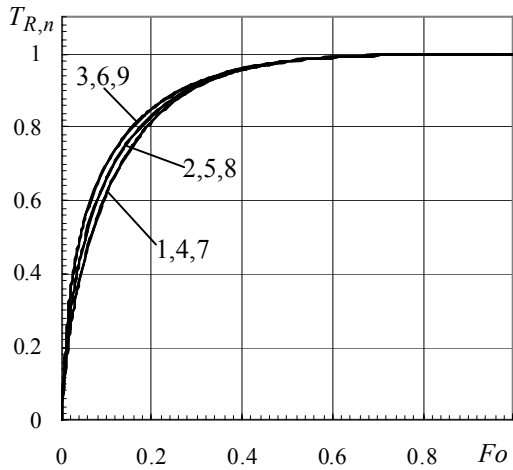


Fig. 7 Variation of a droplet surface temperature, normalized according to the expression (26) in the universal time coordinate. Boundary conditions correspond to Fig. 3

As the velocity of droplet slip in gas decreases, $\Delta T_{e,c} \rightarrow 0$. During unsteady evaporation the gradient of temperature field in a droplet is positive, q_k^- corresponds to the part of the total external heat flux density $q_\Sigma^+ = q_k^+ + q_c^+$ that heats

the droplet. It cools during its equilibrium evaporation; that is conditioned by decreasing velocity of the droplet slip in gas due to drag forces. Therefore convective heating of the droplet turns into conductive one and its temperature approaches to temperature $T_{R,e,k}$.

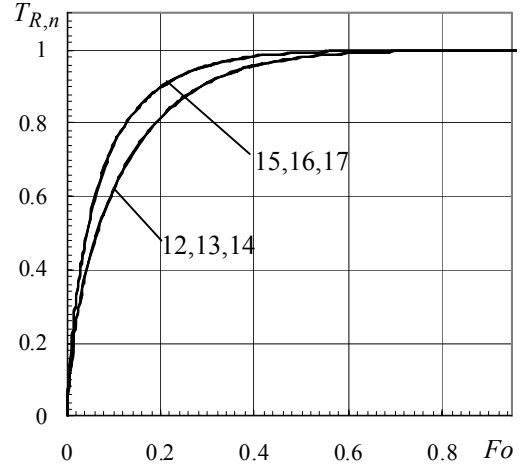


Fig. 8 Variation of a droplet surface temperature, normalized according to the expression (26) in the universal time coordinate. Boundary conditions correspond to Fig. 1

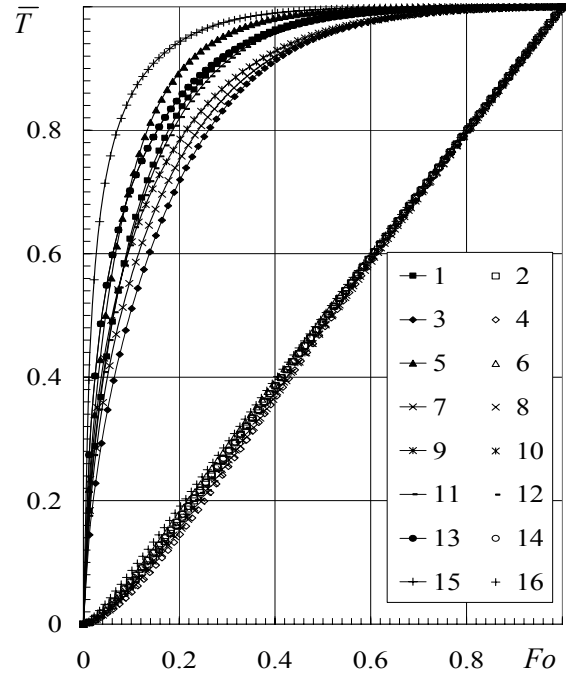


Fig. 9 Change of normalized droplet surface temperature in universal time coordinate. Lines – normalized according to (26); points – according to (27). T_g , K: 1-10 – 1073; 11-16 – 1273. T_{R0} , K: 1-6, 11, 12, 15, 16 – 283; 7, 8 – 303; 9, 10, 13, 14 – 323; $p_{g,\infty} / p$: 7-14 – 0; 1, 2, 5, 6 – 0.1; 15, 16 – 0.2

Interaction of radiation and thermal conduction processes in semitransparent liquid makes an essential influence on the state of an evaporating droplet. This interac-

tion deforms unsteady temperature field and changes heat flux densities. Interaction intensity can be evaluated by the ratios of the components of the external total heat flux density q_{Σ}^{\pm} : in the case of radiant – conductive heating q_r^+ / q_k^+ ; in the case of radiant – convective heating q_r^+ / q_c^+ . Dynamics of these ratios determines the peculiarities of temperature change of evaporating droplet.

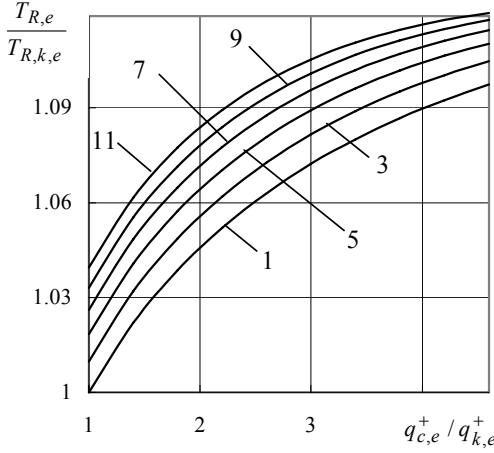


Fig. 10 Dependence of water droplets equilibrium evaporation temperature $T_{R,e}$ on the ratio of the total heat flux density components. $q_{r,e}^+ / q_{c,e}^+$: 1 – 0; 3 – 0.2; 5 – 0.4; 7 – 0.6; 9 – 0.8; 11 – 1

Dependence of water droplet surface temperature on the components of the total external heating flux density in the case equilibrium droplet evaporation is presented in the Fig. 10. The ratios of these components define droplet surface temperature $T_{R,e}$ at the beginning of equilibrium evaporation. When $q_{c,e}^+ / q_{k,e}^+ > 1$ or $q_{r,e}^+ / q_{c,e}^+ > 0$, $T_{R,e}$ of an evaporating droplet surface is higher than $T_{R,e,k}$. In the case of radiant – conductive heating this temperature difference is the function of the ratio $q_{r,e}^+ / q_{k,e}^+$

$$\Delta T_{e,r-k} = T_{R,e,r-k} - T_{R,e,k} = f(q_{r,e}^+ / q_{k,e}^+) \quad (29)$$

In the case of radiant – convective heating case temperature difference is the function of the ratios $q_{r,e}^+ / q_{c,e}^+$ and $q_{c,e}^+ / q_{k,e}^+$

$$\Delta T_{e,r-c} = T_{R,e,r-c} - T_{R,e,k} = f(q_{c,e}^+ / q_{k,e}^+; q_{r,e}^+ / q_{c,e}^+) \quad (30)$$

Heat flux density ratios change during equilibrium evaporation of a droplet (Fig. 11, b), therefore the increment between droplet surface temperature and temperature $T_{R,e,k}$ changes as well (Fig. 11, a). In the case of combined heating the droplet is non-isothermal.

In the case of radiant – convective heating due to the influence of drag forces during the initial and transient heating period velocity of droplet slip in gas decreases significantly. Therefore radiant – convective heating gradually turns into radiant conductive heating. As liquid evapo-

rates, radiant absorption intensity in a droplet decreases [11-17]. Hence, at the final period of droplet evaporation the conditions of droplet conductive heating are obtained. Thus, without reference to the peculiarities of heat and mass transfer between a droplet and its surroundings, at the final stage of evaporation the droplet approaches to an isothermal state at temperature $T_{R,e,k}$.

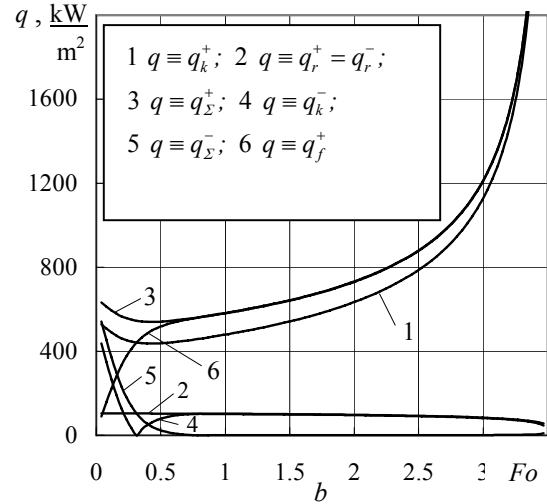
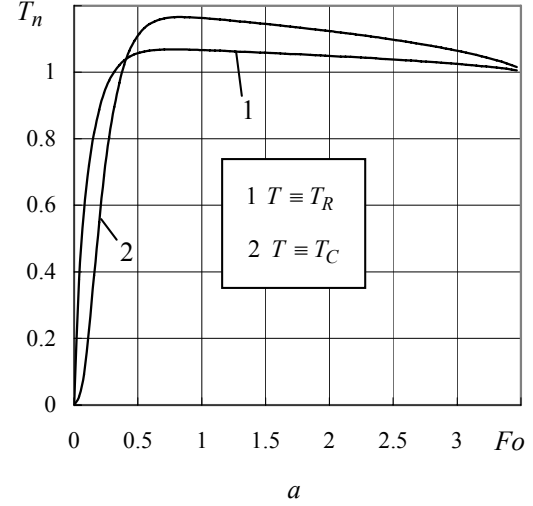


Fig. 11 Variation of water droplet temperature and energy flow densities on its surface. $\Delta w_l = 0$. $R_0 = 0.0001$ m. $T_{R,0} = 283$ K. $T_d = 1273$ K

The velocity of droplet slip in gas has a key influence on thermal convection in a droplet, the intensity of which is defined by the parameter k_c^- . Higher is the initial velocity of droplet slip in gas $\Delta w_{l,0}$, more intensive is relative decrease of this velocity (Fig. 12) and bigger influence of liquid circulation on heat transfer in the droplet.

Droplet evaporation intensity can be defined by vapor flow density on a droplet surface. During unsteady evaporation mode, as droplet surface temperature increases, evaporation intensity increases as well. At the beginning of equilibrium evaporation it can even decrease for some time due to decrease of the ratio q_c^+ / q_k^+ . After that droplet evaporation intensity increases intensively due to rapid growth of heating intensity, as $R \rightarrow 0$.

The experimental results of large water droplet evaporation under radiation influence, presented in the

study [27], prove a significant impact of radiation on the interaction of transfer processes in evaporating water droplets, as well as on the rate of their phase transformations. The experiments were performed with 0.0009-0.002900 m size water droplets, evaporating in equilibrium

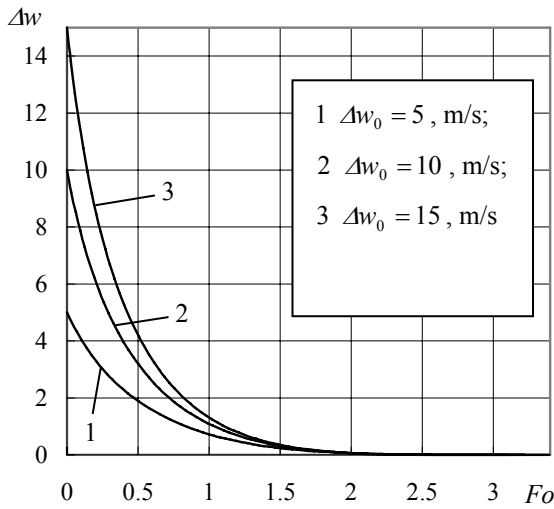


Fig. 12 Variation of velocity of droplet slip in gas. $R_0 = 0.0001$ m. $T_{R,0} = 283$ K. $T_d = 1273$ K

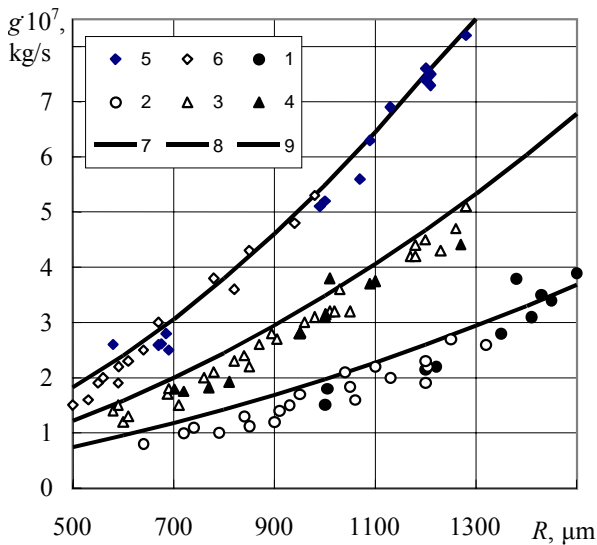


Fig. 13 The rate of water droplet evaporation. (Points – the experiment, lines – numerical investigation). T_g , K: 1,2,7 – 818; 3,4,8 – 981; 5,6,9 – 1133

conditions in dry air at temperature 673-1073 K. This was achieved by a continuous pumping out water vapor at the velocity, not higher than 0.01 m/s. The constancy of droplet size was ensured. Maximal error of droplet evaporation rate did not exceed 5%. The experimental results are well described by the model of radiant-conductive droplet heating, presented in this study, stating that the source of radiation is absolutely black body; its temperature equals to that of air, as $p_{g,\infty}/p = 0$ (Fig. 13, lines). This premise explains theoretically obtained evaporation rate, which is a little bit higher, because in the case of more rigorous investigations emissivity of grey surfaces should be evaluated, which has not been done in the study [27].

4. Conclusions

1. The state change of evaporating droplets depends on their heating manner.
2. Temperature of a conductively heated droplet during its equilibrium evaporation does not depend on its initial state.
3. In the case of combined heating during unsteady evaporation droplets warm more if compared to conductive heating. Temperature increment depends on the ratio of total heat flux density components.
4. During equilibrium evaporation process radiant – convective heating of droplets due to drag forces gradually turns into radiant – conductive heating, which, as radiant absorption is weak in small size droplets, gradually turns into conductive heating. Therefore, without reference to heating manner of droplets, in the moment of droplet existence the droplet temperature is close to equilibrium evaporation temperature in the case of conductive heating.
5. The change of water droplet state can be generalized according to non-dimensional surface temperature. In the case of conductive heating non-dimensional droplet temperature in universal time scale does not depend on gas temperature and is not sensitive to the ratio of liquid vapor densities. That makes premise for further creation of engineering methods of droplet problem solution.

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SAVEIKAUJANTYS PERNAŠOS PROCESAI VANDENS LAŠUOSE

Re z i u m ė

Atlikta lašo uždavinio, apimančio šilumos ir masės pernašos procesų tyrimą laše ir jo aplinkoje, analizė. Sumodeliuota nestacionarioji šilumos pernaša vandens lašuose. Skaitinio tyrimo metodas įvertina spinduliuotės sugėrimo selektyvumą ir pernašos procesų sąveiką. Ištirta lašų būsenos kitimo priklausomybė nuo jų šildymo būdo. Aptarti nestacionariosios pernašos procesų sąveikos dėsniniamai. Skaitinio tyrimo rezultatai koreliuoja su vandens lašų temperatūros ir garavimo greičio žinomų eksperimentinių tyrimų rezultatais.

G. Miliauskas, V. Sabanas

INTERACTING HEAT TRANSFER PROCESSES IN WATER DROPLETS

S u m m a r y

A brief review of a droplet problem research, which covers heat and mass transfer in the droplet and its surroundings, is presented in the study. The unsteady heat transfer in evaporating semitransparent droplets is modeled. The numerical research method evaluates selective radiation absorption and interaction of transfer processes. The influence of the droplet state on its heating process is investigated. The interactions of unsteady transfer processes in droplets are discussed. The results of the numerical research are compared to the known results of the experimental studies of water droplet temperature and evaporation rate.

Г. Миляускас, В. Шабанас

ВЗАИМОДЕЙСТВИЕ ПРОЦЕССОВ ТЕПЛОПЕРЕ- НОСА В КАПЛЯХ ВОДЫ

Р е з ю м е

Проведен анализ исследований в области тепломассообмена в капле воды. Смоделирован нестационарный теплоперенос в каплях воды. Метод численного исследования учитывает селективность поглощения излучения и взаимодействия процессов переноса. Исследована зависимость изменения состояния капель от способа их нагрева. Результаты численного исследования согласуются с известными результатами экспериментального исследования температуры и скорости испарения капель воды.

Received October 17, 2004

DOI: 10.5755/j02.mech.13006