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Stress and strain analysis of layered beams under hygrothermal and mechanical loads

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1. Introduction

Conventionally the used in practice layered construction by joint of the layers can be divided into two groups: the structures with rigidly jointed layers and the structures with partially rigidly jointed layers. The following two differences substantially determine which calculation method can be used for the stress and strain calculation. When the layers are connected using flexible connectors or the joints of the layers are partially rigid for the calculation of the stresses build-up bars theory can be applied [1]. For the calculation of sandwich type layered beam other calculation theories can be used as well [2-5]. Frequently the layers of the layered structure are connected absolutely rigidly. When the joint of the layers is absolutely rigid then all the layers deform together and slip of the layers should not be taken into account. In this case the stresses of the layers are proportional to its stiffness.

For the calculation of hygrothermal stresses of the layered beam with absolutely rigid joint when shear strain is taken into account the following methods can be used [6,7]: classical beam theory, first-order beam theory, second-order beam theory, third-order beam theory and etc. The first and higher order beam theories are applied to thick or deep beam, for which shear deformation is significant. Many engineering tasks can be solved using the classical beam theory. This theory is applicable to thin beams. It is based on the assumption that plane sections that are perpendicular to the neutral layer before bending remain plane and perpendicular to the neutral layer after bending. This assumption is known as plane section hypothesis. The classical beam theory is also based on the assumption that transverse shear and transverse normal strains are equal to zero. For the calculation of such layered beam the classical layered beam theory is widely used [4,8]. However, this theory is complicated and therefore unhandy for engineering applications. Although present numerical modeling methods are universal they take additional time and require specific engineering background.

In [9], proposes a simple engineering method to calculate stress and strain of layered beam caused by external forces. Therefore, this method does not take into account hygrothermal strain which is often generated in the building structures and other laminated beam. These deformations can be a reason of failure of single layer and overall structures. For engineering applications it is necessary to make simple and herewith accurate methods for the calculation of hygrothermal stresses of layered beams.

The main goal of this work is to develop the method proposed in [9] for calculation total stresses and strain of layered beams under hygrothermal and mechanical loads. The developed method is user-friendly to practical engineering application. The obtained relationships are presented in explicit functions form.

2. Main dependences

2.1. Calculation of neutral axis of coordinate

In case of unaxial stresses state the layered beam can be considered as one-dimensional object (Fig. 1). The calculation is performed using the well-known assumption of materials mechanics of bending beams.

Assumptions:

- all assumptions of classical beam theory are taken into account;
- the bond between layers is perfect, i.e. slip of the layers is impossible;
- hygrothermal strain of the beam is independent on z coordinate (Fig. 1), i.e. does not change through width of the beam.

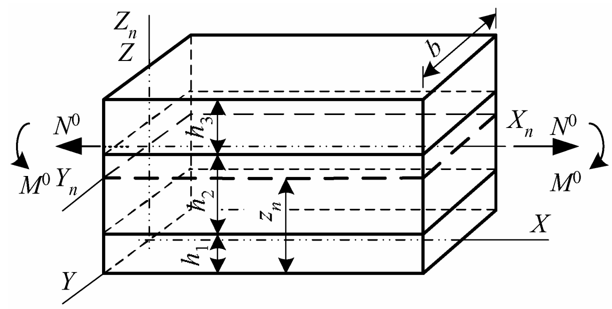

Fig. 1 Scheme of a layered beam, coordinates and forces

The location of neutral axis of the layered beam [9], in pure bending case, was obtained using the condition that the ratio of longitudinal strain at an individual node and distance from the neutral axis of a cross-section is the same. Other methods for calculation of neutral axis coordinate have been proposed in [10,11] articles. In these articles it was taken that a neutral axis coordinate is equal to the ratio between total first moments of layers longitudinal rigidity and an overall beam longitudinal rigidity. However, the location of neutral axis can be obtained using another condition – generalized axial force, in pure bending case, is equal to zero [12]

$$N=\sum_{i=1}^{n}\iint_{A_i}\sigma_i\left(z\right)dydz=0 \quad (1)$$

where σ_i is stress in $X_nY_nZ_n$ coordinates and A_i is cross-sectional area of ith layer. The stress of Eq. (1) can be expressed in terms of strain

$$\sigma_i\left(z\right)=E_i\varepsilon_i\left(z\right)=E_i\kappa z,\ i=1,...,n \quad (2)$$

where z is the coordinate of a node in $X_nY_nZ_n$ system, κ is the curvature. After putting Eq. (2) in to Eq. (1) and taking into account, that elasticity modulus is constant through the depth of each layer and curvature of all layers has the same value we get

$$N = \kappa \sum_{i=1}^{n} E_i \int_{-\frac{1}{2}b_i}^{\frac{1}{2}b_i} \int_{(a_0 - z_n)}^{(a_1 - z_n)} z dy dz = 0 \quad (3)$$

where $a_0 = \sum_{j=1}^{i-1} h_j$, $a_1 = \sum_{j=1}^{i} h_j$; b, h and E are width height and elasticity modulus of ith layer respectively. The curvature $\kappa \neq 0$ therefore the (3) conditions can be satisfied only if the sum of integrals is equal to zero

$$\sum_{i=1}^{n} E_i \int_{-\frac{1}{2}b_i}^{\frac{1}{2}b_i} \int_{(a_0 - z_n)}^{(a_1 - z_n)} z dy dz = \\ = \sum_{i=1}^{n} \left(E_i A_i \left(\sum_{k=1}^{i} h_k - \frac{h_i}{2} - z_n \right) \right) = 0 \quad (4)$$

where z_n is coordinate of neutral axis. Solving the Eq. (4) for z_n we get the expression of the coordinate of neutral axis

$$z_n = \frac{2\sum_{i=1}^{n} B_i \sum_{k=1}^{i} h_k - \sum_{i=1}^{n} E_i b_i h_i^2}{2\sum_{i=1}^{n} B_i} \quad (5)$$

where $B_i = E_i A_i$. It must be emphasized that the obtained (5) relationship haves analogy with the relationship of J. Bareišis [9] of neutral axis coordinate calculation method of layered structural elements. But Eq. (5) was obtained using another condition.

2.2 Stress state of a layered beam

Hygrothermal stress can be obtained subtracting hygrothermal strain of the material from total hygrothermal axial and bending strain of the beam. Multiplying the obtained restrained hygrothermal deformation by elasticity modulus of materials of the layer we get the stress. The total stresses in the layered beam are obtained by summing up the mechanical and hygrothermal stress components, respectively.

Axial force of the beam, from which its axial strain is equal to the hygrothermal strain, can be found from such formula [13]

$$N_b = \iint_A E(z) \varepsilon_b(z) dz dy = \sum_{i=1}^{n} E_i \int_{-\frac{1}{2}b_i}^{\frac{1}{2}b_i} \int_{a_0}^{a_1} \varepsilon_{b,i}(z) dz dy = \\ = \sum_{i=1}^{n} E_i b_i \int_{a_0}^{a_1} \varepsilon_{b,i}(z) dz \quad (6)$$

where A is the area of cross section of the layered beam, $\varepsilon_{b,i}(z)$ is distribution function of hygrothermal strain through the depth of ith layer in XYZ coordinates (Fig. 1). When temperature and humidity through the depth of the layer is uniform i.e. $\varepsilon_{b,i}(z) \rightarrow$const, then $N_b = \sum_{i=1}^{n} E_i b_i h_i \varepsilon_{b,i}$. Total axial strains of a layered beam can be obtained by summing up the strain due to hygrothermal load and axial force respectively

$$\varepsilon = \frac{N^0 + N_b}{B} \quad (7)$$

where N^0 is external force, B is longitudinal rigidity which is calculated according to $B = \sum_{i=1}^{n} E_i A_i$ formula [9]. Owing to hygrothermal strain of each layer the beam is bending. The bending moment in the layered beam caused by hygrothermal strain can be expressed as [13]

$$M_b = \iint_A E(z) \varepsilon_b(z) z dz dy = \\ = \sum_{i=1}^{n} E_i \int_{-0,5b_i}^{0,5b_i} \int_{a_0}^{a_1} \varepsilon_{b,i}(z) z dz dy = \sum_{i=1}^{n} E_i b_i \int_{a_0}^{a_1} \varepsilon_{b,i}(z) z dz \quad (8)$$

When temperature through the depth of each layer is uniformly distributed i.e., $\varepsilon_{b,i}(z) \rightarrow$const, then $M_b = 0$. Owing to axial forces of each layer the layered beam additionally bends because localities of the layers are nonsymmetrical in respect to neutral axis. Due to axial force of each layer its bending moment is calculated

$$M_{N,i} = N_i \mu_i, \, i = 1, \ldots, n \quad (9)$$

where μ_i is the distance between centroid of the ith layer to neutral axis calculated according to formula

$$\mu_i = \sum_{j=1}^{i} h_j - \frac{h_i}{2} - z_n \quad (10)$$

In Eq. (9) N_i is the force of the ith layer due to hygrothermal load and axial force calculated by using Eq. (6) and (7)

$$N_i = \frac{B_i N^0}{B} + E_i b_i \int_{a_0}^{a_1} \varepsilon_{b,i}(z) dz \quad (11)$$

The total bending moment consist of: the external bending moment M^0; bending moment of each layer generated owing to external load and hygrothermal strain; due to nonlinear distribution of hygrothermal strain through the depth of each layer

$$M = M^0 + M_b + \sum_{i=1}^{n} M_{N,i} \quad (12)$$

Then the curvature of layered beam calculated according to this formula

$$\kappa = \frac{M}{D} \quad (13)$$

where D is flexural rigidity of layered beam calculated according to $D = \sum_{i=1}^{n} E_i I_i$ [9]; I_i is the second moment of area to neutral axis calculated according to the well known formula $I_i = b_i\left(h_i\right)^3/12 + A_i\mu_i^2$.

If we have strain of the layered beam then stress calculation at any point is not complicated

$$\sigma_{x,i} = E_i\left(\varepsilon + \kappa z - \varepsilon_{b,i}\left(z\right)\right) \tag{14}$$

where z is the space between neutral axis and the node of considered layer of the beam, $\varepsilon_{b,i}(z)$ is the distribution function of hygrothermal strain of the ith layer in XYZ coordinates (Fig. 1).

In practice may be a case when the layered beam can not bend. For example, plastered or finished in another way external layer of masonry by flexible joint connected to the internal bearing wall with high flexural rigidity. If shear stiffness of the joint is not high then we can consider that only the bending deformation is restrained. In this case the stress can be calculated according to the Eq. (14) formula taking that κ=0. When the ends of the beam are hinged to two fixed bearing then the beam can bend but can not deform in axial direction. In this case stress is calculated also according to (14) but taking that ε=0. If the beam can not bend and deform in axial direction, for example the ends of the beam are rigidly fixed then we consider that κ=0 and ε=0.

3. Analysis of the results

The dependence of neutral axis location of layered beam upon various factors was investigated in [10,11,14]. In the equations describing changes in the stiffness center and neutral layer directions upon various factors have been found [10,11].

In the current analysis, the limiting values of neutral axis location, curvature, hygrothermal stresses and strains upon elasticity modulus and depth of the layers are presented. Also is considered the analysis of bilayer beam stresses upon various factors: diference of layers hygrothermal strains, depth of the hygrothermal strain, layers' materials' elasticity modulus ratio, support of beam.

When material's elasticity modulus of one of the layer is infinitely big or infinitely small in comparison with elasticity modulus of materials of other layers then limiting value of coordinate of neutral axis is as follows

$$\lim_{E_i\to\infty} z_n \to \sum_{j=1}^{i} h_j - \frac{1}{2}h_i \tag{15}$$

With decreased elasticity modulus of kth layer with regard to other layers, the coordinate of neutral axis tend to the following limit

$$\lim_{E_m\to 0} z_n \to \\ \to \frac{2\sum_{i=1}^{n} B_i \sum_{k=1}^{i} h_k - 2B_m \sum_{k=1}^{m} h_k - \sum_{i=1}^{n} E_i b_i h_i^2 + E_m b_m h_m^2}{2\left(\sum_{i=1}^{n} B_i - B_m\right)} \tag{16}$$

Also the following limiting values of curvature, axial strain and stress upon elasticity modulus and depth of the layers are determined. When the elasticity modulus of ith layer increases, the curvature and the axial strain of a layered beam tend to limits

$$\lim_{E_i\to\infty} \kappa \to \frac{12}{h_i^3}\int_{a_0}^{a_1} \varepsilon_{b,i}\left(z\right) z dz \tag{17}$$

$$\lim_{E_i\to\infty} \varepsilon \to \frac{1}{h_i}\int_{a_0}^{a_1} \varepsilon_{b,i}\left(z\right) dz \tag{18}$$

i.e. the curvature and axial strains of a layered beam tend to the curvature and axial strains of a beam which consist only of ith layer.

If hygrothermal strain through the depth of ith layer is uniform, i.e. $\varepsilon_{b,i}(z)\rightarrow$const, and the elasticity modulus as well as the depth of this layer increase, such dependences of a layered beam are determined: curvature tends to 0; axial strains tend to the ith layer's hygrothermal strains; stresses of ith layer tend to 0

$$\lim_{\wp_i\to\infty} \kappa \to 0 \tag{19}$$

$$\lim_{\wp_i\to\infty} \varepsilon \to \varepsilon_{b,i}\left(z\right) \tag{20}$$

$$\lim_{\wp_i\to\infty} \sigma_i\left(z\right) \to 0 \tag{21}$$

where $\wp_i \in \left\{E_i, h_i\right\}$.

When $\varepsilon_{b,k}(z)\rightarrow$const, with increased elasticity modulus and depth of kth layer, stresses of other layers tend to limit

$$\lim_{\wp_k\to\infty} \sigma_i\left(z\right) \to E_i\left(\varepsilon_{b,k}\left(z\right) - \varepsilon_{b,i}\left(z\right)\right), k \neq i \tag{22}$$

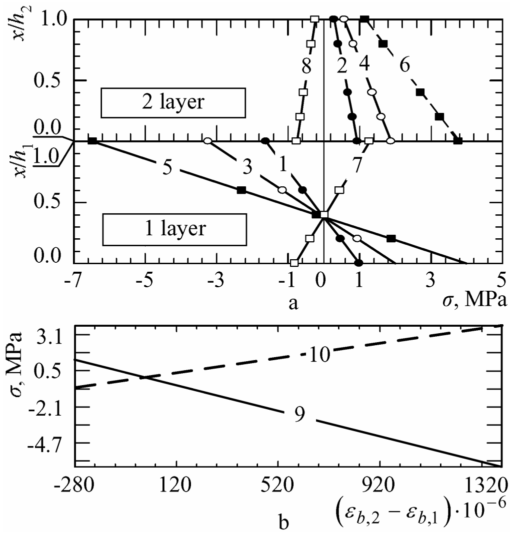

Fig. 2 Stress state of bilayered beam upon the difference of uniformly distributed hygrothermal strain: a - stress distribution through depth of the layers *1, 2* – $\varepsilon_{b,2}$=-350·10^{-6}, $\varepsilon_{b,1}$=0, *3, 4* – $\varepsilon_{b,2}$=-700·10^{-6}, $\varepsilon_{b,1}$=0, *5, 6* – $\varepsilon_{b,2}$=-1400·10^{-6}, $\varepsilon_{b,1}$=0, *7, 8* – $\varepsilon_{b,2}$=280·10^{-6}, $\varepsilon_{b,1}$=0; b - dependence of stress at the joint of layers upon difference of layers hygrothermal strain: *9* – first layer, *10* – 2nd layer, parameters of the beam are given in the table

The stress states upon different distributions of hygrothermal strain through the depth of the beam are investigated using bilayer beam. The parameters of the beam are presented in the Table.

Table

Parameters of bilayered beam

Layer No.	b, m	h, m	E, GPa	$\varepsilon_{b,2}$
1 layer	1	0.05	5	0
2 layer	1	0.1	10	$350\cdot10^{-6}$

As we can see from Fig. 2, owing to the uniformly distributed hygrothermal strain through the depth of the layer maximum stresses are generated at the joint of the layers. Also in this case stresses through the depth of the layer change linearly. In such structures fracture of a layer due to hygrothermal strain begins at joint of the layers. It is found that the dependence of stresses on the difference of hygrothermal strain is linear (Fig. 2, b).

The decreasing depth of hygrothermal strain of 2nd layer results in its stress increasing while the stress of 1st layer has tendency to be decreased (Fig. 3). Hygrothermal strains only in surface layers occur when ambient conditions (temperature and humidity) change sharply. Therefore, in this case sharp temperature and humidity changes increase cracking possibility of the layers.

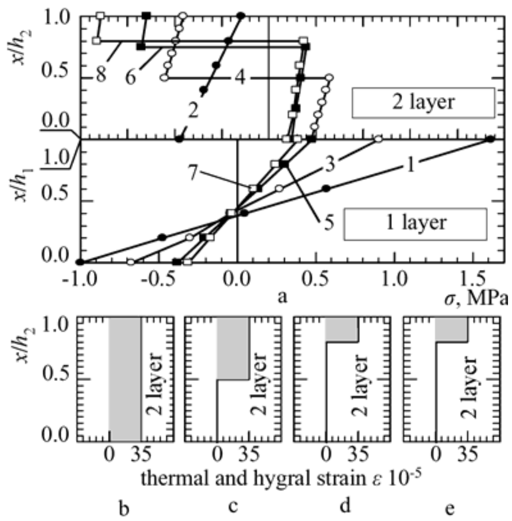

Fig. 3 Dependence of stresses on the depth of 2nd layers hygrothermal strain: a - distribution of stresses through the layers depth: *1*, *2* – when hygrothermal strain are distributed by (b) case, *3*, *4* – when hygrothermal strain are distributed by (c) case, *5*, *6* – when hygrothermal strain are distributed by (d) case, *7*, *8* – when hygrothermal strain are distributed by (e) case; (b-e) - distribution of hygrothermal strain in the 2nd layer; parameters of layers are give in the Table

The dependence of stresses on elasticity modulus is illustrated in Fig. 4 and 5. As we can see with increase of the ratio of elasticity modulus of the layers absolute value of stresses increase, but asymptotically tends to a specific limit Eq. (22). When the beam can not bend and the distribution of hygrothermal strain through the depth of a layer is uniform then stresses through the depth of each layer are distributed uniformly.

Mostly various protective and finishing covers have less depth than the base. Often various coverings have to meet physical requirements such as density, water permeability, abrasion resistance, etc.

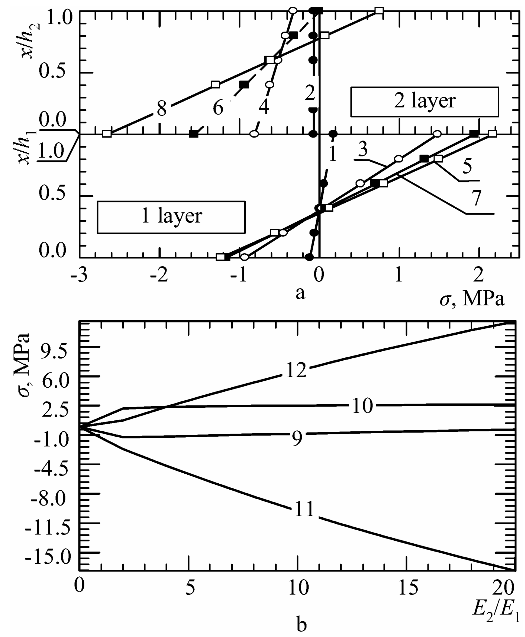

Fig. 4 Dependence of stresses on the ratio of elasticity modulus when E_1=10GPa: a - stress distribution through depth of the layer: *1*, *2* – E_2/E_1=0.02, *3*, *4* – E_2/E_1=0.4, *5*, *6* – E_2/E_1=1, *7*, *8* – E_2/E_1=2; b - stresses dependence on the ratio of elasticity modulus E_2/E_1: *9*, *10* – stresses of the 1st layer at outside (z=0) and at joint of the layers (z=0.1), *11* and *12* – stresses of the 2nd layer at joint of the layers (z=0.1) and outside (z=0.15); parameters of layers are give in the Table

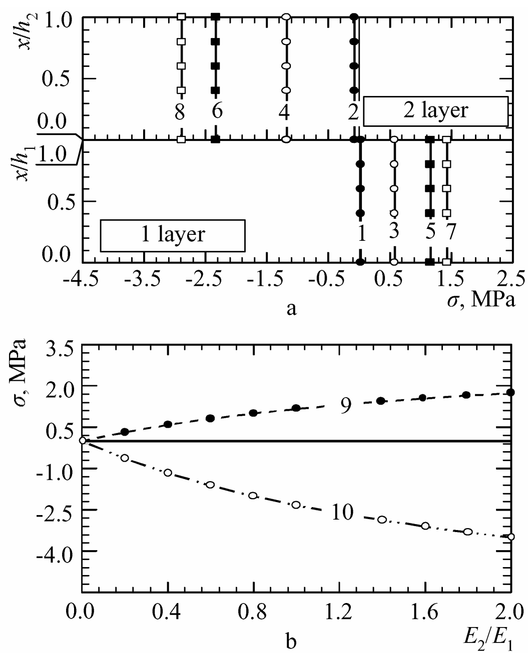

Fig. 5 Dependence of stresses on the ratio E_2/E_1 owing to uniformly distributed hygrothermal strain through the depth of the layers when the beam can not bend when E_1=10GPa: a - distribution of stresses through the layers: *1*, *2* – E_2/E_1=0.02, *3*, *4* – E_2/E_1=0.4, *5*, *6* – E_2/E_1=1, *7*, *8* – E_2/E_1=1.4; b - dependence of stresses on the ratio of elasticity modulus: *9* – 1st layer, *10* – 2nd layer; parameters of layers are give in the Table

If we select the covers with particular physical properties then its elasticity modulus can also change. If due to increasing elasticity modulus of materials strength increase of the material is less than of the stresses, then cracking possibility of covers increases. For thin covers it is possible to take that conditions (22) are valid, therefore such conditions of layers noncracking can be written

$$\lim_{E_k \to \infty} \sigma_i \to E_i \left(\varepsilon_{b,k} - \varepsilon_{b,i} \right) \le f_i, k \ne i \quad (23)$$

Considering derivatives of Eq. (23) inequality in respect to E_i we get such condition

$$\left(\varepsilon_{b,k} - \varepsilon_{b,i} \right) \le \frac{df_i\left(E_i\right)}{dE_i}, k \ne i \quad (24)$$

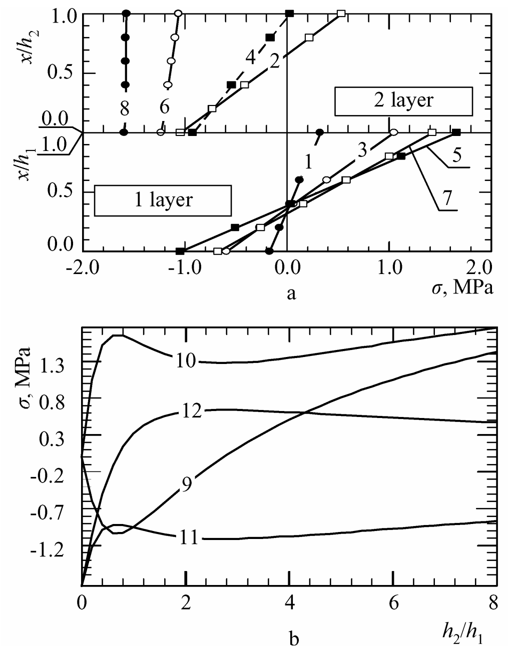

Fig. 6 Dependence of stresses on the ratio h_2/h_1 owing to uniform distribution of hygrothermal strain through the depth of the layer: a - distribution of stresses through the depth of the layers: *1*, *2* – h_2/h_1=0.05, *3*, *4* – h_2/h_1=0.2, *5*, *6* – h_2/h_1=0.7, *7*, *8* – h_2/h_1=1.5; b - dependence of stresses on ratio of depth of the layers: *9* and *10* – stresses of 1st layer at outside (z=0) and at joint of the layers (z=0.1), *11* and *12* – stresses of 2nd layer at joint of the layers (z=0.1) and outside (z=0.15); parameters of layers are give in the Table

If (23) and (24) conditions are satisfied then with increasing of elasticity modulus of 1-st layer its cracking possibility do not increases.

The dependence of bilayered beam stresses upon depth ratio of the layers is shown in Fig. 6. As illustrated in Fig. 6 with the depth increase of the second layer stresses of the first layer asymptotically tend to (22) limit. With increasing h_2/h_1 ratio stresses of the second layer decrease. The irregularities of stresses change with the ratio h_2/h_1 change arise owing to the change of neutral axis location.

When layers cannot bend then stresses of the layer without local extremes tend to (22) limits. This is perfectly illustrated in Fig. 7. By comparing the first layer's stress of simply supported beam (Figs. 2 and 6) with the stress of beam the deformation of which is restrained (Figs. 5 and 7) we notice, that this restraining can highly increase the stress. However, in general case layer‘s stresses at the joint of the layers of bended beam owing to bending moment may exceed the stresses of the beam whose bending deformation is restrained. The performed analysis also shows that the stresses of layers owing to restrained axial strain are greater than the stresses owing to restrained bending deformation.

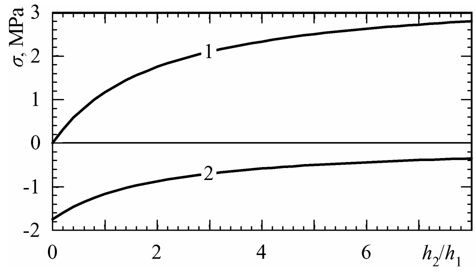

Fig. 7 Dependence of layers stresses upon the ratio of layer depth when the layer can not bend: *1* – 1st layer, *2* – 2nd layer; parameters of layers are give in the Table

4. Conclusions

Owing to uniform distribution through the depth of the layer hygrothermal strain the layer‘s stresses of simply supported beam are generated at the joint of the layers. Therefore at this place the possibility of layer's cracking is the greatest.

With the reduction of the depth of hygrothermal strain stresses of the material at this depth increase. However, stresses of the material decrease at the space which does not deform thermally and hygrally.

Restraining of hygrothermal strain increases the stresses of the layers. Due to the restrained axial strain stresses of the layers exceed the stresses generated owing to restrained bending strain.

In comparison with other layers the increases of elasticity modulus of thin layers increase the probability of cracking.

References

1 **Rzhanitsyn, A.R.** Built-up Bars and Plates.-Moscow: Strojizdat, 1986.-316p. (in Russian).
2 **Gay, D., Hoa, S.V., Tsai, S.W.** Composite Materials. -Design and applications.-Boca Raton: CRC Press, 2003.-552p.
3 **Staab, G.H**. Laminar Composites.-Boston: Butterworth-Heinemann, 1999.-314p.
4 **Berthelot, J.M.** Composite Materials. Mechanical Behaviour and Structural Analysis.-New York: Springer–verlag. 1999.-645p.
5 **Bolotin, V.V., Novichkov, J.N.** Mechanics of Multilayered Structures.-Moscow: Mashinostrojenije, 1980.-375p. (in Russian)
6 **Khdeir, A.A., Reddy, J.N.** Jordan canonical form solution for thermally induced deformations of cross-ply laminated composite beams.-J of Thermal Stresses, 1999, 22, p.331-349.

7 **Ghugal, Y.M., Shimpi, R.P.** A review of refined shear deformation theories for izotropic and anisotropic laminated beams.-J. of Reinforced Plastics and Composites, 2001, 3(20), p.256-268.

8 **Vasiliev, V.V., Morozov, E.V.** Mechanics and Analysis of Composite Materials.-Amsterdam: Elsevier, 2001.-412p.

9 **Bareišis, J.** Static and Cyclic Strength of Structural Composites: Summary of Habilitation Thesis.-Kaunas: Technologija, 1995.-36p.

10 **Bareišis, J., Kleiza, V**. Stiffness center and neutral layer direction investigation method and its application to asymmetric multilayer structural elements. -Mechanika.-Kaunas: Technologija, 2004, Nr.4(48), p.5-12.

11 **Bareišis, J., Kleiza, V.** Influence of multilayer beams cross-section shape on stiffness under bending. Mechanika.-Kaunas: Technologija, 2005, Nr.1(51), p.5-11.

12 **Megson, H.G.** Structural and Stress Analysis.-Oxford: Butterworth-Heinemann, 2000.-641p.

13 **Boley, B.A., Wiener, J.H.** Theory of Thermal Stresses.-New York: Wiley, 1960.-278p.

14 **Garuckas, D.** Investigation of Strength and Stiffness of Laminatwed Structural Elements. PhD thesis.-Kaunas, KTU, 2000.-108p. (in Lithuanian).

D. Zabulionis

IŠORINIŲ APKROVŲ, TEMPERATŪRINIŲ IR DRĖGMINIŲ DEFORMACIJŲ VEIKIAMŲ SLUOKSNIUOTŲJŲ SIJŲ ĮTEMPIŲ IR DEFORMACIJŲ BŪVIO ANALIZĖ

R e z i u m ė

Darbe pateikta sluoksniuotųjų sijų išorinių apkrovų ir temperatūrinių bei drėgminių deformacijų sąlygojamo įtempių ir deformacijų būvio inžinerinė skaičiavimo metodika ir analizė. Sudarytos priklausomybės remiasi klasikine sluoksniuotųjų strypų teorija. Pagal gautas priklausomybes atlikta realios dvisluoksnės sijos įtempių priklausomybės nuo sluoksnių temperatūrinių ir drėgminių deformacijų skirtumo, sluoksnių medžiagų tamprumo modulių santykio, sluoksnių storių santykio ir temperatūrinių bei drėgminių deformacijų pasiskirstymo sluoksnių storyje analizė. Nustatyta, kad didžiausi sluoksnių įtempiai susidaro sluoksnių sąlyčio vietoje. Sijos temperatūrinių ir drėgminių deformacijų suvaržymas gali gerokai padidinti įtempius.

D. Zabulionis

STRESS AND STRAIN ANALYSIS OF LAYERED BEAMS UNDER HYGROTHERMAL AND MECHANICAL LOADS

S u m m a r y

In the paper an engineering method for the calculation of stress and strain of layered beams under hygrothermal and mechanical loads is proposed. The analysis of bilayer beam stresses and strain states is also given. The proposed relationship is based on classical layered beam theory. According to the obtained relationship the analysis of dependences of stress and strain state of bilayerd beam upon the difference of layers‘ hygrothermal strain is performed. Variations of layers elasticity modulus and their depth ratio as well as the distribution of hygrothermal strains through layer depth are also considered in analysis. It has been determined that maximal stresses occur at the layers joint. Restraining of hygrothermal deformations for the beam may significantly increase layer stresses.

Д. Забулёнис

НАПРЯЖЕННО - ДЕФОРМИРОВАННОЕ СОСТОЯНИЕ СЛОИСТОЙ БАЛКИ ОТ ДЕЙСТВИЯ ВНЕШНИХ НАГРУЗОК, ТЕМПЕРАТУРНЫХ И ВЛАЖНОСТНЫХ ДЕФОРМАЦИЙ

Р е з ю м е

В статье предложена инженерная методика расчета напряжений и деформаций слоистой балки от действия внешних нагрузок, температурных и влажностных деформаций. Предложенные зависимости получены на основе классической теории слоистых стержней. С помощью полученных зависимостей сделан анализ напряженно–деформированного состояния реальной двухслойной балки в зависимости от разницы температурных и влажностных деформаций слоев, модуля упругости материалов слоев, толщины слоев и распределения температурных и влажностных деформаций по высоте поперечного сечения балки. Установлено, что максимальные напряжения слоев образуются в стыке. Стеснение температурных и влажностных деформаций балки может вызвать значительный рост напряжений.

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