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Stress concentration in castellated I-beams under transverse bending

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1. Introduction

Although castellated beams are already applied about 100 years, the theory of their stress state still is not still finally elaborated. It can be confirmed, for example, with absence of recommendations on designing of such beams in Eurocode 3. This situation can be explained, first of all, with complexity of problem.

To choice the optimal dimensions of castellated beam it is need to appreciate the maximum level of stresses in it, because this is one of important parameters in structural norms. Stress distribution in castellated beams was investigated in any works [1-13] mainly using FEM and experiments. Analytical relations were analyzed in works [14-15]. However reliable formula for stress level was not obtained.

Very suited instrument for compare of stress state of beams with different web-cutting pattern is the stress concentration factor α_σ (SCF), representing by itself non-dimension magnitude. It is possible determine SCF as ratio of maximum equivalent stresses in zone of opening to maximum stress in flange of beam with solid web under given external load. In this work the determination of coefficient α_σ was performed for case of transverse flexure.

2. Equivalent stresses in beam

For estimation of the stress concentration level under transverse bending it was initially considered simply supported castellated I-beam, performed on unwasted technology from rolled profile #50 (GOST 8239-72), loaded with concentrated force, applied in middle of span. Depth of holes was adopted equal to $h = 0.667H$, as more useful in structural practice. Web-post width was equal to a side of hexagonal hole, i. e. classic scheme of the beam perforation was considered (Fig. 1). In this case dimensions of beams were *l*-75-1-17-1.52 cm-0.667-1. In common case it can be interpreted as $l - H - t_w - b_f - t_f cm - h / H - c / a$, i. e. length – total height of beam – web thickness – flange width - thickness of flange – relative depth of opening – relative width of web-post. Concentrated force *P* = 112.5 kN was constant in all cases of loading.

Accuracy of calculations by FEM is mainly determined with the finite element sizes: the less FE, the more accurate is calculation. But application of all refined elements is not suitable because of the restricted computer memory and essential increasing of computed time. For example, solution of the equation system with 300000 unknowns in computer with 4 Gb RAM demand more the one minute. Reducing the size of the equation system and respectively time of calculation can be achieved using different approaches: application the super element method; taking into account the structure symmetry and considering only a half of beam; using no uniform mesh of finite elements and others. Last two approaches are simplest and rather effective, but appear question, what size of elements will be sufficient for getting of demanded accuracy of calculation? Theoretically to determine optimum sizes of FE is rather complex, that is why in most cases they are chosen on base of calculations with successive reducing of element sizes, until difference in results will be negligible.

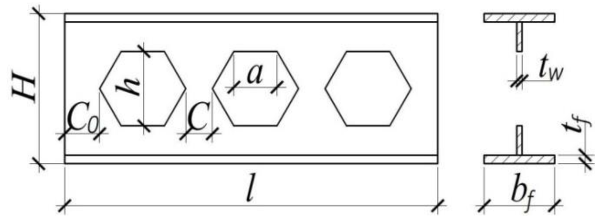


Fig. 1 Calculation scheme of castellated beam

In this work the refined mesh of FE was adopted only in vicinity of one opening, as it gives the least system of equations. Then this refined mesh was displaced in turn to each opening.

It is clear the size of element is to be connected with radius of curvature of hexagonal opening. The smaller fillet radius the lesser elements are to be, otherwise contour of opening takes form not smooth but broken line, and accuracy of calculation will be reduced. Although formally openings in webs performed on unwaste technology are not rounded in reality they have some fillets. In accordance with recommendations of AISC (American Institute of Steel Constructions) in calculations for strength of castellated beams the fillet radius of hexagonal openings is necessary adopt equal to $r = 2t_w$, or to $r = 5/8''$, depending what is bigger. In calculations performing below the radius of fillet was taken $r = 0.04h$ (*h* – depth of opening). This corresponds approximately to condition $r = 2t_w$.

Fulfilled analysis show the satisfactory accuracy is reached under sizes of finite elements equal to 0.05*r*, that is why in calculations sizes of FE near the contour of opening they were equal $\Delta_{FE} = 1$ mm , and in other parts of beam their dimensions were $\Delta = 20$ mm . Depth of openings was *h* = 500 mm-600 mm.

Under transverse bending the important role in value of α_σ play as bending moment *M* so and shear force *V*. The first one determines level of normal stress σ_x, and second is connected with the shear stress τ_{xy} in web. In technical theory of flexure the stress state of beam with solid web is considering taking into account only two stress components σ_x and τ_{xy}. In perforated beams near openings the normal stress σ_y is also achieve big values. That is why in castellated beams under transverse bending takes place complex stress state, the integral parameter of which for evaluation of SCF under joint action of σ_x, σ_y and τ_{xy} can be equivalent stress von Mises σ_{max}^{eqv}. In common form it can be expressed via stress components as:

$$\sigma_{max}^{eqv} = \sqrt{\sigma_x^2 - \sigma_x \sigma_y + \sigma_x^2 + 3\tau_{xy}^2} \ . \tag{1}$$

Appreciate value of α_σ with formula:

$$\alpha_\sigma = \sigma_{max}^{eqv} / \sigma_{max}^{TT} , \tag{2}$$

where σ_{max}^{TT} is maximum stress in flange of beam with solid web, determined on technical theory of flexure as:

$$\sigma_{max}^{TT} = M_{max} / W , \tag{3}$$

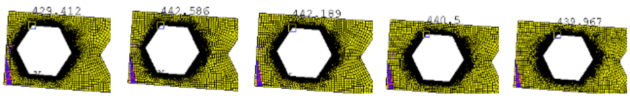
where W is modulus of inertia of beam's cross section with solid web:

$$W \approx b_f t_f H + H^2 t_w / 6 . \tag{4}$$

Of course such approach to calculation of SCF on Eq. (2) has some peculiarity, because the level of maximum stress σ_{max}^{eqv} is measuring in one cross section, and base value σ_{max}^{TT} is taken in another, but it has no important sense, because α_σ is non-dimension magnitude. Basic advantage here is commodity of calculation σ_{max}^{TT}, and correspondence of obtained value α_σ to a physical picture of stress state of web in the concentration zone.

With the aim to distinguish influence of V and M on value α_σ it was performed calculations under constant value V and practically absent moment M and under joint action of V and M.

For that a simply supported castellated I-beam of arbitrary length, loaded with concentrated force $P = Const$ in middle of span was considered. In this case near ended opening under any length of beam the shear force V will be constant and bending moment $M \approx 0$. Near any other opening the joint action of V and M takes place.

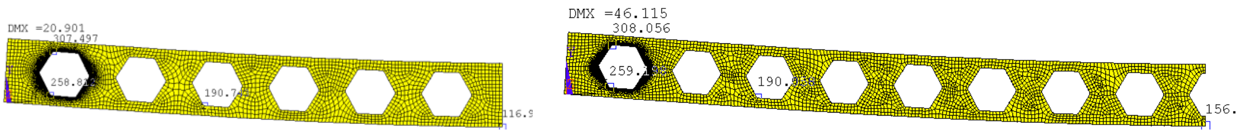


a b c d e

Fig. 2 $\sigma_{\max}^{eqv}$ versus c_0 of ended web-post: a - $c_0 = 0.13H$; b - $c_0 = 0.2H$; c - $c_0 = 0.26H$; d - $c_0 = 0.33H$; e - $c_0 = 0.4H$

First of all define original location of ended opening, i.e. distance c_0 from the opening edge to the support section. Obtained by FEM calculations of σ_{max}^{eqv} versus width of ended web-post are shown in Fig. 2, from which it can be seen the pick of stress σ_{max}^{eqv} takes place in zone (0.2-0.22)H, after that level of stress is stabilizing, although flexure moment is growing. Difference in values σ_{max}^{eqv} (Fig. 2) does not exceed 3%, i. e. this magnitude is rather stable under changing of width of ended web-post. All this allow in further calculations adopt $c_0 = 0.22H$.

Evaluate now influence of beam length l at maximum level of equivalent stress near first opening located near support. As show calculations by FEM (Fig. 3), under constant transverse force V and absent flexure moment M maximum equivalent stress near contour of ended hole will be practically constant. Moreover related length of beam dose not play any role (compare Fig. 3, a and 3, b). From this can be concluded the stress state in vicinity of ended hole of simply supported beam completely determined with value of transverse force V.



a b

Fig. 3 Stress state near ended hole under constant force in beams l-75-1-17-1.52см-0.667-1 of different length: a - l = 15H; b - $l = 20H$

Value of stress σ_{max}^{eqv}, produced with action force V, can be represented with relation:

$$\sigma_V^{eqv} = \alpha_V \frac{V}{Ht_w} , \tag{5}$$

where α_V is numerical coefficient, determined from FE analysis. Calculation with FEM shows the coefficient $\alpha_V = 41$.

For example, for castellated I-beam with dimensions 1125-75-1-17-1.52 cm-0.667-1 under action of concentrated force P = 112.25 kN level of maximum equivalent stress in vicinity of 1-st hole according to Eq. (5) will be:

$$\sigma_V^{eqv} = 41 \times 112.5 \times 10^3 / (2 \times 750 \times 10) = 307.5 \text{ MPa} .$$

Now perform with help of calculations by FEM analysis the influence on magnitude σ_{max}^{eqv} of flexure moment in the same beam with dimensions 1125-75-1-17-1.52 cm-0.667-1, but in zone of second and following openings (Fig. 4). As it can be seen, level of maximum stress grows proportionally to moment M. On base of these results the stress σ_{max}^{eqv} in vicinity of any opening can be represented as sum of two items: one caused by shear force V in accordance with Eq. (5) and second caused by moment M:

$$\sigma_{max}^{eqv} = \alpha_V \frac{V}{Ht_w} + \alpha_M \frac{M}{W}, \qquad (6)$$

where α_M is numerical coefficient, determined by FEM analysis. Flexure moment M for n-th opening can be approximated as:

$$M = Vx \approx V(n-1)s, \qquad (7)$$

where s is step of openings; n is ordinal number of opening, in vicinity of which the stress value σ_{max}^{eqv} is determining.

In common case the step of openings under any perforation can be written as:

$$s = (2+\xi)a = (2+\xi)\beta H / \sqrt{3}, \qquad (8)$$

where $\xi = c/a$ is relative width of web-post; $\beta = h/H$ is relative depth of opening.

Substitution of Eq. (4), Eq. (7) and Eq. (8) in Eq. (6) bring to expression:

$$\sigma_{max}^{eqv} = \left(\alpha_V + \alpha_M \frac{(n-2)(2+\xi)\beta}{6b_f t_f / Ht_w + 1}\right)\frac{V}{Ht_w}, \quad n \geq 2 \qquad (9)$$

In Eq. (9) value (n - 2) is written instead of (n - 1), because proportional growth of stresses is appearing only beginning from 2nd opening. Taking into account the coefficient $\alpha_M = 6.4$ is constant for all calculated below beams Eq. (9) can be rewritten as:

$$\sigma_{max}^{\text{экв}} = \left(\alpha_V + 6.4\frac{(n-2)(2+\xi)\beta}{6b_f t_f / Ht_w + 1}\right)\frac{V}{Ht_w}. \qquad (10)$$

Verify Eq. (10) for castellated I-beam 1125-75-1-17-1.52 cm-0.667-1 loaded with concentrated force $P = 112.5$ kN, applied in mid-span. For this variant it was adopted value $\alpha_V = 41$. Then for 6-th hole under $\xi = 1$ and $\beta = 0.667$ in accordance with Eq. (10) it can be obtained $\sigma_{max}^{eqv} = 432.8$ MPa.

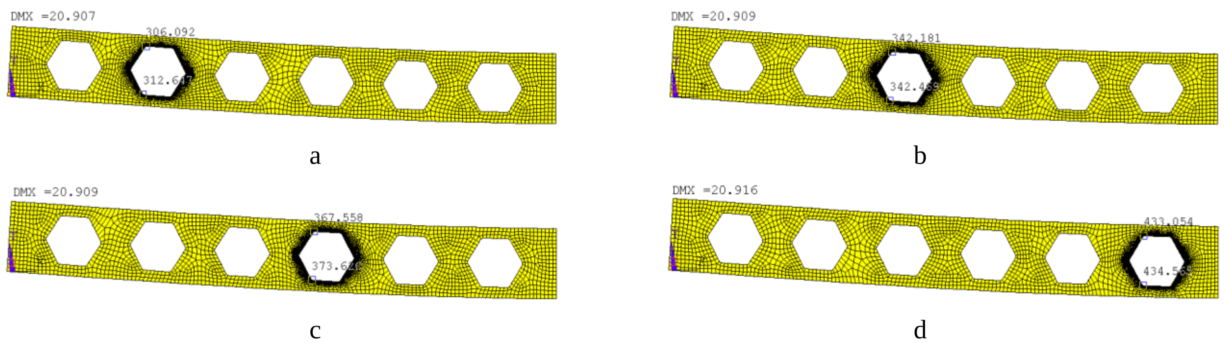


Fig. 4 Stress state of I-beam with dimensions 1125-75-1-17-1.52 cm-0.667-1 under action of concentrated force P = 112.25 kN: a - 2nd; b - 3rd; c - 4th; d - 6th holes

According to result of FEM, as shown in Fig. 4, d for 6th opening value $\sigma_{max}^{eqv} = 433$ MPa. It indicates on practically full coincidence with result of Eq. (11). For other openings of this beam the values σ_{max}^{eqv} obtained analytically by Eq. (10) are shown in Table 1.

It is need to note that Eq. (10) remains applicable for any length of beam under unchangeable parameters of perforation. So for ratio of beam length $l/H = 20$ in accordance with Eq. (10) value $\sigma_{max}^{eqv} = 495$ MPa, and calculation by FEM gives $\sigma_{max}^{eqv} = 489.8$ MPa, that indicate on divergence approximately in 1%.

For other beam with dimensions 1350-90-1.2-19-1.78 cm-0.667-1 under the same load the stress state of beam obtained by FEM is represented in Fig. 5.

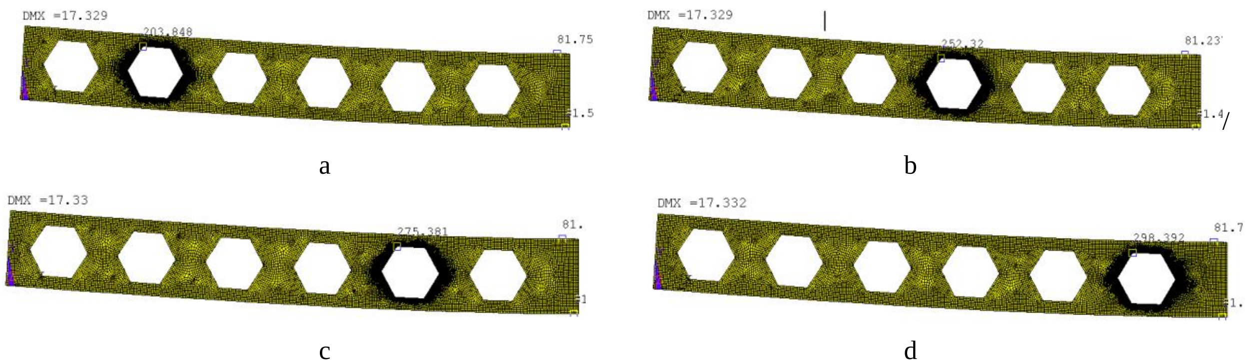


Fig. 5 Stress state of I-beam with dimensions 1350-90-1.2-19-1.78 cm-0.667-1 under transverse bending: a - 2nd; b - 4th; c - 5th; d - 6th holes

Calculation according to Eq. (10) under $\xi = 1$, $\beta = 0.667$ and $\alpha_V = 41$ for 6th opening gives $\sigma_{max}^{eqv} = 306.2$ MPa and according to FEM (Fig. 5, d) result is $\sigma_{max}^{eqv} = 298.4$ MPa (the divergence does not exceed 2.6%). Such result is satisfying to engineering accuracy of calculations. But it is possible to obtain practically full co-incidence of results by FEM and by Eq. (10) for this beam, if perform correction of coefficient α_V, reducing it from 41 to 39.5 and remaining second coefficient α_M unchangeable and equal to 6.4. In this case results obtained by FEM and by Eq. (10) practically coincide. Values of stresses for other openings of this beam and for beams of other sizes are shown in Table 1.

In Table 1 it were considered castellated beams fabricated on unwasted technology from rolled profiles #45, 50, 55 and 60. They are quite similar in proportions of height and web thickness but main difference is in relations of web-area to flange-area. This difference can be seen in variation of coefficient α_V which is changing in very narrow range: from 38.8 to 41.

Table 1

Stress $\sigma_{\max}^{eqv}$ in zone of hexagonal openings with radius of fillet $r = 0.04h$ in I-beams of different profile under concentrated load P = 112.5 kN applied in mid-span

Number of hole	1	2	3	4	5	6
Parameters of beam	$\alpha_V = 41$; $\alpha_M = 6.4$; 1125-75-1-17-1.52 cm-0.667-1					
Stress by FEM	307	306	342	368	399	433
Stress by Eq. (10), MPa	-	307	339	370	401	433
Divergence, %	-	0.3	0.9	0.5	0.5	0.2
Parameters of beam	$\alpha_V = 39.5$; $\alpha_M = 6.4$; 15H-90-1.2-19-1.78 cm-0.667-1					
Stress by FEM	214	204	230	252	275	298
Stress by Eq. (10), MPa	-	206	229	252	275	298
Divergence, %	-	1	0.4	0	0	0
Parameters of beam	$\alpha_V = 39.2$; $\alpha_M = 6.4$; 15H-82.5-1.1-18-1.65 cm-0.667-1					
Stress by FEM	254	254	272	297	324	350
Stress by Eq. (10), MPa	-	243	270	297	323	350
Divergence, %	-	4.4	0.4	0	0.3	0
Parameters of beam	$\alpha_V = 38.8$; $\alpha_M = 6.4$; 15H-67.5-0.9-16-1.42 cm-0.667-1					
Stress by FEM	370	354	396	430	466	500.5
Stress by Eq. (10), MPa	-	359	396	432	469	505
Divergence, %	-	1.5	0	0.5	0.6	0.9

3. Influence of depth of opening at the stress level

Evaluate now influence of depth of opening at the stress σ_{max}^{eqv}. Consider beams with relative depth of opening $\beta = 0.7$ and $\beta = 0.73$. Results of calculation of beam with dimensions 1125-75-1-17-1.52 cm-0.7-1, performed by Eq. (10) with value of $\alpha_V = 46.5$ and computed by FEM are shown in Table 2. Stress state of web near the different openings of this beam under concentrated load P = 112.5 kN is also shown in Fig. 6.

It can be seen from Table 2 and Table 1 the in-creaseing of the depth of opening leads to growth of coeffi-cient of force α_V, i. e. role of shear force in value of equiva-lent stresses is increasing. Dependence α_V on magnitude β is proportional and can be approximated with expression:

$$\alpha_V = 172.3\beta - 73.9. \tag{11}$$

Similar lineal dependence there is and for beams with other dimensions.

Table 2

Stresses $\sigma_{max}^{экв}$ in vicinity of hexagonal opening of different depth in I-beams under concentrated load P = 112.5 kN

Number of hole	1	2	3	4	5	6
Parameters of beam	$\alpha_V = 46.5$; $\alpha_M = 6.4$; 1125-75-1-17-1.52 cm-0.7-1					
Stress by FEM	356.5	351	380	412	446	481
Stress by Eq. (10), MPa	-	349	382	414	447	480
Divergence, %	-	0.6	0.5	0.5	0.2	0.2
Parameters of beam	$\alpha_V = 55.3$; $\alpha_M = 6.4$; 1125-75-1-17-1.52 cm-0.73-1					
Stress by FEM	421	404	446	482	517	553
Stress by Eq. (10), MPa	-	414	449	483	518	552
Divergence, %	-	2.5	0.7	0.2	0.2	0.2

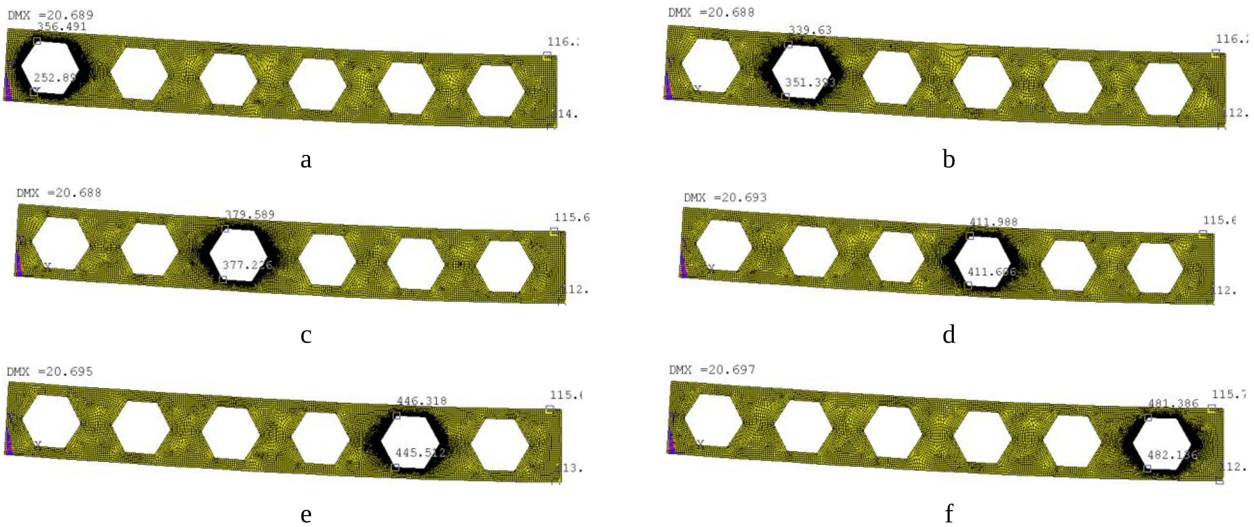


Fig. 6 Stresses in simply supported I-beam 1125-75-1-17-1.52cm-0.7-1 under transverse bending: a - 1st; b - 2nd; c - 3rd; d - 4th; e - 5th; f - 6th holes

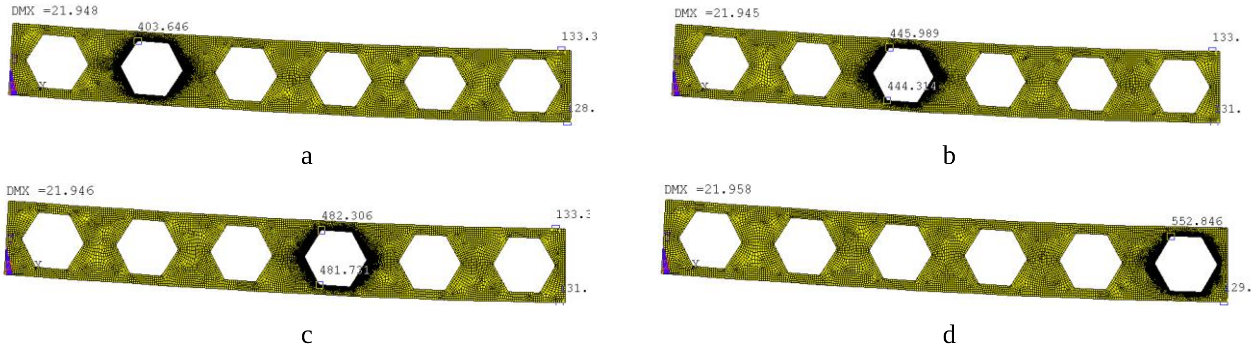


Fig. 7 Stresses in simply supported I-beam 1125-75-1-17-1.52 cm-0.73-1 under transverse bending: a - 2nd; b - 3rd; c - 4th; d - 6th holes

For related depth of openings $\beta = 0.73$ coefficient α_V takes value 55.3 but α_M remains unchangeable. Values of σ_{max}^{eqv} for I-beam with dimensions 1125-75-1-17-1.52 cm-0.73-1 are shown in Table 2, from which it is seen the divergence in different calculations does not exceed 2.5%. Results of calculation by FEM are shown in Fig. 7.

Calculations performed for beams 1350-90-1.2-19-1.78 cm-0.7-1 and 1350-90-1.2-19-1.78 cm-0.73-1 show that if adopt coefficients $\alpha_V = 47.3$ and $\alpha_V = 56.2$ respectively divergence does not exceed 1% for third and following openings (see Table 3). It is important the high accuracy takes place near the most loaded openings, located in the middle part of beam. Stress distribution in beam 1350-90-1.2-19-1.78 cm-0.7-1 is shown in Fig. 8.

Table 3

Stresses $\sigma_{\max}^{eqv}$ near hexagonal openings of different depth in I-beams

Number of hole	1	2	3	4	5	6
Parameters of beam	$\alpha_V = 47.3$; $\alpha_M = 6.4$; 1350-90-1.2-19-1.78 cm-0.7-1					
Stress by FEM	247	239	269	293	319	345
Stress by Eq. (10), MPa	-	246	271	295	319	344
Divergence, %	-	2.8	0.7	0.7	0.1	0.3
Parameters of beam	$\alpha_V = 56.2$; $\alpha_M = 6.4$; 1350-90-1.2-19-1.78 cm-0.73-1					
Stress by FEM	292	285	316	343	370	397
Stress by Eq. (10), MPa	-	293	319	344	368	394
Divergence, %	-	2.8	0.9	0.3	0.5	0.8

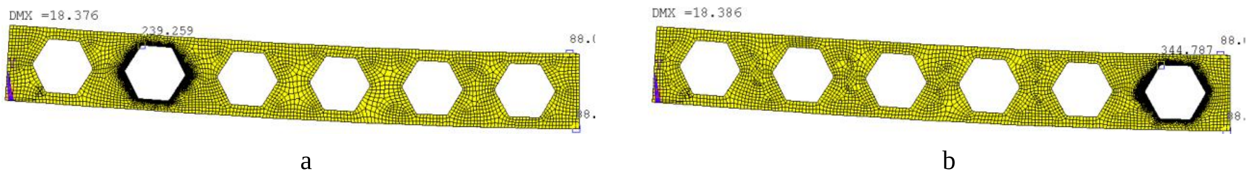


Fig. 8 Stresses in simply supported I-beam 1350-90-1.2-19-1.78 cm-0.7-1 under transverse bending: a - 2nd; b - 6th holes

4. Influence of relative width of web-post at stress level

As it is known, developing of perforated beams is directed on lightening of web with different ways: increasing the depth of openings; increasing the length of opening, by performing them with elongated form such as oval, rectangular or sinusoidal; reducing the width of web-posts. Author proposed technology of fabrication of castellated beams with regular hexagonal openings under any width of web-posts [16]. That is why influence of relative width of web-post at stress σ_{max}^{eqv} is considering below. Results of calculation by FEM on program ANSYS of I-beam 1350-90-1.2-19-1.78cm-0.667-0.5 with relative width of web-post $\beta = 0.5$ with sequent displacement of small mesh are shown in Fig. 9. Due to reducing of width of web-posts the number of openings at half of the beam length increased to 7. Analytical calculation of equivalent stress allows confirm, that according to Eq. (10) reducing of β lead to less level of σ_{max}^{eqv}. In Eq. (10), as in previous variants with $\beta = 1$ factor of influence of moment remains the same $\alpha_M = 6.4$, but factor of shear force takes value $\alpha_V = 37.6$. Results of calculation of indicated beam by Eq. (10) are shown in Table 4.

Reduce the relative web-post width till $\beta = 0.3$ and calculate again the same simply supported beam with $H = 90$ cm under action of concentrated load $P = 112.5$ kN. The values of stresses σ_{max}^{eqv} obtained by FEM are shown in Fig 10. Due to reducing the web-post width the number of openings at half of a beam length increased to 8. Calculations in according to Eq. (10) are performed under the same values of $\alpha_Q = 37.6$ and $\alpha_M = 6.4$.

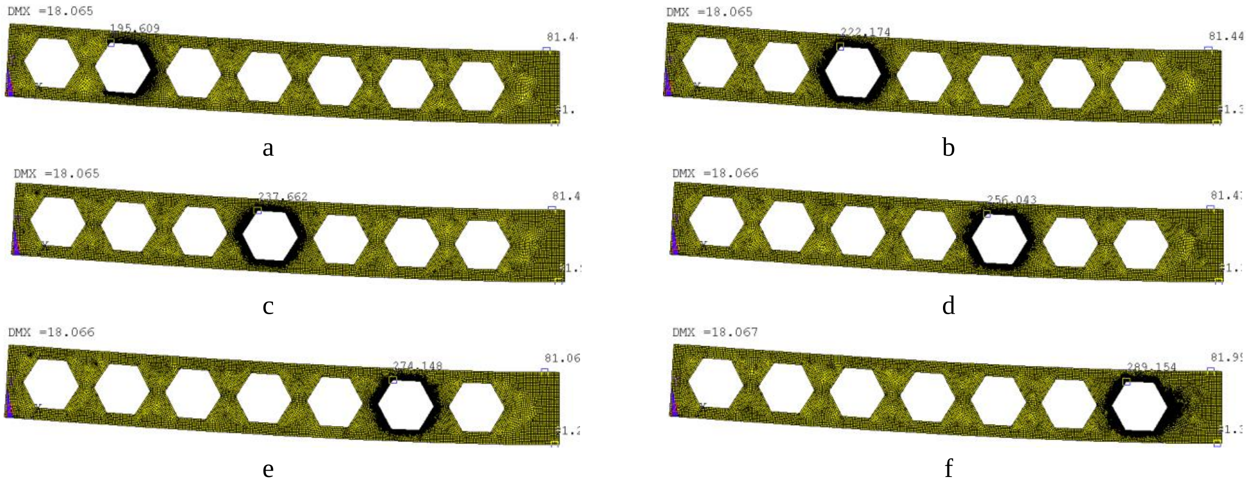


Fig. 9 Stress state of I-beam with dimensions 1350-90-1.2-19-1.78 cm-0.667-0.5 under transverse bending: a - 2nd; b - 3rd; c - 4th ; d - 5th ; e - 6th ; f - 7th holes

Table 4

Stress $\sigma_{\max}^{eqv}$ near hexagonal openings with radius of fillet $r = 0.04h$ and different width of web-posts in simply supported I-beam under transverse bending

Number of hole	1	2	3	4	5	6	7
Beam's parameters	$\alpha_V = 37.6$; $\alpha_M = 6.4$; 1350-90-1.2-19-1.78 cm-0.667-0.5						
Stress by FEM, MPa	220	196	222	238	256	274	289
Stress by Eq. (10), MPa	-	196	226	235	254	273	292
Divergence, %	-	0	1.8	1.3	0.8	0.4	1.0
Beam's parameters	$\alpha_V = 37.6$; $\alpha_M = 6.4$; 1350-90-1.2-19-1.78 cm-0.667-0.3						
Stress by FEM, MPa	228	195	220	233	250	266	282
Stress by Eq. (10), MPa	-	196	214	231	249	267	285
Divergence, %		0.5	2.7	0.9	0.4	0.4	1.1

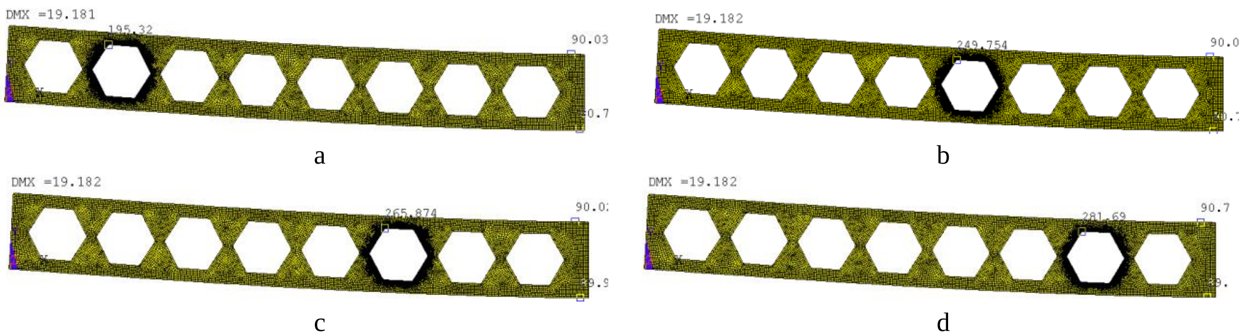


Fig. 10 Stress state of I-beam with dimensions 1350-90-1.2-19-1.78 cm-0.667-0.3 under transverse bending: a - 2nd; b - 5th; c - 6th; d - 7th holes

5. Experimental investigation

In order to verify Eq. (10) it was put an experiment on steel model in form of double cantilevered I-beam with dimensions 410-38-0.6-12-1 cm-0.667-1, loaded by two concentrated forces $V = 10$ kN applied at the ended sections via dynamometers DR-20 (Fig. 11). Material of beam was steel S345. Installation had two rigid posts located at distance 1 m from each other. Length of each cantilever was 1.5 m. During loading the level of stresses in vicinity of openings was measured by strain gauges with base 1mm located on web in form of strain rosettes. Readings of gauges were registered with Data Acquisition Controller of English firm Schlumberger. Gauges were glue near the fillet corner openings in places, determined with calculation by FEM. Beam was simply supported and loaded symmetrically at ends.

Fig. 11 Test set-up with castellated I-beam model 410-38-0.6-12-1 cm-0.667-1

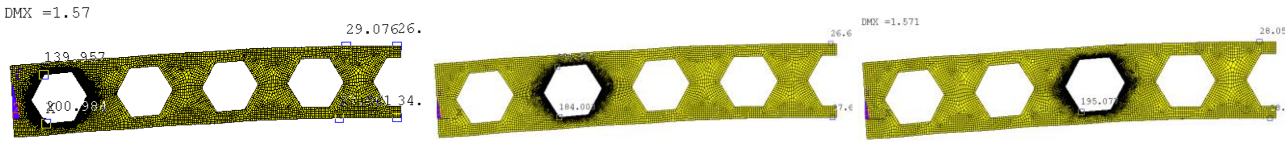
Results of tests show the measured maximum equivalent stress σ_{max}^{eqv} near contour of 2-nd opening was equal to 181 MPa and in vicinity of 3-rd opening it was 190 MPa. Calculation of beam with finite element method (Fig. 12, b and 11, c) indicate values of σ_{max}^{eqv} in the same locations equal to 184 MPa and 195 MPa respectively. Difference in values reach 2.5%.

Determination of equivalent stress σ_{max}^{eqv} appearing in vicinity of third opening in accordance with Eq. (10) for tested beam gives:

$$\sigma_{max}^{eqv} = \left(42 + 6.4\frac{(3-2)(2+1)0.667}{6\times 120\times 10/(380\times 6)+1}\right)\times$$

$$\times\frac{10\times 10^3}{380\times 6} = 198\ \text{MPa}. \qquad (12)$$

Obtained results indicate the stresses calculated by Eq. (12), by FEM and registered in experiment under transverse bending are in good correlation. As it can be seen coefficient $\alpha_M = 6.4$ is constant for all dimensions of beams and different perforation.



a b c

Fig. 12 Stress distribution in double cantilevered simply supported I-beam model 410-38-0.6-12-1 cm-0.667-1 under transverse bending: a - 1st; b - 2nd; c - 3rd openings

6. Stress concentration factor

Evaluate now stress concentration appearing in web under action of flexure moment M and transverse force V. For this purpose it will be using Eq. (2) in which we substitute value of maximum equivalent stress in arbitrary section Eq. (10) and stress σ_{max}^{TT}, determined on technical theory of flexure as:

$$\sigma_{max}^{TT} = \frac{Vl}{2\left(b_f t_f H + H^2 t_w/6\right)}. \qquad (13)$$

Substituting Eq. (10) and Eq. (13) in Eq. (2), the stress concentration coefficient α_σ is determined as follows

$$\alpha_\sigma = \left(\alpha_V \omega^* + 6.4(n-2)(2+\xi)\beta\right)/3\eta, \qquad (14)$$

where $\omega^* = 6b_f t_f / Ht_w + 1$ and $\eta = l / H$.

As it can be seen from Eq. (14) SCF does not depend on load factors but is determined only with geometry of beam, relative length $\eta = l / H$ and parameters of perforation in non-dimension form ξ and β.

Calculated by Eq. (14) coefficients α_σ for beams with dimensions 1125-75-1-17-1.52 cm-0.667-1 and 1350-90-1.2-19-1.78 cm-0.73-1 will be equal $\alpha_\sigma = 433 / 110 = 3.93$ and $\alpha_\sigma = 298 / 81.4 = 3.67$ respectively. The less value α_σ for beam with height $H = 90$ cm compare with beam with $H = 75$ cm can be explained by reduction of related area of flange: if in beam with $H = 75$ cm value $\omega_f / \omega_w = 0.345$, then in beam with $H = 90$ cm it will be $\omega_f / \omega_w = 0.313$. The flange effect can be compared with increasing the plate dimensions under evaluation of stress concentration in vicinity of alone opening under plane stress state.

It is need to remember the obtained results are applicable to castellated beams with fillet radius of opening $r = 0.04h$.

7. Conclusions

1. Analytical expression for SCF for case of transverse bending is obtained as sum of two components reflecting influence of shear force V and flexure moment M respectively.

2. Obtained relations for α_σ and for equivalent stresses $\sigma_{\max}^{eqv}$ are applicable for relative depth of openings in diapason $0.667 \le \beta \le 0.73$ and for relative width of web-posts in diapason $0.3 \le \xi \le 1$ under fillet radius $r = 0.04h$.

3. Factor of influence of moment $\alpha_M = 6.4$ does not depend on the relative values ξ and β.

4. Factor of influence of shear force α_V grows with increasing of depth openings and is almost proportional to value β.

5. Stress concentration factor near hexagonal openings under transverse bending can reach value $\alpha_\sigma = 4$.

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STRESS CONCENTRATION IN CASTELLATED I-BEAMS UNDER TRANSVERSE BENDING

S u m m a r y

In the work on base of calculations by FEM of castellated I-beams the approximate relations for evaluation of stress level and stress concentration factor in vicinity of hexagonal fillet openings under transverse bending are derived. Calculation of simply supported castellated I-beams under action of one concentrated force applied in mid-span and two symmetrically applied forces was performed. Proposed relation for equivalent stress near openings differentiate role of each force factor V and M and allow determine level of stresses in castellated beams in wide diapason of the opening parameters under different length ratio with engineering accuracy. Obtained results were verified with experiment test on steel castellated beam with 4 m length.

Keywords: stress concentration factor, castellated I-beams, hexagonal openings, von Mises stress, FEM, experiment.

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