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On direct and stepped optimization problems of flexural steel frame and their numerical realization considering strength and stiffness conditions

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1. Introduction

A lot of methods, algorithms, approaches have been developed for optimization methods in engineering optimal design of structures in the last three decades and earlier [1-7]. One can mention that recently widely applied genetic algorithms, based on the implementation of biological principles to computational optimization, have been used to solve structural optimization problems [4, 5, 8]. They employ the principle of the fittest in structural design. Attractive feature is their compatibility with discrete optimization, it does not require derivatives of functions as in classical optimization case. Most often in actual engineering design cross-sections of the members are discrete ones, chosen from standard sections or constructed combining them. This is conditioned by the availability of standard sizes due to manufacturing, construction, etc. reasons.

The most effective optimization methods employ mathematical programming methods. The applied methods range from simple optimization techniques to variational, energy principles employed in structural optimization problem [6, 7, 9, 10]. An evaluation of dissipative features of elastic-plastic vs elastic material of the structure enables significant economy of material resources in optimization of the structures [6-17]. Ductility of engineering structures (e.g. steel) is effective structural response feature in optimal design and subsequent safe maintenance of actual ones. However, the evaluation only of strength constraints (i.e. as in rigid-plastic structural optimization [6,7,9-11]) does not ensure the optimal elastic-plastic structure to response the loading by admissible displacements prescribed in design codes, therefore stiffness requirements must be introduced into optimization problem [5,14-17]. Structural optimization problem formulated as mathematical programming problem and including stiffness constraints can be formulated and solved directly [15] or applying the proposed method of certain optimization cycles, i.e. stepped optimization [16,17]. Both methods employ structural elastic response values as input data of optimization problem, those finally conditioning the optimized parameters of the structure. In practical realization this feature enables the problem to be solved iteratively, recalculating above values for running optimization iteration till the problem's solution convergence. Stepped optimization problem formulation via optimization cycles, containing subsequent solution of analysis and optimization problems, allows avoiding an evaluation of complementarity conditions. The latter are included in direct multi-extremum optimization problem. They complicate significantly numerical solution of the problem or sometimes make it to be unsolvable one, often in the cases of larger structures.

A noncorrect choosing of bounding constraints, that of starting point values complicates the direct and stepped optimization procedures or even makes the problem the unsolvable one. Stepped process procedures handling ensures an efficient and successful convergence of the solution process.

The aims of the investigation are: to develop mathematical models for direct and stepped flexural frame optimization; perform analysis of numerical solution aspects of structural optimization problems. The proposed techniques are illustrated via a solution of ten-storey single bay flexural steel frame, subjected by vertical and lateral loads. The optimization is performed taking into account relations, valid for standard steel section properties.

2. Mathematical model of frame direct optimization problem and its numerical realization

2.1. Mathematical model

The direct frame optimization problem of minimum "theoretical weight" considering strength, stiffness and constructional constraints is stated as nonlinear mathematical programming problem with linear objective function. Mathematical model for the frame FEM model reads:
find:

$$\boldsymbol{L}^T \boldsymbol{M}_0 \rightarrow min \tag{1}$$

subject to:

$$\boldsymbol{M}_r - \boldsymbol{M}_0 \leq -\boldsymbol{M}_e\,;\; -\boldsymbol{M}_r - \boldsymbol{M}_0 \leq \boldsymbol{M}_e \tag{2}$$

$$\left[C\right]^T \left[\overline{A}\right] \boldsymbol{M}_r = \boldsymbol{0} \tag{3}$$

$$\left.\begin{array}{l} \left[\mathcal{K}\right]^{-1} \boldsymbol{M}_r + \boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2 + \left[\overline{A}\right]^T \left[C\right] \boldsymbol{u}_r = \boldsymbol{0}, \\ \boldsymbol{\lambda}_1 \geq \boldsymbol{0},\; \boldsymbol{\lambda}_2 \geq \boldsymbol{0} \end{array}\right\} \tag{4}$$

$$\boldsymbol{\lambda}_1^T \left(\boldsymbol{M}_r - \boldsymbol{M}_0 + \boldsymbol{M}_e\right) + \boldsymbol{\lambda}_2^T \left(-\boldsymbol{M}_r - \boldsymbol{M}_0 - \boldsymbol{M}_e\right) = 0 \tag{5}$$

$$\boldsymbol{u}_d^- \leq \left(\boldsymbol{u}_r + \boldsymbol{u}_e\right) \leq \boldsymbol{u}_d^+ \tag{6}$$

$$\boldsymbol{M}_e = \left[\mathcal{K}\right]\left[\overline{A}\right]^T \left[C\right]\left[K\right]^{-1} \boldsymbol{F} \tag{7}$$

$$\boldsymbol{u}_e = \left[K\right]^{-1} \boldsymbol{F} \tag{8}$$

$$\boldsymbol{M}_0 \geq \boldsymbol{M}_0^{min} \tag{9}$$

Here $\boldsymbol{L}^T\boldsymbol{M}_0$ is frame linear optimality criterion; $\boldsymbol{L}=\left(L_{01},L_{02},...,L_{0n_0}\right)^T$ is the vector of total lengths of the frame constant cross-section members (via n_0 denoting a number of frame member types) compatible with the vector of frame limit bending moments (further in text moments instead of bending moments for simplicity); $\boldsymbol{M}_0=\left(M_{01},M_{02},...,M_{0n_0}\right)^T$, $M_0=W_y\sigma_y$, where W_y is cross-sectional plastic modulus and σ_y is material yield strength; $\boldsymbol{M}_e=\left(M_{e,j}\right)^T=\left(M_{e,1},M_{e,2},...,M_{e,j},...,M_{e,n}\right)^T$ is the vector of moments of elastic solution; $\boldsymbol{M}_r=\left(M_{r,j}\right)^T$ is vector of residual moments; $\boldsymbol{u}_e=\left(u_{e,i}\right)^T$ is the vector of displacements of elastic solution; $\boldsymbol{u}_r=\left(u_{r,i}\right)^T$ is the vector of residual displacements; $\boldsymbol{\lambda}_1=\left(\lambda_{1,j}\right)^T$ and $\boldsymbol{\lambda}_2=\left(\lambda_{2,j}\right)^T$ are the vectors of Lagrange multipliers of yield conditions (2); $\boldsymbol{F}$ is the vector of external loads; $\boldsymbol{u}_d^+$ and $\boldsymbol{u}_d^-$ are the vectors of prescribed upper and lower displacement variation bounds, respectively; $\boldsymbol{M}_0^{min}$ is the vector of minimum limit moments. For instance, it's components can be chosen as maximum values obtained by rigid-plastic structural optimization, from constructional requirements, etc.; $[C]$ is $(6s\times m)$- dimensional configuration matrix of local $\overline{\boldsymbol{u}}$ and that of global $\boldsymbol{u}$ displacements, containing unit and zero components (via s denoting a total number of finite elements and via $m=$ DOF denoting a number of global displacements); $[\overline{A}]$ is $(6s\times 2s)$-dimensional fictitious matrix of frame elements equilibrium eqns $[\overline{A}]\boldsymbol{M}_e=\overline{\boldsymbol{F}}$ in global coordinate system, where vector $\overline{\boldsymbol{F}}$ couples nodal forces of frame elements ends. Note that $[C]^T[\overline{A}]\boldsymbol{M}_e=\boldsymbol{F}$; $[\mathcal{K}]$ is $(2s\times 2s)$-dimensional quasi-diagonal frame stiffness matrix of elemental flexural stiffnesses EI_p/l_p $(p=1,2,...,s)$ in case of alike material, i.e. alike elasticity modulus E; $[K]=[C]^T[\overline{K}][C]$ is $(m\times m)$- dimensional stiffness matrix of the structure, where $[\overline{K}]$ is $(6s\times 6s)$- dimensional quasidiagonal matrix of the structure, obtained by assemblage of the elements stiffness matrices $[\overline{K}_p]$ in global coordinate system. Introducing the notations $a=\cos\alpha$, $b=\sin\alpha$, via α denoting an angle between finite element axis and horizontal global coordinate axis, and $k_2=-6l_p$, $k_3=4l_p^2$, $k_4=2l_p^2$, the element p stiffness matrix reads (up to multiplier EI_p/l_p)

$$[\overline{K}_p]=\begin{bmatrix} 12a^2 & -12ab & k_2a & -12a^2 & 12ab & k_2a \\ -12ab & 12b^2 & -k_2b & -12ab & -12b^2 & -k_2b \\ k_2a & -k_2b & k_3 & -k_2a & k_2b & k_4 \\ -12a^2 & 12ab & -k_2a & 12a^2 & -12ab & -k_2a \\ 12ab & -12b^2 & k_2b & -12ab & 12b^2 & k_2b \\ k_2a & -k_2b & k_4 & -k_2a & k_2b & k_3 \end{bmatrix}$$

The problem (1)-(9) solution yields the optimal distribution of limit moments, residual moments, residual displacements and Lagrange multipliers.

An elimination of residual moments and residual displacements from above mathematical model enables to reduce the set of unknowns in optimization problem up to limit moments $\boldsymbol{M}_0$ and Lagrange multipliers $\boldsymbol{\lambda}_1$, $\boldsymbol{\lambda}_2$. Having introduced the matrices

$$[H]=\left([C]^T[\overline{A}][\mathcal{K}][\overline{A}]^T[C]\right)^{-1}[C]^T[\overline{A}][\mathcal{K}] \quad (10a)$$

$$[G]=[\mathcal{K}][\overline{A}]^T[C][H]-[\mathcal{K}] \quad (10b)$$

residual moments and displacements are expressed by

$$\boldsymbol{M}_r=[G](\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2),\ \ \boldsymbol{u}_r=[H](\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2) \quad (11)$$

Then the modified mathematical model (1)-(9) reads: find:

$$\boldsymbol{L}^T\boldsymbol{M}_0\rightarrow min \quad (12)$$

subject to:

$$\left.\begin{aligned} &[G](\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2)-\boldsymbol{M}_0\le-[\mathcal{K}][\overline{A}]^T[C][K]^{-1}\boldsymbol{F} \\ &-[G](\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2)-\boldsymbol{M}_0\le[\mathcal{K}][\overline{A}]^T[C][K]^{-1}\boldsymbol{F} \end{aligned}\right\} \quad (13)$$

$$\left.\begin{aligned} &\boldsymbol{\lambda}_1^T\left([G](\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2)+[\mathcal{K}][\overline{A}]^T[C][K]^{-1}\boldsymbol{F}-\boldsymbol{M}_0\right)+ \\ &+\boldsymbol{\lambda}_2^T\left(-[G](\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2)-[\mathcal{K}][\overline{A}]^T[C][K]^{-1}\boldsymbol{F}-\boldsymbol{M}_0\right)=0 \\ &\boldsymbol{\lambda}_1\ge\boldsymbol{0},\ \boldsymbol{\lambda}_2\ge\boldsymbol{0} \end{aligned}\right\} \quad (14)$$

$$\boldsymbol{u}_d^-\le[H](\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2)+[K]^{-1}\boldsymbol{F}\le\boldsymbol{u}_d^+ \quad (15)$$

$$\boldsymbol{M}_0\ge\boldsymbol{M}_0^{min} \quad (16)$$

The moments and displacements of elastic solution (7) and (8) represent the sums of products of certain influence coefficients and loading components. These coefficients combine certain flexural stiffnesses of elements EI (here and further missing element index p for simplicity), where I is cross-sectional second moment of area A. Cross-sectional limit moment M_0 also depends on it's plastic modulus W_y being the double magnitude of cross-section first moment of area A. So, the cross-sectional input data parameter – moment of inertia I and optimization result parameter M_0 are functionally related via it's area A. To identify the actual relations between above parameters one must perform certain functional analysis in respect of A. In case of standard sections (e.g. of certain ones being employed in usual engineering practice) one must perform a relational approximation of above section characteristics. The above values for usual standard cross-sections can be approximated with sufficient accuracy by:

$$I=a_1A^{b_1},\quad M_0=\sigma_ya_3A^{b_3} \quad (17)$$

where a_1, b_1, a_3, b_3 are the identified constants, depending on the class of cross-sections to be manufactured.

It is obvious that direct incorporation of expression for the moment of inertia I vs M_0 (combining (17) and the expression of stiffness matrix $[\mathcal{K}]$ for subsequent obtaining of matrices $[G]$ and $[H]$) leads to rather complicated expressions. Analogously, obtaining of displacements and moments of elastic solution expressed via M_0 in matrices of relations (7) and (8) of mathematical model of optimization problem seems to be also very complicated. Thus, keeping in mind numerical difficulties even met when solving the above presented optimization problem formulations, a direct introduction of above relations into the mathematical model of optimization problem formulations and their subsequent solution is rather difficult even for small simple structures.

Seeking to avoid the above described difficulties the optimization problem is solved iteratively obtaining per each iteration an exact optimization problem solution in respect of introduced elastic response values (input data for problem). As for starting point a certain collection of cross-sections is chosen to identify elastic response values for the beginning of optimization problem solution process. Having obtained the optimal limit moments of cross-sections (then the corresponding to them A and I) the new elastic values are recalculated for subsequent optimization problem resolving. Such process is repeated until the optimization problem's convergence.

2.2. Numerical realization peculiarities of the optimization problem

Efficiency and even successful performing of iterative process depend upon many factors, coupling introduction of correct starting point and restrictions, solution processing, namely: choosing good primary set of design values; introduction bounds for minimal limit moments and extreme displacements, compatible with elastic-plastic structural behaviour; identifying lower and upper bounds of design values for running calculation procedures; optimization process handling aiming to avoid or overcome singularity or process "hanging" cases, etc.; correction of obvious deviation of the process direction from optimal trajectory when seeking for faster process convergence.

Starting point. Performed numerical experiments in respect proved that as starting distribution of limit moments the solution of rigid-plastic structural optimization [9, 10] can be employed. The starting point values are obtained as of increased proportionally by certain coefficient ones after checking that the resulting total displacements of starting structure are sufficiently less than those of the introduced by stiffness constraints.

Fixing lower bounds of limit moments for iterative procedures. Extended numerical experiments viewed that optimal solution finally yields the values greater than obtained by rigid-plastic optimization. Physically it can be explained that total structural ductility resource of elastic-plastic structure, being in elastic-plastic state, is maximally employed, resulting the objective function minimal magnitude. Generally, these values can be limited as nonzero ones. This case either increases the required number of solution iterations till problem convergence or often makes it unavailable at all. If certain constructional requirements in respect of limit moments are introduced, one must follow them to be no less than the ones from rigid-plastic optimization.

Introducing constraints for displacement magnitudes. Actually, the structure is created to response in elastic-plastic way in order to employ it's carrying resources conditioned by its ability to deform plastically. Therefore it makes sense to choose the values larger the ones of elastic solution components (at least of one, to allow development of plastic deformations). The magnitudes of residual displacements can be obtained having solved analysis problem [6,9] in respect of slightly increased limit moments, obtained by rigid-plastic optimization.

Iteration process handling. Iterative solution of direct optimization problem results a distribution of limit moments (note, that at least one displacement constraint is satisfied as equality [17] – in this case it is structure behavior dominant) if constraints both in respect of limit moments and displacements are introduced correctly. Find that having recalculated actual stiffness properties and solved analysis problem in respect of each problem iteration (solution) some displacements constraints can be violated or satisfied with certain reserve. Obviously an objective function and dominating displacement constraints deviate per iterations. If the process stops (results governing matrices to be singular/(close to singular), rank deficiency appears, etc.) one must perform artificial intervention ("impact") by reducing/increasing slightly accordingly the input/(previous iteration) limit moments. One can meet that the process obviously deviates from optimal trajectory. Then aiming to reduce computational efforts the input data for subsequent iterative procedures are to be analogously "impacted". A potential of residual moments can serve as a criterion of optimality for the structure states close to optimal. For the state close to optimal it is greater than compared with others (only one must check the displacement constraint to be not violated and limit moments to be greater than those of the rigid-plastic optimization). As a specific peculiarity one can mention the feature, that the iterative process being close to the optimal solution complicates: it stops or "hangs "quite often, the significant increment of the computational time for iterations is observed.

The principle algorithm scheme of direct flexural frame optimization is presented in Fig. 1.

3. Mathematical model of frame stepped optimization and it's numerical realization

Aiming to avoid the numerical solution difficulties, conditioned by complementarity conditions (5) or (14) when solving direct optimization problem (1)-(9) or its modification (12)-(16), a problem solution method per certain optimization cycles, realizing subsequent solutions of certain sub-problems or stages is employed [16, 17]. The cycles (optimization problem solution steps) are continued until the problem's solution converges. The cyclic problem solution process is also conditioned by previously described reason - relation between input data and final result of solution cycle (distribution of limit moments). The main stages of frame optimization cycle υ are:

1. Determining elastic response values $\boldsymbol{M}_{e,\upsilon}$ and $\boldsymbol{u}_{e,\upsilon}$ due to primary (starting point) chosen cross-sections or due

to cross-sections (obtained from limit moments $\boldsymbol{M}_{0,\upsilon-1}$) of previous optimization cycle $\upsilon-1$.

2. Defining residual response values $\boldsymbol{M}_{r,\upsilon}$ and $\boldsymbol{u}_{r,\upsilon}$ of the structure, being in state prior to plastic collapse, according fixed in stage 1 moments of elastic solution (i.e. solving structural analysis problem).
3. Determining an optimal distribution of limit moments $\boldsymbol{M}_{0,\upsilon}$ to satisfy strength, stiffness and constructional requirements in respect of total response values $\boldsymbol{M}_{\upsilon}=\boldsymbol{M}_{e,\upsilon}+\boldsymbol{M}_{r,\upsilon}$ and $\boldsymbol{u}_{\upsilon}=\boldsymbol{u}_{e,\upsilon}+\boldsymbol{u}_{r,\upsilon}$.

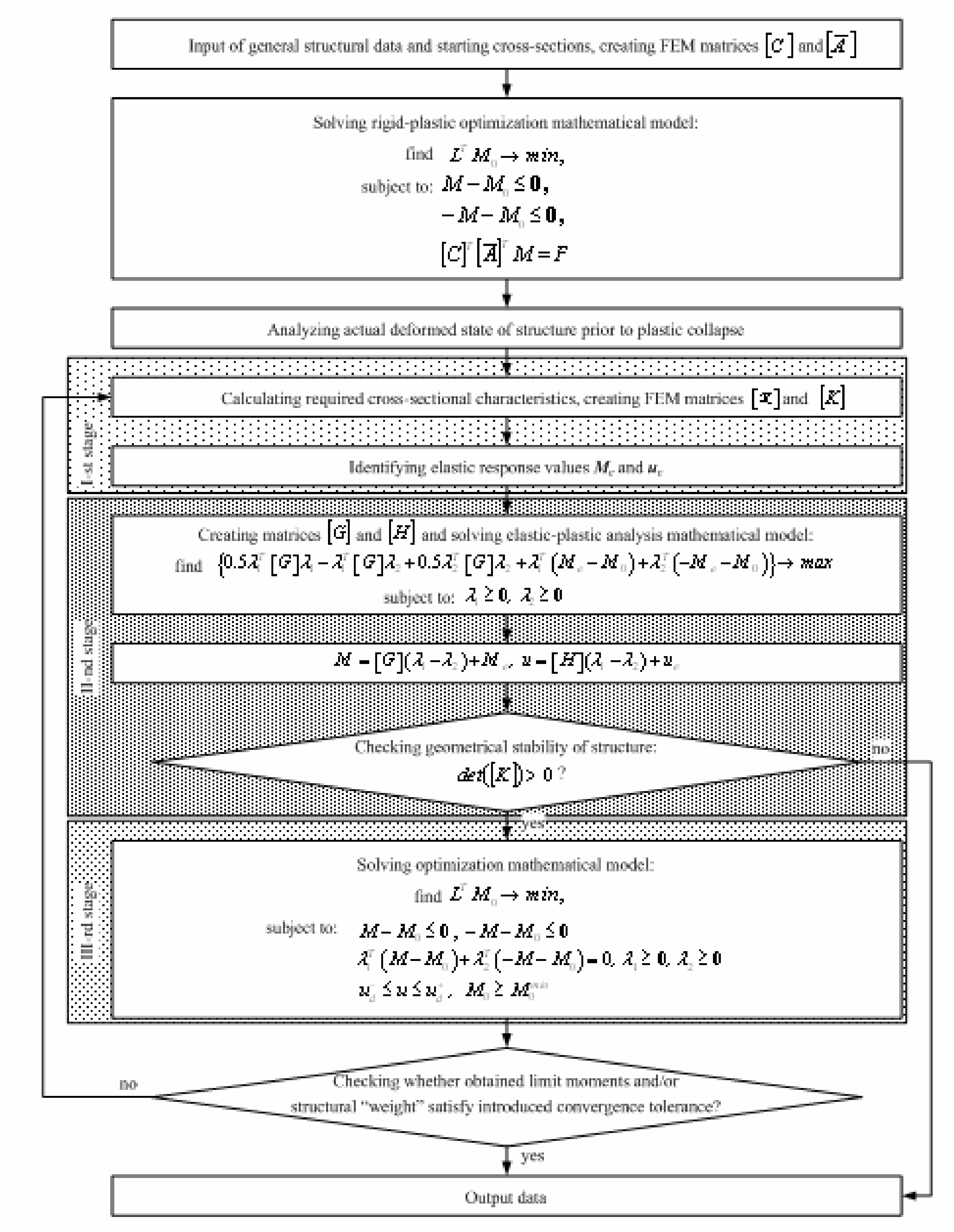

Fig. 1 Algorithm principle stages of direct structural optimization

Frame analysis problem (stage 2) is solved applying the mathematical model (for simplicity neglecting the cycle index υ), reading
find:

$$\left\{\begin{matrix} 0.5\boldsymbol{\lambda}_1^T[G]\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_1^T[G]\boldsymbol{\lambda}_2+0.5\boldsymbol{\lambda}_2^T[G]\boldsymbol{\lambda}_2+ \\ \boldsymbol{\lambda}_1^T(\boldsymbol{M}_e-\boldsymbol{M}_0)+\boldsymbol{\lambda}_2^T(-\boldsymbol{M}_e-\boldsymbol{M}_0) \end{matrix}\right\}\rightarrow max \quad (18)$$

subject to:

$$\boldsymbol{\lambda}_1 \geq \mathbf{0},\ \boldsymbol{\lambda}_2 \geq \mathbf{0} \quad (19)$$

Residual response values via Lagrange multipliers $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ are obtained applying formulae (11).

The structure of optimality criterion is as explained in previous section. Under necessity it can be transformed to actual total structural volume or total weight, having replaced $\boldsymbol{M}_0$ by corresponding vector of members areas $\boldsymbol{A}$ (applying formula (17)), or vector of the areas and material density ρ product. Then the optimality criterions are expressed by functions $\boldsymbol{L}^T\boldsymbol{A}\rightarrow min$ or $\rho\boldsymbol{L}^T\boldsymbol{A}\rightarrow min$. The mathematical model of determining limit moments (see stage 3) at cycle reads (for simplicity omitting the cycle counter υ):
find:

$$\boldsymbol{L}^T\boldsymbol{M}_0\rightarrow min \quad (20)$$

subject to:

$$-\boldsymbol{M}_0\leq-\boldsymbol{M},\ -\boldsymbol{M}_0\leq\boldsymbol{M} \quad (21)$$

$$u_{t,d}^{-}\leq u_t\leq u_{t,d}^{+},\ t=1,\ 2,\ ...,\ m_t \quad (22)$$

$$\boldsymbol{M}_0\geq\boldsymbol{M}_0^{min} \quad (23)$$

Here u_t is total displacement being constrained in certain direction t; m_t is the number of constrained displacements, $m_t\leq m$; $u_{t,d}^{+}$ and $u_{t,d}^{-}$ are the design upper and lover variation bounds of displacement u_t.

Applying the virtual displacement principle the displacement u_t can be expressed by

$$u_t=\sum_{p=1}^{s}\left(\boldsymbol{u}_{t,p}\right)^T\left[\bar{K}_p\right]\bar{\boldsymbol{u}}_p\frac{M_{0p,\upsilon-1}^{1/b3}}{M_{0p,\upsilon}^{1/b3}} \quad (24)$$

where $\boldsymbol{u}_{t,p}$ is the vector of actual end joint displacements of p-th member, obtained from elastic-plastic analysis of the frame; $\bar{\boldsymbol{u}}_p$ is the vector of virtual displacements of p-th member, corresponding to a virtual unit load applied in the direction of the *t*-th restricted displacement of the frame, being in the state prior to plastic collapse; $M_{0p,\upsilon}$ is the member cross-sectional limit moment of running optimization cycle υ.

Find that constraints (21)-(23) express strength, stiffness and constructional requirements, respectively.

Some notes on numerical realization peculiarities of stepped optimization problem. Generally, numerical realization and peculiarities are the same as described in section 2.2. However the solution process of stepped optimization problem is more turned to singularities and “hanging” when comparing with that of the direct optimization problem’s one. One must more often “impact” the solution process, following the governing criterion – approaching of constrained displacement to admitted magnitudes and/or observing deviations of potential of residual moments vs objective function magnitudes of the structure. As specific peculiarity one must mark an influence of displacement constraints (22) which employ function (24). They play a significant role in solution process. One must note that function must be convex to obtain a reliable solution of the problem (20)-(23). Thus, the Hessian of the function must be positively semidefined. The stepped solution process can lead to the situation when the Hessian of the function is not defined in the running cycle. It means that the domain of unknown limit moments $M_{0p,\upsilon}$ is a nonconvex one. This reason conditions the further iterative solution complications. The input data for cycle beginning, i.e. limit moments $M_{0p,\upsilon-1}$, must be “impacted” to overcome this singularity accordingly when continuing optimization process, or one must stop it. When analyzing computer resources, employed for optimization problem solution, one must mark that direct optimization method requires hundreds or even more times of computational efforts vs problem solution by direct optimization method.

4. Numerical simulations

To compare the computational efforts and an efficiency of direct and stepped optimization problem solution as well as to perform numerical solution process analysis, a ten-storey single bay flexural steel frame (see Fig. 2, a) subjected by vertical and lateral loads was investigated. Direct optimization method via the mathematical model (12)-(16) and the stepped one – combining the mathematical models (18)-(19) and (20)-(23) were realized.

The structure FEM model contains 32 nodes, $s=40$ members and $n=80$ unknown bending moments, it’s DOF is $m=50$. An optimal distribution of $n_0=6$ limit moments of standard steel IPE sections, coupled in $\boldsymbol{M}_0=\left(M_{01},M_{02},\ M_{03},M_{04},M_{05},M_{06}\right)^T$ are to be identified to satisfy linear optimality criterion $\boldsymbol{L}^T\boldsymbol{M}_0$ and introduced nodal linear displacement constraints: for vertical ones – 5 cm and lateral ones – 20 cm (in absolute values), respectively. No constructional requirements in respect of minimum magnitudes of limit moments $\boldsymbol{M}_0^{min}$ are introduced. Steel properties are described via elasticity modulus E = 206 GPa and yield strength σ_y = 235 MPa.

Then the cross-sectional moment of inertia (in cm^4) and limit bending moment (in kNcm) are expressed via area A (in cm^2) by

$$I=0.791876A^{2.319975}$$

$$M_0=23.5\left(0.8294723A^{1.660459}\right)$$

Fixing lower bounds of limit moments. In order to fix for frame members the lower bounds of limit moments and to check correctness of limitations for displacement magnitudes (aiming to employ structural ductility resources the structure is designed to response in elastic-plastic range) the rigid plastic optimization [6] was performed to identify $\boldsymbol{M}_0^{pl}$. It yielded :

$$\boldsymbol{M}_0^{pl} = \left(22564,\ 16339,\ 13465,\ 10336,\ 32678,\ 23801\right)^T$$

resulting the objective function magnitude $\boldsymbol{L}^T \boldsymbol{M}_0^{pl} = 3.5701 \times 10^8$ kNcm^2.

The solution of analysis problem for obtained limit moments in respect of reduced loading (reduction factor $\gamma_{red} = 0.999998$) resulted the extreme linear structural displacements (in cm) to develop:

in horizontal direction – in upper beam, namely:

$$u = u_r + u_e = 56.996 + 25.753 = 82.749;$$

in vertical direction – in middle span of the 6-th beam (floor), namely:

$$u = u_r + u_e = 17.303 + 2.508 = 19.810.$$

An investigation of elastic response limit of the above structure resulted in load reduction factor $\gamma_{red} = 0.66322$, yielding extreme displacements (in cm) to develop in upper beam - $u_e = 17.080$, and vertical ones in the middle span of 8-th floor - $u_e = 1.663$. Thus, introduced displacement constraints for vertical (5 cm) and horizontal (20 cm) ones do not restrict the development of plastic deformations – thus, they are chosen in correct way.

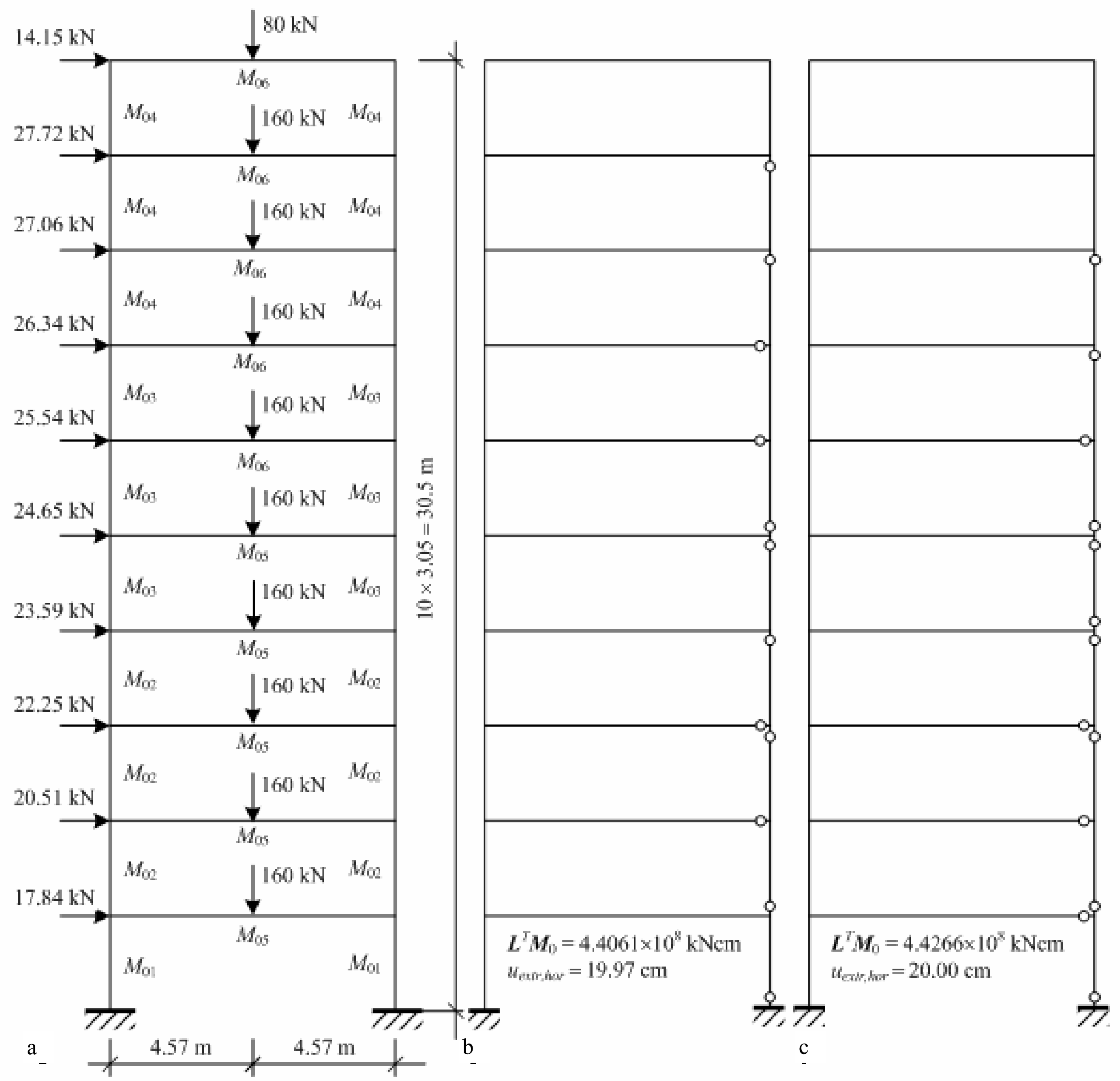

Fig. 2 10-storey frame: a - design scheme; b - positions of plastic hinges due optimal solution in Table 2; c - positions of plastic hinges due optimal solution in Table 4

Choosing starting point. The distribution of primary limit moments was taken to be of $\boldsymbol{M}_0^{pl}$, multiplied by correction factor $\gamma_{cor} = 1.5$. This distribution ensures the sufficiently small displacements vs introduced constraints: extreme total horizontal displacement $u = 14.620$ (with residual counterpart 0.0056) (in cm) developed in upper beam, and that of vertical one - $u = 1.582$ (no valuable residual counterpart fixed) fixed in the middle span of 9-th floor. Only one cross-section of the structure was in plastic state – minimal residual structural response is observed.

An optimal solution via direct optimization problem (12)-(16) was obtained in 13 iterations (see last row of Table 1) under above described conditions. Further iterations, deviating towards this solution required the significant computational resources for calculations and resulted in no essentially better result. Only one "impact" (an increment of limit moments by 0.1%) to overcome the process singularity was necessary after the 6-th iteration.

Table 1

The optimum "weight" 10-storey frame solution convergence per iterations. Direct optimization problem. Starting point 1.5 $\boldsymbol{M}_0^{pl}$, $\boldsymbol{M}_0 \geq \boldsymbol{M}_0^{pl}$

Iteration number	M_{01}	M_{02}	M_{03}	M_{04}	M_{05}	M_{06}	$\boldsymbol{L}^T\boldsymbol{M}_0 \times 10^8$	$u_{extr,hor}$
0	33846	24509	20198	15504	49017	35701	5.1799	14.62
1	26323	21752	19008	11454	38689	24683	4.0122	29.26
2	70832	25482	19380	13216	46136	31858	5.0593	15.12
3	33482	18899	18537	10410	43435	28548	4.3694	23.79
4	24614	23102	18605	10814	45326	29418	4.5271	19.01
5	24150	21282	18710	11620	42564	28348	4.3325	21.32
6	27982	23023	18670	11081	44932	29368	4.5319	18.63
7	23.746	24832	19237	10603	46502	27800	4.6502	19.84
8	22564	21491	18592	10440	43639	29261	4.4086	20.77
9	27261	22288	18665	11513	42818	29041	4.4104	19.93
10	26480	21360	18554	11767	42599	29353	4.3955	20.12
11	25870	21500	19232	11656	42867	29370	4.4178	19.87
12	28925	22529	18899	11691	43142	28220	4.4098	20.02
13	**25158**	**21984**	**18308**	**11786**	**43402**	**28949**	**4.4130**	**20.00**

Table 2

The optimum "weight" 10-storey frame solution convergence per iterations. Direct optimization problem. Starting point 1.5 $\boldsymbol{M}_0^{pl}$, $\boldsymbol{M}_0 \geq \boldsymbol{0}$

Iteration number	M_{01}	M_{02}	M_{03}	M_{04}	M_{05}	M_{06}	$\boldsymbol{L}^T\boldsymbol{M}_0 \times 10^8$	$u_{extr,hor}$
0	33846	24509	20198	15504	49017	35701	5.1799	14.62
11	23975	21960	23769	10683	43476	28865	4.4846	20.33
21	25554	22009	18651	11705	42761	29214	4.4035	20.04
66	**29654**	**22563**	**18603**	**11664**	**42399**	**28901**	**4.4061**	**19.97**

To check an influence of limiting bounds for limit moments $\boldsymbol{M}_0 \geq \boldsymbol{M}_0^{min}$ to the problem's solution convergence, this problem was resolved for constraints $\boldsymbol{M}_0 \geq \boldsymbol{0}$. An optimal solution was obtained only in 66 iterations (see last row of Table 2), that obviously illustrates the necessity to employ the limiting bounds, determined by rigid-plastic optimization. One "impact" (increment of limit moments by 0.1%) to overcome the process singularity was necessary to introduce after the 25-th iteration. The direct optimization problem was also solved for another starting point taken as 1.3 $\boldsymbol{M}_0^{pl}$. An optimal solution was obtained by 16 iterations (see last row of Table 3).

Analyzing three above optimal solutions one can find that under slight deviation of the objective function magnitude only two first limit moments vary more in respect of each other, when the other ones change insignificantly (in percentage). Actually, the final selection of cross-sections/limit moments is performed from prescribed discrete set of sections, therefore a precise convergence (requiring relatively large number of iterations, i.e. computational effort) is not of actual necessity. From the other hand possessing of a certain set of solutions in an area close to the optimal solution (direct criterion – objective function magnitude, indirect criterions – active limiting displacement constraints and magnitude of potential of residual moments) gives a certain space for engineering decisions of the structure, close to "expected exact" optimum.

The frame was solved by stepped optimization method via optimization cycles (see section 3) under the same input conditions, i.e. primal distribution of limit moments 1.5 $\boldsymbol{M}_0^{pl}$ and $\boldsymbol{M}_0 \geq \boldsymbol{M}_0^{min}$. An optimal solution was obtained by 14 iterations (see last row of Table 4). One must note that the stepped optimization via cycles is more sensitive when handling convergence and successful solution processing. The introduced slight "impacts" to the limit moments of previous cycles were conditioned by the

Table 3

The optimum "weight" 10-storey frame solution convergence per iterations. Direct optimization problem. Starting point 1.3 $\boldsymbol{M}_0^{pl}$, $\boldsymbol{M}_0 \geq \boldsymbol{M}_0^{pl}$

Iteration number	M_{01}	M_{02}	M_{03}	M_{04}	M_{05}	M_{06}	$\boldsymbol{L}^T \boldsymbol{M}_0 \times 10^8$	$u_{extr,hor}$
0	29333	21241	17505	13437	42481	30941	4.4893	19.61
5	22597	24711	18400	11384	44181	29103	4.4842	19.32
10	23349	21094	18467	11490	42188	29008	4.3303	21.48
15	29994	21345	18370	11815	41923	29507	4.3969	20.10
16	**32486**	**22602**	**18678**	**11930**	**41698**	**29158**	**4.4100**	**19.85**

Table 4

The optimum "weight" 10-storey frame solution convergence per cycles. Stepped optimization problem. Starting point 1.5 $\boldsymbol{M}_0^{pl}$, $\boldsymbol{M}_0 \geq \boldsymbol{M}_0^{pl}$

Iteration number	M_{01}	M_{02}	M_{03}	M_{04}	M_{05}	M_{06}	$\boldsymbol{L}^T \boldsymbol{M}_0 \times 10^8$	$u_{extr,hor}$
0	33846	24509	20198	15504	49017	35701	5.1799	14.63
1	29533	24263	18984	13368	45639	31738	4.7523	16.40
2	28756	24506	18947	13218	45891	31715	4.7591	16.53
3	28468	24261	18758	13086	45432	31398	4.7115	16.92
4	28184	24018	18570	12955	44978	31084	4.6644	17.33
5	27902	23778	18385	12826	4528	30773	4.61789	17.78
6	27623	23540	18201	12697	44083	30466	4.5716	18.31
7	27346	23305	18019	12570	43642	30161	45259	18.26
8	27620	23538	18199	12696	44079	30463	4.5711	18.31
9	27344	23303	18017	12569	43638	30158	4.5254	18.89
10	27070	23070	17837	12443	43201	29856	4.4802	19.34
11	26935	22954	17748	12381	42985	29707	4.4578	19.72
12	26800	22840	17569	12319	42770	29559	4.4355	19.99
13	26773	22817	17641	12307	42728	29529	4.4310	20.00
14	**26747**	**22794**	**17624**	**12295**	**42685**	**29499**	**4.4266**	**20.00**

following criterions: reduction of objective function magnitude and an approach of extreme displacement magnitude to admitted bound. Note that of all displacement constraints only one - horizontal displacement of upper beam - was governing in all above described optimization cases (including their inner iterations) in all considered cases of the frame optimization presented in last columns of above mentioned Tables.

Analyzing response (via solution of analysis problem (18)-(19)) of optimal structure (see last rows of Tables 1-4). In all cases extreme horizontal displacements $u_{extr,hor}$ develop in the upper beam of the frame and vary from 19.97 cm to 20.00 cm. The optimum structural "weight" vary from 4.4061×10^8 kNcm^2 till 4.4266×10^8 kNcm^2. Bounding magnitudes for vertical displacement are not achieved. Maximum magnitude develops in the middle span of 8-th flour and varies from 2.16 cm (for Table 4 solution) till 2.31 cm (for Table 2 solution). In all cases plastic counterparts of bending moments and displacements appear. Analyzing collections of plasticized cross-sections due to the presented solutions, one can find that only part of them differs (partial variation of certain number of plasticized cross-sections are possible for the solutions close to expected "exact theoretical" one). As two principle collections of plasticized cross-sections (compatible with plastic hinges activated) can be treated the ones corresponding to solutions of Table 2 (see Fig. 2, b) and Table 4 (see Fig. 2, c), respectively.

When comparing direct and stepped optimization results of the frame under identical constraints and starting point one can find only slight difference (0.308%) in respect of objective function magnitude and relatively small difference ranging from 0.037% (in respect of M_{02}) till 6.32% (in respect of M_{01}) vs significant computational effort savings in case of stepped optimization.

5. Discussion and concluding remarks

In this investigation, two possible ways to solve elastic-plastic frame optimization problem, considering strength and stiffness constraints are investigated. The structural optimization is aimed to find an optimal distribution of cross-sectional areas (limit moments), sufficient for subsequent selection of the actual ones from discrete set of standard steel sections.

Numerical realization aspects taking into account identified actual physical relations of standard steel sections properties were analyzed. The investigation proved that the solution process is sensitive to starting point, introduced lower bounds for designed variables. Numerical processing analysis viewed a necessity of solution trajectory handling, aiming to reduce computational efforts and/or either to avoid the process singularities or it's "hanging".

It was defined that increased magnitude of the potential of residual moments is compatible with reduced objective function magnitude of structure for it's state close to an optimal one. Then corresponding limit moments exceed/are equal the ones of rigid-plastic optimization. This feature should be directly incorporated in further developments of iterative optimization methods of elastic-plastic structures.

Finally, one can conclude that:

1. Stepped optimization method via optimization cycles illustrates the sufficient accuracy and significant savings of computational resources vs direct optimization method. This is especially important in case of larger structures.

2. Sufficient accuracy of the method is conditioned by small deviations of possible solutions from "expected exact" one, keeping in mind certain variation of limit moments (i.e. sections areas) under small change of structural optimal "weight". Moreover a final actual selection of profiles for the structure actually is performed from discrete set of manufactured sections. This usually envelopes a variation of cross-sectional areas per available solutions of structural optimal "weight".

3. The convenient relation of displacement vs limit moments, employed in stepped optimization problem solution process, is proposed.

4. The reliability of obtained results is ensured by the application of extreme energy principle for actual structural state definition (direct optimization) and of the actual structural state checking/evaluation in each optimization cycle (stepped optimization). The proposed principles of stepped optimization can be employed for optimum design of more complicated structures.

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PLIENINIO LENKIAMO RĖMO TIESIOGINIO IR PAKOPINIO OPTIMIZAVIMO UŽDAVINIAI, ATSIŽVELGIANT Į STIPRUMO IR STANDUMO APRIBOJIMUS, IR JŲ SKAITINIS SPRENDIMAS

R e z i u m ė

Nagrinėjamas lenkiamo plieninio tampriai plastiško rėmo optimizavimas tiesioginiu ir pakopiniu metodais. Abiem atvejais naudojami konstrukcijos tampraus atsako dydžiai, kurie savo ruožtu yra sąlygojami optimizuojamų konstrukcijos parametrų. Todėl uždavinys sprendžiamas iteraciniu būdu, kiekvieną kartą perskaičiuojant šiuos parametrus pagal gautus konstrukcijos optimizavimo rezultatus, kol uždavinio sprendimas konverguoja. Taikant pakopinį metodą, optimizavimo uždavinys formuluojamas nuosekliais ciklais, apimančiais analizės ir optimizavimo uždavinius. Tai leidžia tiesiogiai nenaudoti papildomumo sąlygų, įeinančių į tiesioginio optimizavimo uždavinį. Nekorektiškas apribojimų dydžių ir pradinių parametrų parinkimas skaičiavimo procesui pradėti, sprendžiant tiek tiesioginio, tiek pakopinio optimizavimo uždavinius, iš esmės komplikuoja iteracines/pakopines sprendimo procedūras arba net padaro uždavinius neišsprendžiamus. Sumaniai reguliuojant pakopinį procesą užtikrinama efektyvi ir sėkminga uždavinio konvergencija, daug mažiau naudojami kompiuterio ištekliai, palyginti su tiesioginės optimizacijos metodu. Siūlomi metodai yra iliustruojami skaičiuojant 10 aukštų plieninį rėmą, projektuojamą panaudojant standartinius plieno profilius.

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ON DIRECT AND STEPPED OPTIMIZATION PROBLEMS OF FLEXURAL STEEL FRAME AND THEIR NUMERICAL REALIZATION CONSIDERING STRENGTH AND STIFFNESS CONDITIONS

S u m m a r y

Optimization of flexural steel frame responding to loading in elastic-plastic range via direct and stepped methods is considered. Both methods employ structural elastic response values, finally conditioning the optimized parameters of the structure. It enables the problem to be solved iteratively by recalculating the above values for running optimization problem solutions and is continued until problem's convergence. The formulation of stepped optimization problem via optimization cycles, containing subsequent solution of analysis and optimization problems, allows to avoid direct evaluation of complementarity conditions being included in the direct optimization problem. A noncorrect choosing of bounding constraints, the starting point values essentially complicate the iterative and/or stepped procedures or even makes the problem to be the unsolvable one. An intelligent handling of stepped process procedures ensures an efficient and successful convergence of the problem with significant computing resources savings in respect of direct optimization procedures. The proposed techniques are illustrated via the solution of ten-storey steel frame, designed from standard steel sections.

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О ЗАДАЧАХ ПРЯМОЙ И ШАГОВОЙ ОПТИМИЗАЦИИ ИЗГИБАЕМОЙ СТАЛЬНОЙ РАМЫ И ИХ ЧИСЛЕННОЙ РЕАЛИЗАЦИИ ПРИ ПРОЧНОСТНЫХ И ЖЁСТКОСТНЫХ ОГРАНИЧЕНИЯХ

Р е з ю м е

Представлены методы прямой и шаговой оптимизации стальной изгибаемой рамы в упруго-пластичной стадии работы. В реализации обоих методов используются характеристики упругого поведения конструкции, которые в свою очередь обуславливают оптимизируемые параметры конструкции. Поэтому задачи оптимизации решаются итерационно, пересчитывая упругие параметры конструкции до достижения сходимости процесса расчёта. Используя шаговый метод, задача оптимизации формулируется в виде последовательных циклов, объединяющих в себе задачи анализа и оптимизации. Такой подход позволяет избежать непосредственного использования условий дополняющей нежесткости, входящих в задачу прямой оптимизации. Некорректный выбор ограничений задачи, а также исходных параметров значительно осложняет процесс решения и даже может сделать задачу неразрешимой. Направленное воздействие, используемое в процессе шаговой оптимизации, значительно улучшает сходимость решения по сравнению с методом прямой оптимизации, что в свою очередь снижает затраты расчетных ресурсов. Предложенные методы иллюстрируются численным примером расчета стальной десятиэтажной рамы, проектируемой из стандартных металлических профилей.

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