

# Yield stresses in compressed and bended columns and beams

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## 1. Introduction

Caused by loading columns and other structures deform elastically plastically. A column is not only compressed, but also bended, if the column is eccentrically loaded or a force perpendicular to longitudinal axis is applied. Earthquake ground motion can be one of the reasons for extreme horizontal loading to building structures, presented by D. V. Val and F. Segal [1]. The influence the plastic deformations on structures and low cycle fatigue is discussed by A. Teran-Gilmore, E. Avila, G. Rangel [2]. Some simple expressions are found to calculate the input energy that is transformed into plastic energy of the system by A. M. F. Cruz, O. A. Lopez [3]. Research undertaken in various related disciplines (engineering seismology, soil and mechanical system dynamics, mechanics of materials) is reviewed by A. M. Chandler, N. T. K. Lam [4]. An investigation of beams yield deformation is presented by V.V. Sokolovskij [5]. Response of inelastic systems, effects of damping and yielding in structures are presented by A. K. Chopra [6]. In parallel with earthquake engineering wind-induced vibrations of structures, man-made motion of various mechanical systems also can be investigated applying these methods. In general more comprehensive analysis is essential when extremely high loading is applied and yielding in cross-sections of a structure arise.

The main objective of this paper is to present strength of a double-tee section column when yield stresses emerge in some parts of cross-section. Load carrying capacity of the whole structure can be determined if this dependence is solved out. An investigations of the specific structures are to be carried out in the future.

## 2. Single-sided yield in cross-section

Double-tee cross-section is one of the most universally employed column shapes (Fig. 1). For the solution to be less complicated the web width  $\delta_0$  is assumed infinitesimal as compared with  $h$ , therefore  $\delta_0 \rightarrow 0$ , but the web area  $A_1 = \delta_0 b = const$  (Fig. 2). The whole area of cross-section  $A = 2A_1 + 2\delta h$ . The compression-tension is assumed to be positive. The longitudinal forces  $N$  in Fig. 1 and Fig. 2 coincide if no other external longitudinal forces are applied.

In this chapter yield stress is assumed to be only in one web, so stress in the other  $|\sigma_k| < \sigma_y$ , where  $\sigma_y$  – yield stress. If  $\eta$  is the distance from cross-section centre of gravity C to the yield point B, and  $e$  is the distance from C to the neutral axis E, then

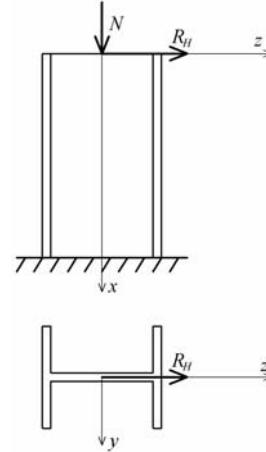


Fig. 1 Forces  $N$ ,  $R_H$  exerted on the column, and coordinate axes  $x$ ,  $y$ ,  $z$

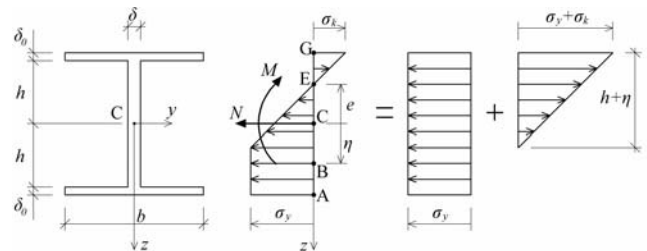


Fig. 2 A column cross-section and normal stresses for the single-sided yield case

$$\sigma_k = \sigma_y \frac{h-e}{\eta+e}; \quad \sigma_y + \sigma_k = \sigma_y \frac{\eta+h}{\eta+e}$$

and longitudinal internal force

$$N = \sigma_y A - \frac{\sigma_y (\eta+h)^2}{2} \frac{\delta}{\eta+e} - \sigma_y \frac{\eta+h}{\eta+e} A_1$$

Internal resisting moment about center C

$$M = \frac{\sigma_y (\eta+h)^2}{2} \frac{\delta (2h-\eta)}{3} + \sigma_y \frac{\eta+h}{\eta+e} A_1 h$$

If dimensionless load parameters

$$\alpha = \frac{N}{A\sigma_y}; \quad \beta = \frac{M}{hA\sigma_y} \quad (1)$$

and cross-section parameter

$$q = 2A_1 / A \quad (2)$$

are defined, these equations can be presented as

$$\begin{cases} \frac{1-q}{4} \frac{(1+\zeta)^2}{\zeta+\theta} + \frac{q}{2} \frac{1+\zeta}{\zeta+\theta} = 1-\alpha \\ \frac{1-q}{4} \frac{(1+\zeta)^2}{\zeta+\theta} - \frac{2-\zeta}{3} + \frac{q}{2} \frac{1+\zeta}{\zeta+\theta} = \beta \end{cases} \quad (3)$$

Dimensionless coordinates  $\zeta = \eta/h$ ,  $\theta = e/h$  of yield point B and neutral axis E are used. If  $\alpha$  and  $\beta$  are assumed given values the system of Eqs. (3) can be rearranged to one equation

$$\frac{1-\alpha}{1-\alpha-\beta} (1+\zeta)^2 - 3(1+\zeta) - \frac{6q}{1-q} = 0 \quad (\alpha \neq 1, \zeta \neq -1),$$

quadratic with respect of unknown  $(1+\zeta)$ . The solution of this equation

$$\zeta = \frac{3}{2} \frac{1-\alpha-\beta+\sqrt{D_0}}{1-\alpha} - 1 \quad (4)$$

where

$$D_0 = (1-\alpha-\beta)(q_c - \beta) \quad (5)$$

and the constant

$$q_c = \frac{1-\alpha}{3} \frac{3+5q}{1-q}$$

Treating  $D_0$  as a function of  $\beta$  the minimum

$$D_{0min} = - \left( \frac{4q}{3} \frac{1-\alpha}{1-q} \right)^2 \leq 0$$

can be found when

$$\beta = \frac{1-\alpha}{3} \frac{3+q}{1-q}$$

Quadratic polynomial  $D_0$  has two roots: the first  $\beta = 1-\alpha = \beta'$  and the second  $\beta = q_c = \beta'' \geq \beta'$ . The negative minimum  $D_{0min}$  exists between these roots, so  $D_0$  is positive if  $\beta < \beta'$  or  $\beta > \beta''$ . For positive  $D_0$  a real value of  $\zeta$  can be calculated from Eq. (4) and from the system of Eqs. (3)

$$\theta = \frac{1-q}{1-\alpha-\beta} \frac{(1+\zeta)^3}{12} - \zeta \quad (6)$$

If  $\beta = 1-\alpha$  then  $D_0 = 0$  and value  $\zeta = -1$  follows from Eq. (4). The function (6) is not defined when  $\beta = 1-\alpha$  for both numerator and denominator are 0, but the  $\lim \theta = -1$

exists when  $\varepsilon$  approaches 0, and  $\beta = 1-\alpha-\varepsilon$ . As positive  $e$  and  $\eta$  are defined in opposite directions (Fig. 2), limit positions of the neutral line E and the yield point B coincide at the same web.

### 3. Double-sided yield in cross-section

Since  $e + \eta = \eta^* - e$  (Fig. 3) the distance  $\eta^* = \eta + 2e$ , therefore longitudinal internal force and moment

$$N = \sigma_y \delta 2e$$

$$M = \sigma_y A_1 2h + \frac{\sigma_y \delta}{3} (3h^2 - 4e^2 - 2e\eta - \eta^2)$$

If dimensionless parameters (1) are used the equations

$$\theta = \frac{\alpha}{1-q} \quad (7)$$

$$\frac{1}{3} (\zeta + \theta)^2 = 1 + 2 \frac{q-\beta}{1-q} - \theta^2 \quad (8)$$

can be deduced. If  $\alpha$  is known value then  $\theta$  can be obtained from Eq. (7), and then  $\zeta$  from Eq. (8).

If  $\zeta^* = \eta^*/h$  then  $\zeta^* = \zeta + 2\theta$ . The stress diagram is really double-sided if necessary conditions  $|\zeta| < 1$  and  $|\zeta^*| < 1$  are hold.

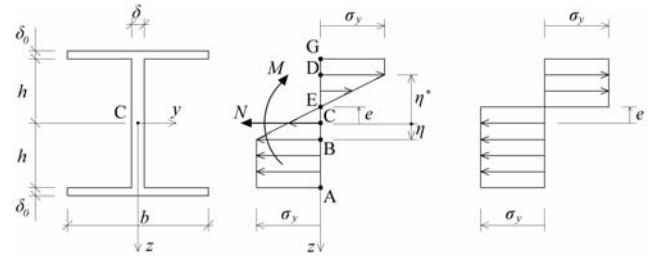


Fig. 3 A column cross-section and normal stresses for the double-sided yield case. Plastic hinge is shown on the right

Shear stresses are neglected in this investigation, therefore in the case of plastic hinge (Fig. 3)  $\eta^* = e$ ,  $\eta = -e$  and  $M = \sigma_y 2A_1 h + \sigma_y \delta (h^2 - e^2)$ ,  $N = \sigma_y 2\delta e$ . The same Eq. (7) for  $\theta$  is deduced and

$$\beta_3 = \frac{1-q^2 - \alpha^2}{2(1-q)} \quad (9)$$

This value of the dimensionless plastic hinge moment can be deduced alternatively from Eq. (8) if  $\zeta = \eta/h = -e/h = -\theta$  and Eq. (7) for  $\theta$  are used.

It should be noted that  $\theta = e/h$  and  $e$  do not depend on  $\beta$  when yield is double-sided, so position of the neutral line in cross-section is constant when bending mo-

ment  $M$  alters. Completely different situation is for single-sided yield. Eq. (6) provides support for this statement.

When plastic hinge Eq. (9) is inserted the general Eq. (8) can be presented

$$\zeta = \sqrt{6 \frac{\beta_3 - \beta}{1-q}} - \frac{\alpha}{1-q}$$

here condition  $\beta_3 \geq \beta$  has to be observed.

#### 4. Single-sided and double sided yield regions

When the bending moment and the longitudinal force are known stress distribution in cross-section can be solved from Eqs. (4), (6) or (7), (8). Nevertheless in these equations no sufficient additional conditions are included to assess single-sided or double-sided stress distribution in the cross-section. When some values of  $\alpha$  and  $\beta$  are specified the real solutions (i.e. not complex) can be found from both (4), (6) or (7), (8) equations. Complementary conditions are necessary to determine regions of single-sided yield, double-sided yield and elastic stresses.

The yield in a web starts when  $\eta = h$  in the diagram of Fig. 2. The expressions in the chapter 2 are simplified for this case

$$\frac{N}{\sigma_y} = \frac{e}{h+e} A; \quad \frac{M}{\sigma_y} = \frac{I_x}{h+e}; \quad I_x = \frac{2\delta h^3}{3} + 2h^2 A_1$$

A dividing line between the completely elastic region and the single-sided region can be deduced from these equations

$$\theta_1 = \frac{\alpha}{1-\alpha}; \quad \beta_1 = (1-\alpha) \frac{1+2q}{3} \quad (10)$$

The line, dividing single-sided region and double-sided region, can be deduced submitting  $\sigma_k = \sigma_y$  in Fig. 2 or  $\eta^* = h$  in Fig. 3

$$\frac{N}{\sigma_y} = 2\delta e; \quad \frac{M}{\sigma_y} = 2h A_1 + \frac{2\delta}{3} (h^2 + eh - 2e^2)$$

If dimensionless parameters (1) are inserted, the line is presented by

$$\theta_2 = \frac{\alpha}{1-q}; \quad \beta_2 = q + \frac{(1-q-\alpha)(1-q+2\alpha)}{3(1-q)} \quad (11)$$

When  $q=0$ , i.e. when cross-section of the column is rectangle

$$\theta_1 = \frac{\alpha}{1-\alpha}; \quad \theta_2 = \theta_3 = \alpha$$

$$\beta_1 = \frac{1-\alpha}{3}; \quad \beta_2 = \frac{1-\alpha}{3} (1+2\alpha); \quad \beta_3 = \frac{1-\alpha^2}{2}$$

So if  $0 < \alpha < 1$ , then  $0 < \beta_1 < \beta_2 < \beta_3 < 1$ , and if  $\alpha=1$ , then  $\beta_1 = \beta_2 = \beta_3 = 0$ . Therefore if  $q=0$ , then completely elastic region  $\beta < \beta_1$  is succeeded by single-sided yield region  $\beta_1 < \beta < \beta_2$  and then follows double-sided region  $\beta_2 < \beta < \beta_3$  (Fig. 4).

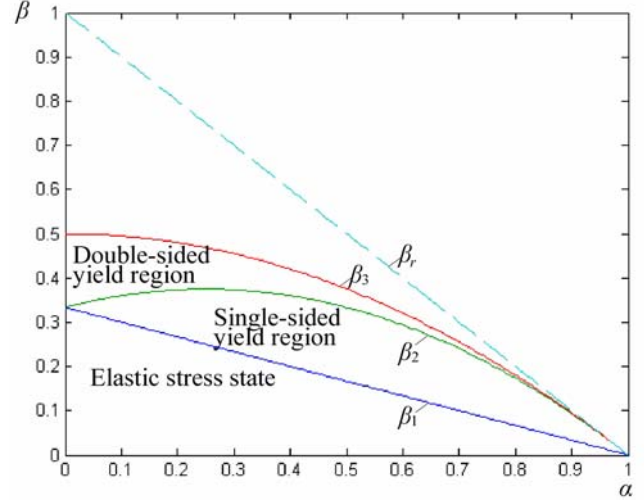


Fig. 4 The regions of elastic stress state and yield stress state when  $q=0$  (cross-section of the column is rectangle)

Location of these regions is more complicated when  $q \neq 0$ . If, for example,  $q=0.5$  then

$$\beta_1 = 2 \frac{1-\alpha}{3}; \quad \beta_2 = \frac{2+\alpha^2-4\alpha^2}{3}; \quad \beta_3 = 0.75 - \alpha^2$$

So when  $\alpha=0.8$  values  $\beta_1=0.133$ ;  $\beta_2=0.080$ ;  $\beta_3=0.110$ , ( $\beta_2 < \beta_3 < \beta_1$ ) can be calculated. If  $\alpha=0.9$  values of  $\beta_2$  and  $\beta_3$  are negative. This example proves that single-sided and double-sided yield regions are quite differently located when  $q$  and  $\alpha$  are sufficiently large.

The dividing line of single-sided and double-sided regions  $\beta = \beta_2$  is investigated. Identities

$$1-\alpha-\beta_2 = \frac{2(1-\alpha-q)^2}{3(1-q)}$$

$$q_c - \beta_2 = \frac{2(1-\alpha+q)^2}{3(1-q)}$$

can be proved inserting Eq. (11), therefore for  $\beta = \beta_2$  from Eq. (5) it follows that

$$D_0 = \frac{2(1-\alpha)^2 - q^2}{3(1-q)}$$

and solution (4) is  $\zeta + 1 = 2E$ , where  $E = \frac{1-\alpha-q}{1-q}$ . If this

expression is used Eq. (6) gives  $\zeta + \theta = \frac{2}{3} E^2$ , hence

$$\zeta^* = \frac{\eta^*}{h} = \zeta + 2\theta = \frac{4}{3}E^2 - 2E + 1$$

This solution is deduced from single-sided yield formulae, but complementary condition  $\eta^* > h$  or  $\zeta^* > 1$  has to be satisfied. This inequality can be transformed to  $(1-\alpha-q)(q-1-2\alpha) > 0$ . As  $1-q > 0$  for all cross-sections the inequality can be satisfied only when  $1-\alpha-q < 0$  or  $\alpha > 1-q$ . If this condition is fulfilled and  $\beta = \beta_2$ , then single-sided yield is not transformed to double-sided yield, Eq. (9) loses its meaning as the condition of plastic hinge. The solution of single-sided yield (4) is real if  $D_0 \geq 0$ , so the maximal value of dimensionless moment is  $\beta_r = 1-\alpha$ . Identity

$$\beta_r - \beta_3 = \frac{(1-\alpha-q)^2}{2(1-\alpha)}$$

can be proved, so  $\beta_3 \leq \beta_r$  for any  $\alpha$  and  $q$ , and equality  $\beta_r = \beta_3$  is possible only if  $\alpha = 1-q$ . It can be proved, that  $\beta_2 = \beta_3 = \beta_r = 1-\alpha = q$  if  $\alpha = 1-q$ . Furthermore, derivatives of functions (9) and (11)

$$\frac{d\beta_2}{d\alpha} = \frac{1-q-4\alpha}{3(1-q)}, \quad \frac{d\beta_3}{d\alpha} = -\frac{\alpha}{1-q}; \quad \frac{d\beta_r}{d\alpha} = -1$$

are equal when  $\alpha = 1-q$ , so the straight line  $\beta_r = 1-\alpha$  is tangent of the curved lines  $\beta_2 = \beta_2(\alpha)$ ,  $\beta_3 = \beta_3(\alpha)$  at the point  $\alpha = 1-q$ . In this way the double-sided yield region ends at the point  $\alpha_c = 1-q$ ,  $\beta_c = q$  (Fig. 5). When  $\alpha > 1-q$  the single-sided yield region is restricted not by the parabola  $\beta_2 = \beta_2(\alpha)$ , but by the straight line  $\beta_r = 1-\alpha$ . The lower boundary of the region is the straight line  $\beta_1 = \beta_1(\alpha)$  given by Eq. (10). If  $q = 0$  then restriction of the double-sided yield region loses its meaning because condition  $\alpha > 1-q = 1$  can not be fulfilled. If  $\alpha \neq 1$ ,  $q \neq 1$ , then  $\beta_r < \beta_1$  so single-sided yield is reached for every increasing  $\beta$  with exception of the only case  $\alpha = 0$ , i.e. when there is no compression force.

## 5. Plastic piston

If  $\alpha < 1-q$  and  $\beta \rightarrow \beta_3$ , then stress diagram in the cross-section approaches plastic hinge. Neutral line of the double-sided yield does not depend on  $\beta$  (Eq. (7)), and  $\theta < 1$  when  $\alpha < 1-q$ . If  $\alpha > 1-q$  the double-sided yield region is absent and neutral line in cross-section changes its position (Eq. (6)). The limit of stress diagram of the cross-section can be deduced when  $\alpha > 1-q$  and  $\beta \rightarrow \beta_r$ . Let  $\beta \rightarrow \beta_r - \varepsilon = 1-\alpha - \varepsilon$  and  $\varepsilon \rightarrow 0$ . If

$$q_c - (1-\alpha) = \frac{8q}{3} \frac{1-\alpha}{1-q} = b$$

then  $D_0 = \varepsilon(b + \varepsilon) \approx \varepsilon b$ . Approximate values

$$\zeta + 1 \approx b_0 \sqrt{\varepsilon}, \quad b_0 = \frac{3\sqrt{b}}{2(1-\alpha)}$$

can be deduced from Eq. (4) if infinitesimals of higher order are neglected. Thus from Eq. (6)

$$\theta = \left( b_0^2 \frac{1-q}{12} - 1 \right) b_0 \sqrt{\varepsilon} + 1$$

and

$$\begin{cases} -\zeta = -b_0 \sqrt{\varepsilon} + 1 \\ \zeta^* = \frac{\alpha + q - 1}{1-\alpha} b_0 \sqrt{\varepsilon} + 1 \\ \theta = \frac{\alpha + 0.5q - 1}{1-\alpha} b_0 \sqrt{\varepsilon} + 1 \end{cases}$$

Value  $-\zeta$  is presented because positive  $\eta$  and  $\zeta$  are directed in opposition to  $\eta^*$  or  $e$ , and  $\zeta^*$  or  $\theta$  accordingly (Figs. 2 and 3). It follows from these three formulae that B, C, and D approach point G at the upper web. The lower yield point B remains below than G, and neutral line E can be higher or below than G. This depends on the sign of the number  $\alpha + 0.5q - 1$ : if  $\alpha > 1-q$ , then  $\alpha + q - 1 > 0$ , but  $\alpha + 0.5q - 1$  can be positive and negative. It will be not a plastic hinge in any case because stress diagram approaches yield compression everywhere over the cross-section. May be such cross-section can be named as plastic piston.

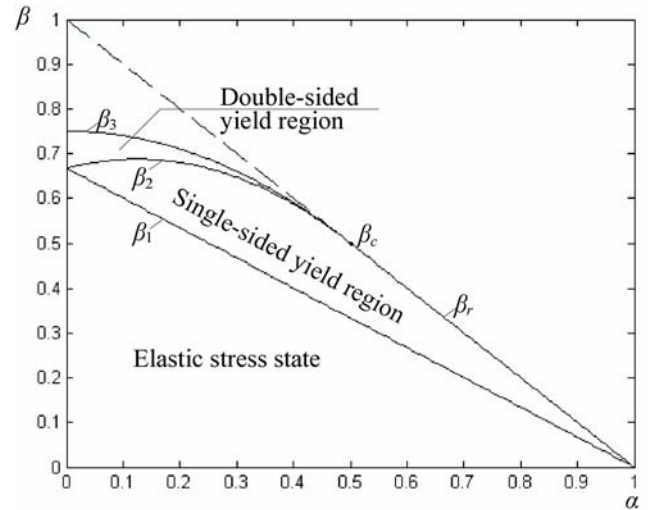


Fig. 5 The regions of elastic stress state and yield stress when  $q = 0.5$ . The double-sided yield region ends at the point  $\beta_c = 1-\alpha = q$

The longitudinal section of the column is presented in Fig. 6 when compression force is relatively small ( $\alpha < 1-q$ ). In this case single-sided yield, further double-sided yield and plastic hinge develop when  $\beta$  increases. The case of relatively large compression force ( $\alpha > 1-q$ ) is presented in Fig. 7. The double-sided yield domain is absent in this longitudinal section and plastic piston forms

when  $\beta = \beta_r$ . The neutral line is lower than the upper web because  $\alpha + 0.5q - 1 = -0.15$ .

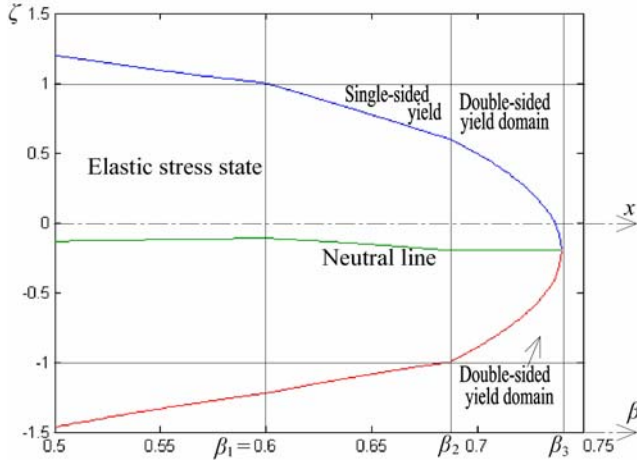


Fig. 6 Longitudinal cross-section of the column when  $q = 0.50$ ,  $\alpha = 0.10$ . The dimensionless moment values are  $\beta_1 = 0.600$ ;  $\beta_2 = 0.6867$ ;  $\beta_3 = 0.740$ ;  $\beta_r = 0.900$ . Plastic hinge develops when  $\beta = \beta_3$

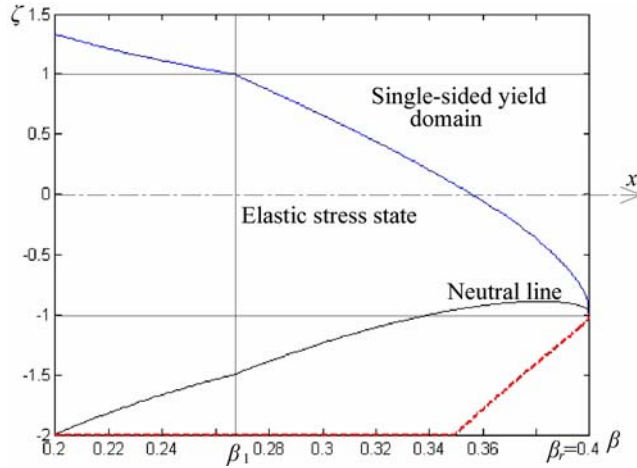


Fig. 7 Longitudinal cross-section of the column when  $q = 0.50$ ,  $\alpha = 0.60$ . The dimensionless moment values are  $\beta_1 = 0.2667$ ;  $\beta_2 = 0.3867$ ;  $\beta_3 = 0.390$ ;  $\beta_r = 0.40$ . Plastic piston develops when  $\beta = \beta_r$ , while the constants  $\beta_2$  and  $\beta_3$  are of no significance

The lines, depicted for  $|\zeta| > 1$ , are simply an abstract mathematical image. These lines do not present real stresses, but suggest an explanation of the stress diagrams for  $|\zeta| \leq 1$ .

The web width  $\delta_0$  (Fig. 1) in this investigation is neglected and because of that the formulae for stress diagrams are relatively simple: only two alternatives have to be presented, the single-sided and the double-sided yield. If there is no  $\delta_0 \rightarrow 0$  assumption the different formulae for every case is required. For single-sided yield no less than three formulae have to be deduced: when yield point B (Fig. 2) is located  $h + \delta_0 > \eta > h$ ,  $h > \eta > -h$ ,

$-h > \eta > -(h + \delta_0)$ . For double-sided yield the same conditions should be represented for the distance  $\eta^*$ .

However, when the value  $\delta_0$  is assumed infinitesimal and the web area  $A_1$  a constant, the investigation is more sophisticated in mathematics. The contradiction between the statements that  $\delta_0 \rightarrow 0, |\sigma| \leq \sigma_y$ , but  $\sigma \delta_0 b = const$  can be resolved applying contemporary theory of derivative and integral, generalized functions (or distributions) [7]. An example of such contradiction can be the case when  $\alpha > 1 - q$  and the limit of increasing bending moment  $\beta \rightarrow \beta_r$  is not a plastic hinge, but uniform stress of the same direction. This outcome, named for plastic piston, is impossible when web area  $A_1 = 0$ . The issue may be simplified by adding two forces  $S_y = \sigma_y A_1$ ,  $S_k = \sigma_k A_1$  applied at the point A and G (Fig. 2) or opposite forces  $S_y$  (Fig. 3). When  $q$  is reasonably large these forces can be more important than stresses over the whole interval  $-h \leq z \leq h$ .

For determinate structures as simply supported beams, cantilevered beams the bending moment diagrams can be calculated first, and then elastic stress, single-sided yield or double sided yield domains deduced. When the structure is indeterminate the bending moment diagrams depend on the elastic stress, single-sided or double-sided yield in the structure and can not be depicted in advance. The whole problem is to be solved as interconnected.

## 6. Conclusions

1. Structural strength calculation is different when stresses do not exceed the elastic limits, and when yielding begins and progresses in some places of the compressed and bended columns.

2. Dimensionless parameter  $\alpha$  depends on the axial force, the area of the column cross-section and yield stress; parameter  $q$  is equal ratio of the web cross-section area to the entire column cross-section area. If  $\alpha < 1 - q$  then elastic stress region is followed by the single-sided yield region, next is the double-sided yield region, and then the plastic hinge can be reached when bending moment is increasing.

3. If  $\alpha \geq 1 - q$  then elastic stress region is followed by the single-sided yield region, and this region concludes by plastic piston. The evaluation of plastic piston is quite different than the evaluation of plastic hinge.

4. Neutral line position in cross-section does not depend on bending moment in double-sided yield domain, but the distance from neutral line to the web changes if yield is single-sided. Calculation of the yield domain boundary is different for the double-sided and the single-sided yield.

5. The yield stresses in a longitudinal section of the column affect the deformation of the column and force-deformation relation.

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## TAKUMO ĮTEMPIAI GNIUŽDOMOSE IR LENKIAMOSE KOLONOSE IR SIJOSE

### Reziumė

Veikiant didelėms apkrovoms plieninėse konstrukcijose atsiranda takumo įtempiai. Jei kolona ar kita konstrukcijos dalis ne tik lenkiama, bet ir gniuždoma, tai takumo įtempiai gali atsirasti vienoje skerspjūvio pusėje arba abiejose. Tokie įtempių būviai skiriasi vienas nuo kito ir nuo tampriųjų įtempių būvio. Straipsnyje parodoma, kaip keičiasi takumo įtempiai dvitėjėse kolonose. Daroma prielaida, kad lentynų storis yra labai mažas, ir apskaičiuojami ribiniai būviai didėjant lenkimo momentui. Įrodoma, kad, esant pakankamai dideliame lentynų skerspjūvio plotui, dvipusio takumo esant bet kokiam lenkimo momentui gali nebūti. Tyrimai taikytini esant ekstremalioms konstrukciją veikiančioms apkrovoms.

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## YIELD STRESSES IN COMPRESSED AND BENDED COLUMNS AND BEAMS

### Summary

Yield stresses develop in steel structures when high loading is applied. If a column or other element of the structure is compressed, not only bended, then yield stresses can appear on one side of the cross-section, or on both sides. These states of stresses differ from one another and from elastic state also. Variation of the yield stresses in double-tee columns is presented in this paper. Thickness of the column webs is assumed infinitesimal and limit stress states are determined when bending moment is increasing. When the area of cross-section is sufficiently large the absence of double-sided yield region for any bending moment is proved. These investigations are applicable in the case of extreme structure loading.

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## НАПРЯЖЕНИЯ ТЕКУЧЕСТИ В СЖИМАЕМЫХ И ИЗГИБАЕМЫХ КОЛОННАХ И БАЛКАХ

### Резюме

Напряжения текучести появляются в стальных конструкциях под действием больших нагрузок. Если колонна или другой элемент конструкции не только изгибается, но и сжимается, то напряжения текучести могут появиться в одной стороне поперечного сечения или в обоих. Такие напряженные состояния отличаются один от другого, как и от упругого напряженного состояния. В статье представлено изменение напряженного состояния в двутавровых колоннах. Принимается допущение о бесконечной малости толщины полок и определяются предельные состояния при увеличении изгибающего момента. Доказывается, что при достаточно большой площади поперечного сечения полка двухсторонняя текучесть может не наступить ни при каком изгибающем моменте. Исследования могут быть применены при экстремальных нагрузках, действующих на конструкцию.

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