# Modelling of precision line scale comparator

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#### 1. Introduction

Optical measurements have become essential in the modern metrology thanks to their higher accuracy and enormous expansion of the optical technologies. Improvements and application of microscopy, modern lithography, laser interferometry increase the required accuracy level and the efficiency of line standards calibration as well as set very high requirements on the stability and accuracy of geometrical dimensions of these components since, lengths must be known with uncertainties of the nanometer range. Modern technical vision systems with CCD (charge coupled device) cameras are capable to achieve high spatial resolution and high sensitivity measurements of signals in the optical microscope.

Error-related problems specific to length measurements are caused primarily by geometrical and thermal deviations of the comparator components and the scale. One of critical tasks in the dynamic calibration of the scales is microscope image acquisition, i.e. bringing the specimen into focus before taking an image and measuring any feature. Due to imperfections of the stage, i.e. inaccuracy of the motion of the scanning mechanism and vibrations, the microscope slide is not perfectly perpendicular in regard to the optical axis of the imaging system. Measuring even a slightly vibrating structure with any degree of accuracy is the prone to an error with an optical microscope. Any deviation in the distance of microscope lens with respect to the scale - for example, when the surface is in motion during the data acquisition process - introduces measurement errors. The magnitude of the resulting error can range from a few nm to several um depending on the magnitude of such disturbances and the measurement setup. The most common error associated with small vibrations is the error in detection of the line scale graduation.

The paper describes FEM techniques used for behaviour simulation of the CCD microscope and calibrated line standard itself under the influence of dynamic and thermal factors, like variations of environmental temperature, vibrations in the structure caused by seismic excitation. A finite element model has been developed in order to investigate influences of thermo-mechanical processes in the length comparator structure and to evaluate possible variations of geometrical dimensions of the line scale and the microscope that lead to increasing uncertainty of line scale measurements. The calculations of measurement error due to defocusing of the scanning mechanism and angular misalignment between the microscope and the scale were performed using MATLAB. They are also presented here.

#### 2. Modelling of comparator

A precision displacement measuring system is a very complex structure consisting of various sophisticated geometric shapes. The design of such systems according to traditional strength criteria often does not ensure the elimination of very small displacements (range from several microns to a few nanometers) of some components caused by elastic vibrations of the construction and temperature deformations.

The state-of-the-art FE technique has been applied in order to evaluate the possible influence of dynamic and thermal factors upon the inaccuracies of measurement. Two basic physical phenomena are of interest:

1. displacements of the structure as a consequence of applied dynamic excitations, as well as, nonhomogeneities of the temperature field;

2. heat transfer inside of the structure caused by an external temperature field.

All necessary aspects of the dynamic behavior of the comparator can be investigated by employing small displacement elastic structural models as [1]

$$[M]\{\dot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{F(t)\} + \{Q(t)\}$$
(1)

[K], [M], [C] are stiffness, mass and damping matrices of the structure;  $\{F\}$  is nodal vector of external excitation forces;  $\{Q\}$  is nodal force vector caused by the temperature propagation effect;  $\{U\}$ ,  $\{\dot{U}\}, \{\ddot{U}\}$  are nodal displacement, velocity and acceleration vectors.

FE models enable us to simulate all 3D displacement or vibration patterns of the structure. Vertical vibrations of the microscope may lead to defocusing. Vibrations in the direction of motion may cause detection errors in determining positions of graduation lines.

In practice the excitation vectors  $\{F\}$  caused by external dynamic effects or by moving parts of the structure are not known explicitly, but often are subjected to external excitation propagating through the base and supports of the comparator structure. A spectrum analysis is one in which results of a modal analysis are used with a known spectrum to calculate displacements and stresses in the structure modeled. The model is capable of predicting the system's behavior under thermal load and enables us to investigate the thermo-mechanical processes in the system, by taking into account both static and dynamic disturbances and parameter deviations.

It can be assumed that the foundation of the microscope holder is vibrating according to a known random distribution in a wide frequency range, see Fig. 1. For precise length measurements systems the displacements of certain points of the system as a consequence of construction deformations are very important. Referring to the equation (1) the model has been adapted in order to analyse the construction of CCD microscope.

The equivalent stiffness of threaded junctions of the holder has been modelled and analysed since it can be influenced by temperature changes, geometrical tolerances and other factors. The model of the whole system and the submodel of the microscope developed in ANSYS are depicted in Fig. 1.



Fig. 1 Principal set-up of the length comparator and the microscope

#### 2.1. Dynamic behavior

The dynamic response of the comparator as a result of seismic excitation can be investigated by employing small displacement elastic structural models

$$[M] \{ \dot{U}_{rel} \} + [C] \{ \dot{U}_{rel} \} + [K] \{ U_{rel} \} = -\begin{cases} m_1 \\ m_2 \\ \vdots \\ m_n \end{cases} \times a(t) + \{ F(t) \}$$

$$(2)$$

where  $\{U_{rel}\}\$  is nodal vector of relative displacement with respect to moving foundation that is accelerated by  $a(t) = \ddot{s}(t)$ . The displacement vectors can be expressed as nodal vector of displacement of non-fixed foundation points  $\{U_N(t)\} = \{U_{Nrel}\} + \{U_{Nk}\}\$  and nodes that vibrate according to a known distribution  $\{U_K(t)\} = \{U_K\}s(t)$ . Then equation (1) becomes

$$\left[M_{NN}\right]\left\{\dot{U}_{Nrel}\right\} + \left[C_{NN}\right]\left\{\dot{U}_{Nrel}\right\} + \left[K_{NN}\right]\left\{U_{Nrel}\right\} = \left[\hat{M}\right]\left[\ddot{U}_{K}\right]$$
(3)

where  $\left[\hat{M}\right] = \left[M_{NN}\right] \left[K_{NN}\right]^{-1} \left[K_{NK}\right] - \left[M_{NK}\right]$ 

The left side of the equation represents the construction that is fixed at points of seismic excitation and the right side – inertia forces that act in every degree of freedom when the vibrations of the foundation are



Fig. 2 Displacements of the microscope structure in response to applied seismic excitations: a - maximum displacements of the construction during seismic excitation; b - their dependence (point A); on different equivalent stiffness at coupling points of the construction

Vibrations of the microscope structure have been calculated using linear spectral method of analysis that treats the synthesis of wave forms as the superposition of many frequency components. According to the linear spectral method approach maximal displacement of every construction point is calculated knowing the maximal displacements in normalized coordinates. Response spectrum of the fixed foundation is determined from the equation of linear oscillator with natural frequency  $\omega$ 

$$\ddot{x} + 2\omega \vartheta \dot{x} + \omega^2 x = a(t) \tag{4}$$

The acceleration response spectrum  $A(\omega)$  is determined after estimating the maximum deviation from the nominal position at a certain time for every natural frequency calculated for the seismic excitation period. The maximal displacement along *i* th normal coordinate

$$z_{i\,max} = \frac{A(\omega_i)}{\omega_i^2} \{y_i\}^T \left[\hat{M}\right] \{V\}$$
(5)

where  $\{y_i\}$  is natural mode and  $\{V\}$  is a known vector for certain structure.

Analyzing microscope construction it is necessary to calculate the response spectrum for the area that is used for fixing of the microscope, see Fig. 1. Then FE model of the structure that has relatively rough elements is simulated and a spring of stiffness w with mass unit is placed at the fixing point of the substructure. Thus the acceleration response spectrum is recalculated for the fixing point of the microscope.

Analysing very small displacements of the microscope the equivalent stiffness of thread junctions is to be considered that are influenced by global temperature deformations, geometric tolerances. Calculations are made with several values of the ratio between equivalent elastic modules of thread junction layer  $E_i$  and the material of

the construction E, 
$$\frac{E}{E_j} = 1 \div 1000$$
.

Maximal displacements of the structure in x direction during seismic excitation are presented in Fig. 2. It is clearly seen that the largest displacement is at the bottom of the microscope objective. Maximal displacements of that point during the excitation for different equivalent stiffness of thread junctions in the structure are shown in Fig. 2, b.

The deviation of the microscope objective from the nominal position due to possible seismic excitations can reach more than 100 nm.

#### 2.2. Temperature deformations

It is impossible to avoid very small mechanical deformations of the comparator components resulting by external dynamic effects or temperature fluctuation that are accuracy limiting factor in the length metrology. This factor becomes very important while calibrating long scales. Because of high requirements for geometrical stability of calibration system, temperature deformations induced by the changes of some hundredth parts of Kelvin must be considered. For high precision calibration systems, it is essential to evaluate an average volume temperature of some parts of a mechanical comparator as well as the temperature of the scale.

Vector  $\{Q\}$  in equation (1) can be easily determined if the temperature field inside of the structure is known. If only the surrounding transient temperature field is known, temperatures inside of the structure can be obtained by solving the thermal conductivity problem

$$\begin{bmatrix} C_{th} \end{bmatrix} \left\{ \dot{T} \right\} + \begin{bmatrix} K_{th} \end{bmatrix} \left\{ T \right\} = \left\{ S_{\infty} \right\}$$
(6)

where  $[C_{th}]$ ,  $[K_{th}]$  are thermal capacity and conductivity matrices of the structure;  $[S_{\infty}]$  is nodal vector of heat sources determined by heat convection across the surface of the structure;  $\{T\}$  is nodal vector of temperature field inside the structure.

Under real calibration conditions, temperature

measurement is possible only at certain points. The response time of temperature sensors is also rather long (from several seconds to several minutes). Therefore, fast temperature changes cannot be detected and consequently, the measurement uncertainty increases [2].

The impact of temperature on mechanical deformation of the line scale can be simulated in several ways:

1. temperature values at certain points of the construction can be detected experimentally and used for the calculation;

2. temperature field can be calculated depending on the assigned non homogeneity of the environment temperature by taking into account heat convection processes between parts of the structure and its surrounding.

FEM techniques have been applied in order to evaluate the possible influence of thermal factors upon the inaccuracies of measurement. The displacements of the structure as a consequence of applied non homogeneities of the temperature field were investigated. Simulations of the line scale have shown that temperature changes of 1K can introduce deformations of the midline along the scale, see Fig. 3.



Fig. 3 Deformations of line scale model due to nonhomogeneity of ambient environment temperature

Due to these deformations the position between the scanning system and the scale changes as well as angular deviations can cause defocusing of the microscope.

#### 2.3. Defocusing

Line scales are calibrated by making carriage displacement, as measured by an interferometer corresponding exactly to the spacing between lines. The key to this correspondence is the line centering action of the photoelectrical microscope that occurs when the carriage is moving (or stopped) and a graduation is in view.

For high precision quantitative analysis of line profile images high quality digital images are necessary. The accuracy of images captured by photoelectrical microscopes is limited by the accuracy and stability of geometrical dimensions of optical components as well as the capabilities of digital imaging. The specimen must be brought into focus, i.e. the focal plane of the microscope must coincide the plane of measured line profile, before taking an image and measuring any feature. Due to imperfections of the stage, i.e. inaccuracy of the motion of the scanning mechanism, mechanical vibrations, thermal deformations the microscope slide is not perfectly perpendicular with respect to the optical axis of the imaging system what cause possible contributions to uncertainty budget from line position sensing, such as influence of adjustment of measurement line, influence of focus variation (defocusing), microscope axis alignment and other factors [3].

A common problem in length metrology of line standards is the determination of the exact location of an edge. In practice, however, exact edge information is generally impossible to obtain. Since line standards have to be calibrated within the range of some parts of a micron it is essential to locate the edge with very high precision through the use of sub-pixeling techniques.

The image of complex structures can be expressed as the linear superposition of the image of a single point source. The performance of a microscope is described not only by the magnification and the numerical aperture of the objective. A more complete approach to describe the imaging system is to determine the point spread function (PSF) in spatial domain or optical transfer function (OTF) in frequency domain [4]. If the image of a point source is known, it is possible to predict the image of any object imaged with the system. The image of the point source gives information about the brightness that is transmitted through the optical system, the distortion introduced by the optics, and the maximal possible resolution.

The image acquisition model of the microscope and CCD element is presented as

$$I(\omega) = \frac{1}{2\pi} \{ O(\omega) OTF(\omega) B(\omega) \} \otimes S(\omega)$$
(7)

where *O*, *OTF*, *B*, *S* and *I* are intensity of the object in Fourier domain, optical transfer function of the microscope, transfer function of a CCD pixel, sampling function of CCD and image intensity sampled with CCD respectively [4].

The microscope projects the line profile onto CCD element; the back focal plane of the microscope objective lens contains the Fourier transform of the image plane and the finite aperture of the microscope acts as a low-pass filter. The projected image is sampled by the square pixels of the CCD element. The magnification of the microscope is included in the CCD element by reducing the size and spacing between pixels.

The image of the profile is read as discrete output image i[n,m], where *n* and *m* are the number of rows and columns of CCD matrix. The efficiency of capturing images generated by an optical microscope onto the pixel array of a charge coupled device (CCD) is dependent upon several factors, ranging from the degree of microscope defocusing  $\Delta z$ , objective magnification *M* and numerical aperture *NA* to the CCD array size and the dimensions of individual pixels within the array.

When we want to measure any feature of images recorded by a camera it is important to know how the original image is sampled by the camera pixels and digitized in order to covert the amplitude of a signal into finite number of discrete levels. The number of pixels in the horizontal and vertical directions  $S(\omega)$  and size of indi-

vidual pixel  $B(\omega)$  indicate the resolution of the camera.

With reference to equation (7) the simulation of acquisition of the line profile image has been conducted using MATLAB and assuming that the size of CCD camera is 512x512 pixels and pixel size of  $6.8x6.8 \mu m$ . The intensity profiles from graduation line images have been calculated varying defocus parameter  $\Delta z$  of the microscope and the midline position of the profile detected.

In general focused images contain higher frequency components than defocused images of a scene. The effect of defocusing on optical image of the line profile creates blurring in the image, it gradually cuts out parts of the object as they move away from the focal plane. The practical consequence is that these parts become darker and eventually disappear. This degraded image is further being processed by CCD camera, which in turn is effected by various noise sources such as photon noise, readout noise, dark current, quantization noise and other factors. Although modern CCD cameras are mainly limited by photon noise, quantization noise that is inherent to the quantization of the pixel amplitudes into a finite number of discrete levels by the analog-to-digital converter also can contribute to the uncertainty of localization of profile edges. Suppression of high frequency components in the image due to defocusing leads to a certain misrepresentation of the amplitude of optical signal.

Simulated images and intensity profiles of a graduation line are depicted in Fig. 4.



Fig. 4 Modelled images of graduation line: a, c – images without defocusing of the microscope and with defocusing of 10 µm; b, d – profiles used for the determination of the middle point

In the actual measurements the line image is analyzed and the average profile of graduation line is obtained by summing picture element intensities of each row from the area of a window defined by CCD matrix. The middle point of the line profile is obtained by averaging of rising and falling edges from the interval indicated by lower und upper threshold values; typically it ranges from 0.45 to 0.8. This method allows good measurement results if the profile of the line is symmetrical [5, 6].

The results of modeling and calculations performed in order to evaluate both the influence of defocusing of scanning mechanism and angular alignment errors resulting from mechanical deformations of line scale or microscope construction are presented in Fig. 5. It can be seen that relatively small angular misalignment increase the error of line centre detection significantly when microscope is out of focus. Therefore the influence of geometrical errors on performance of the measurement system and its nonlinearities can be reduced by a proper arrangement of the scanning system and the line scale.



Fig. 5 Error of midline position of line profile due to defocusing of the microscope and angular deviations between the line scale and microscope

The microscope is usually adjusted perpendicular to the scale plane within  $0.2^{\circ}$  and that can be quite large error if the microscope is not properly focused.

High precision length comparators for precise positioning of the microscope usually employ automatic focusing systems. Modern photoelectrical microscopes allow detecting the position of the line centre with an uncertainty of 2 nm depending also on the quality of graduation lines. High quality lines have equal width and sharp edges in the range from 1  $\mu$ m to 10  $\mu$ m.



z-position, µm

Fig. 6 Measurement error in x and y directions depending on the z position of the microscope

Measurements conducted at German National Metrology Institute (PTB) using high precision microscope positioning system with double spring mechanism (see Fig. 6) showed that errors of the microscope focus position in x and y directions caused by the movement displacement of the microscope in z direction are quite small and amount to about several nanometers [7].

## 3. Conclusions

Error-related problems specific to a precision length comparator were investigated and advanced FE

modeling techniques were applied in order to represent the structural behavior of the comparator. A novel FE model has been developed and influences of thermo-mechanical processes in the length comparator structure and possible variations of geometrical dimensions of the line scale and the microscope that lead to increasing measurement uncertainty of line scale measurements has been evaluated. Modelling of seismic excitations in the comparator structure has shown that maximum displacements are expected at the bottom plane of the microscope objective and can amount more than 100 nm. The image acquisition model of optical microscope and CCD camera is presented and the influence of microscope defocusing on line detection accuracy has been evaluated. Deformations induced by temperature changes and dynamic processes can significantly reduce the measurement accuracy of the system and therefore has to be eliminated by proper alignment of the scanning system and the line scale or the use of error compensating algorithms.

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# PRECIZINIO BRŪKŠNINIŲ ILGIO MATŲ KOMPARATORIAUS MODELIAVIMAS

## Reziumė

Straipsnyje nagrinėjamos precizinio brūkšninių matų komparatoriaus paklaidos, taikant jų modeliavimui baigtinių elementų metodą. Tikslumo reikalavimai tokioms sistemoms yra gana aukšti, todėl svarbi netgi šimtųjų laipsnio dalių ar dinaminių procesų sukeltų mažų deformacijų įtaka komparatoriaus geometrinių matmenų stabilumui. Sudarytas naujas BE modelis leido įvertinti galimus konstrukcijos poslinkius dėl seisminio pobūdžio virpesių, veikiančių per konstrukcijos pamatą ir atramas, bei aplinkos temperatūrinių pokyčių, kurie sąlygoja mikroskopo konstrukcijos bei matuojamos skalės padėties ar kampinius nuokrypius. Atlikti skaičiavimai parodė, jog komparatoriaus tikslumui turi įtakos optinės brūkšnio detektavimo sistemos fokusavimo paklaidos bei galimi kampiniai optinių elementų svyravimai, todėl projektuojant tokias sistemas turi būti minimizuota šias paklaidas sukeliančių veiksnių įtaka bei išnagrinėtos paklaidų kompensavimo galimybės.

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# MODELLING OF PRECISION LINE SCALE COMPARATOR

#### Summary

The paper deals with errors of precision line scale comparator investigated by employing finite element modeling. Accuracy requirements for such systems are high therefore the influence of small deformations on geometrical dimension stability of the comparator caused actually by some hundredth parts of Kelvin or dynamic processes is important. A novel FE model enabled us to evaluate possible displacements of the system due to seismic vibrations transmitted by the construction base and supports as well as ambient temperature changes that determine position and angular deviations between the microscope and the measured scale. The performed calculations have proved that the accuracy of the comparator is influenced by focusing errors of the optical line detection system and possible angular deviations of optical components therefore the weight of the factors inducing these errors must be minimized as well as compensation possibilities of these errors have to be investigated in the design of such systems.

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# МОДЕЛИРОВАНИЕ ПРЕЦИЗИОННОГО КОМПАРАТОРА ДЛЯ КАЛИБРОВКИ ЛИНЕЙНЫХ ШКАЛ

## Резюме

В статье рассматриваются погрешности прецизионного компаратора для калибровки линейных шкал с применением метода конечных элементов (КЭ). Требования точности к системам такого рода высоки, поэтому влияние даже малых отклонений конструкции, вызванных динамическими процессами или изменениями температуры даже на сотые доли Кельвина, является существенным. Разработана новая модель системы, основанная на КЭ, позволила оценить возможные смещения конструкции из-за вибраций сейсмического происхождения, воздействующих через фундамент конструкции и опоры, а также из-за изменения температуры окружающей среды, обусловливающие линейные и угловые отклонения конструкции микроскопа и измеряемой шкалы. Проведенные расчеты позволяют заключить, что на точность компаратора влияют погрешности фокусировки оптической системы определения положения штриха измеряемой шкалы, а также угловые колебания оптических компонентов, поэтому при проектировании таких систем влияние факторов, порождающих эти погрешности, должно быть минимизировано; следует также проанализировать возможности компенсации рассматриваемых погрешностей.

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