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Effects of elastic and damping features of mechanical systems on amplitude frequency characteristics

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1. Introduction

In the industrial environment, operators of machinery are often exposed to low frequency vibrations and noise that could create many problems. Manual operation of various agricultural equipment and machinery could have a negative effect and risk to human health and safety (hearing degradation or loss, vibration induced ailments [1]) when vibrations are excessive and noise levels are high.

The qualitative index of a mechanical system, as vibration characteristics of its parts or units (for instance, amplitudes of force or intensity parameters of mechanical vibrations, deformations, etc.), can be selected as quality criteria of the mechanical system $F(\vec{q})$, where $\vec{q}$ is the generalised parameter, for instance, a coordinate. The vibration characteristics can be related to the quantitative index – reliability characteristic of a technical system as well. Excluding determination of minimal permissible dynamic loads, from conditional reliability of an exploratory mechanical system one can obtain information about limiting condition of the whole system as well as of its single link or a joint. The reliability rate of the mechanical system increases when the quality index of its variable elements decreases. Its minimal condition, $\min F(\vec{q})$, is fulfilled at limiting rates of system design and operation. This condition is presented in algorithmic form [1].

The mechanical system oscillations are characterized by frequency spectrum of force $F(\omega)$ and quality criterion of mechanical system, i.e. vibration characteristics is determined by dynamic loads (force F exacting vibrations) and by vibration-acoustical properties of structural elements (amplitude - frequency characteristics).

Contact layer of moving parts in a mechanical system is a variable element of the system. By control of its qualitative criterion $F(\vec{q})$, reliability of a link (or a join) can be ensured. According to [1], conditions of limiting value, min $F(\vec{q})$, of the controlled parameter ($\vec{q}$) are fulfilled, when

$$q_j^i \le q_j \le q_j^{ii}, \; j=1, 2, \dots, i \qquad (1)$$

$$\varphi_i(\vec{q}) \le 0, \; i=1, \dots, s \qquad (2)$$

in which φ_i is the phase between input and output.

For the estimation and control of mechanical systems generating vibro-acoustic signals, mathematical models of mechanical oscillation process can be developed. In this procedure, mechanical vibrations can be sufficiently characterized by the vectors of force F_r and velocity $\overline{v}_2$, that are obtained from the oscillation amplitude frequency characteristics of system elements [2, 3]. Vibration radiation from a source to outlet points vibro–acoustics signal is influenced by damping of various mechanical elements. In various investigations, the mass – damping characteristics were used in mathematical, experimental and analytical methods to develop practical solutions [3-5]. The advantage of these methods is that they provide more possibilities (two-channel quick-acting Fourier transformation [4]) for the investigation of realistic mechanical systems utilizing amplitude frequency characteristics of construction elements [6-8]. The relation between signal parameters of input and output is expressed by the complex frequency function $H_v(\omega)$. The inverse $(1/H_V(\omega))$ is the complex mechanical resistance (mechanical impedance) $Z(\omega)$:

$$Z_{12}(\omega) = \frac{1}{H_v(\omega)} = \frac{F_1(\omega)}{v_2(\omega)} \qquad (3)$$

where $Z_{12}(\omega)$ is the transitive complex mechanical resistance.

The impedance Z can be written as

$$Z = \left(j\omega m - \frac{k}{j\omega} \right) + c = Im\,Z + Re\,Z \qquad (4)$$

where m, k, c are the mass, elastic stiffness and damping of the mechanical system, respectively; $\omega = 2\pi f$ is angular frequency in rad/sec, f is frequency in Hz.

When velocity amplitude of mechanical oscillations is estimated by the excitation force (F_1) at a point, the impedance of mechanical system is denoted by ($Z_{11}(\omega)$) from which damping features can be estimated.

In a real mechanical system (construction), direct and indirect contacts between moving (kinetic couples) and immobile elements exist. Inter-elements (gaskets, oil layers) are often used in connecting various mechanical elements. There are also elements with negligeable mass ($j\omega m \Rightarrow 0$) but having elastic or damping features [6, 7]. However, the interaction of these elements will have an effect on the output amplitude frequency characteristics of vibro-acoustics signal.

The aim the present investigation is to estimate the effect of mechanical system links of inter-elements on amplitude frequency characteristics.

2. Methods and investigation results

For a system that contains uniform mechanical elements, an analytical model is often used to determine the relationship in a resilient system between static forces and generalised coordinates q_k, k=1,2, … ,n, for application to dynamic analysis. For the definition of interaction between system elements in a mechanical link, a partial system comprised of two elements – a mass element M and elastic element K_i is used (Fig. 1). The element of their connection in mechanical link - K_K, is the elastic element.

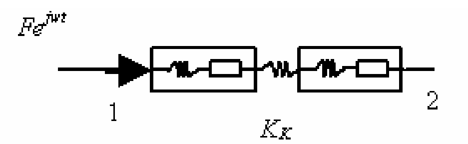


Fig. 1 Scheme of two-elements in a partial mechanical system

It can be shown that the frequency response function is

$$H(\omega)=\frac{K_1}{K_{K-2}}\left|\frac{\dfrac{K_{K-2}}{M_2}-\omega^2}{\dfrac{K_1}{M_S}-\omega^2}\right| \tag{5}$$

where M_S is the element mass of the first dynamic system and K_{K-2} is the total elastic value of K_K and K_2.

$$M_S = M_1 + \frac{M_2}{1-\dfrac{M_2}{K_{K-2}}\omega^2} \tag{6}$$

When the initial conditions of the investigated partial mechanical system are estimated, see Fig. 1, M_1=M_2=M, K_1=K_2=K, the expressions are obtained

$$M_S = M\left(1+\frac{1}{1-\left(\dfrac{\omega}{\omega_0}\right)^2}\right) \tag{7}$$

$$H(\omega)=\left(1+\frac{K}{K_K}\right)\left|\frac{\dfrac{1}{1+\dfrac{K}{K_K}}\omega^2-\omega^2}{\omega_0^2-\omega^2}\right| = \left(1+\frac{K}{K_K}\right)\frac{\dfrac{1}{1+\dfrac{K}{K_K}}-\left(\dfrac{\omega}{\omega_0}\right)^2}{1-\left(\dfrac{\omega}{\omega_0}\right)^2} \tag{8}$$

where $K_{K-2}=\dfrac{K\cdot K_K}{\left(K+K_K\right)}$.

During impacts in a mechanical system between various elements whole contact at surface microsection is not achieved. The condition of impact depends on surface roughness and material plastic characteristics. The micro–roughness plastic deformation at contact surface S_K, during force F_2 application, has on influence on rigidity at the beginning of force action. At a later stage, the elastic parameter of contact couples K_K) is increased [6]. This is influenced by the quality of geometrical characteristic of the surface, material elasticity modulus, Poisson's ratio, the area of contact surface and pressure force:

$$K_K=\frac{2\upsilon_m+1}{2h_m}\left[\frac{b_m k_2 \lambda_{E_0} S_K\sqrt{h_m}}{1{,}5\pi\left(1-\nu^2\right)\sqrt{r_m}}\right]F_K^{\left(\frac{2\upsilon_m-1}{2\upsilon_m+1}\right)} \tag{9}$$

where h_m, b_m, k_2, υ_m, r_m indicate the quality of geometric characteristics of the surface; λ_{E_0} is active part of elasticity modulus; S_K is the area of contact surface; F_K is active pressure force; ν is the Poisson's ratio.

The Eqs. (4) and (9) are obtained under condition, that to define amplitude frequency characteristics of microroughness layer of contact surface is sufficient to have the damping mechanical impedance:

$$Z_K(\omega)=K_K / j\omega \tag{10}$$

where $Z_K(\omega)$ is complex mechanical resistance of elastic mechanical element, K_K the damping coefficient of contact surface microroughness layer.

The dependency of mechanical impedance $Z_K(\omega)$ on the parameters given in Eq. (9) is presented in Fig. 2.

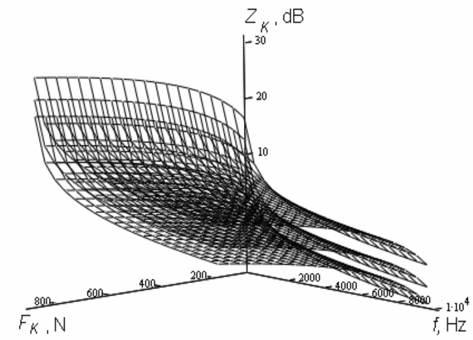


Fig. 2 Mechanical resistance dependence $\left(K/j\omega\right)$ of contact elastic layer of surfaces roughness Δ2, ..., Δ10 (class) on pressure force F_K and mechanical oscillation frequency (f) $10lg\left(K/\omega\right)=F\left(F_{K,}f\right)$

The value of mechanical impedance of elastic layer $Z_K(\omega)$ of mechanical elements (details) depends on the oscillation frequency f and has a linear character of 3 dB/octave per degree (Fig. 2). The pressure (tightness) force F_K influence of the elastic element on mechanical impedance module $|Z_K|$ is less and it is in the range at 2.0–2.4 dB/octave (it has a linear character only up to specific value f_f and it depends on the details of contact surface roughness class). Surface roughness has significant influence on mechanical impedance module of the elastic element.

In real construction, elements (details) of mechanical system are joined tightly (pressure force F_K), using additional elements (thread, screw, etc.), where the elastic coefficients $K_K \neq K_i$, and K_i is elastic coefficient of the same detail. The strength of details joint is typically $K_V < K_i$.

Mechanical impedance of the active component ReZ_K at resonance frequencies $\omega_0^{(n)}$ is, [2, 8]

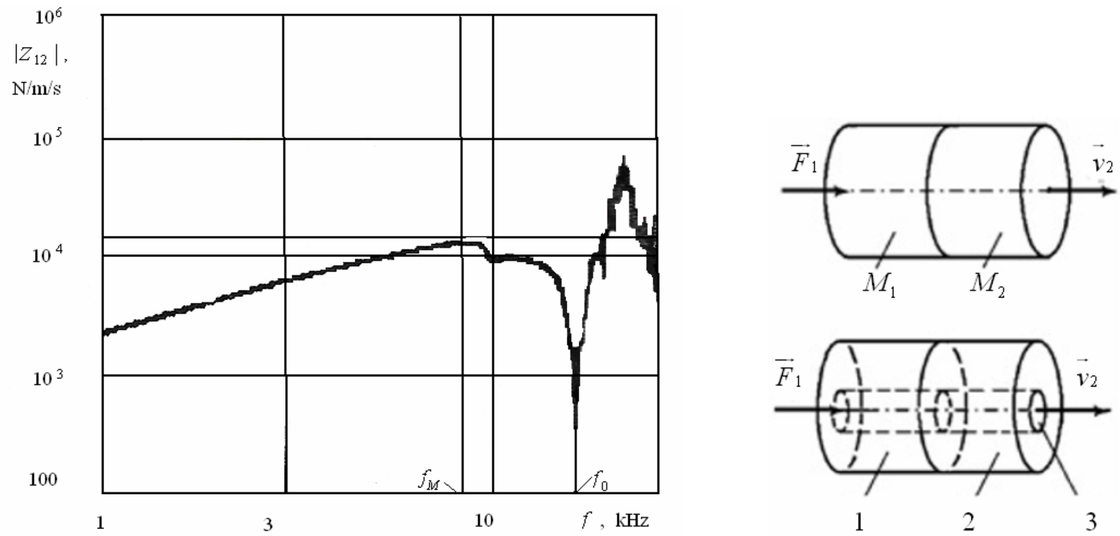
$$R_K = \eta_K Z_K^{(n)} = \eta K_K / \omega_0^{(n)} \quad (11)$$

where η is acoustic loss coefficient; $\omega_0^{(n)}$ is natural oscillation frequency, i=1, 2, 3, … , n.

For the investigation of the influence of contact surface microroughness on vibro-acoustic signal propagation, the method of electro-mechanical analogy can be used [8, 9]. For the contact of two elements with the concentrated masses M_1 ir M_2, the equation can be written as follows

$$\begin{bmatrix} F_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} 1 & j\omega(M_1 + M_2) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j\omega / K_{KNT} & 1 \end{bmatrix} \begin{bmatrix} F_2 \\ v_2 \end{bmatrix} \quad (12)$$

There are limitations of using Eq. (12) is obtaining transfer functions of mechanical systems, when the concentrated mass or elastic characteristics of the mechanical elements system are important. Accordingly, to the cases given in [6, 8], the influence of mechanical impedance of contact microroughness layer on the radiation of vibro-acoustic signals of mechanical network are used to investigate the experimental complex frequency characteristics.



a b

Fig. 3 Mechanical impedance of elastic layer $Z_k(\omega)$ of mechanical elements: a – frequency spectrum $|Z_{12}| = F(f)$ of transient mechanical impedance $|Z_{12}|$ module of the systems; b - the samples (∅35mm, l=100 mm steel pivot)

Considering the details *1* and *2*, where the amplitude frequency characteristics is of the type as in Fig. 3, a, the system behaviour up to the frequency f_m was analysed. These results are presented in Fig. 3, b and that can be expressed by the equations

$$\begin{bmatrix} F_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} 1 & j\omega(M_1 + M_2) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j\omega / K_K & 1 \end{bmatrix} \begin{bmatrix} F_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 & j\omega(M_1 - M_2) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} F_2 \\ v_2 \end{bmatrix} \Rightarrow (-j\omega / K_K)$$

$$\begin{bmatrix} F_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} 1 & j\omega(M_1 + M_2) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j\omega / K_K & 1 \end{bmatrix} \begin{bmatrix} 1 & j\omega M_r \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -j\omega / K_K & 1 \end{bmatrix} \begin{bmatrix} F_2 \\ v_2 \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} 1 & j\omega(M_1 + M_2 + M_r) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} F_2 \\ v_2 \end{bmatrix} \Rightarrow -j\omega / K_K = 0, \; -j\omega / K_K = 0 \quad (13)$$

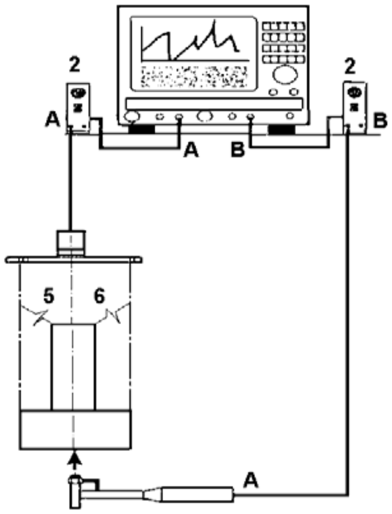
The coefficient K_K of microroughness layer could become large due to the thread oil layer. When the elastic resilience of thread elements $K_V < K_i$, the K_K is increased by better grip in the area S_K. When K_K achieves a large value, vibro–conduction ($Y_{12} = j\omega / K_K$) of that inter–element increase. Using given equations, the following expressions are obtained

$$v_2 = Y_{12} F_2 + Y_{21} F_1 \quad (14a)$$

or

$$v_2 = (F_1 - Z_{12} v_1) / Z_{12} \quad (14b)$$

where F_1, v_1 and F_2, v_2 are the amplitudes of the system mechanical oscillation force F_i and velocity v_i, respectively, at the input point (F_1, v_1) and output point (F_2, v_2);



A, B are the electro-mechanical links of vibration velocity and generation force;
2 is the load amplifier;
5-6 are the elastic construction hanger

Fig. 4 Experimental setup for measurement of mechanical impedance (using the instrumentation of the Brüel&Kjær firm)

Z_{12} is mechanical impedance of transitive microroughness layer.

The complete information on the mechanical system and amplitude frequency characteristics of the elements can be obtained by experiments for determining active and reactive components, see Eq. (4), of the mechanical impedance module $|Z|$ and phase angle (φ) between the vectors of force ($\vec{F}$) and velocity ($\vec{v}$). The designated or transient mechanical impedance module $|Z_i|$ and phase angle φ_i, between the vectors of force and velocity depend on the excitation frequency f. Measurements of these parameters were performed using special instrumentation, where vibro-acoustic signals were analyzed by the Fast Fourier Transform (FFT), see Fig. 4.

The mechanical system (element) is excited either by impulse (F_{pul}) or periodic ($Fe^{j\omega t}$) force and the response vibrations are measured by an accelerometer (impedance hammer type 8202 and the vibration sensors type 8200 of the Brüel&Kjær firm). The signal analysis, see Eq. (3), was carried out using dual-channel analyzer (type 2034). Using the calibrated measurement apparatus, see Fig. 4, measurement accuracy of ±0.1 dB is achieved.

3. Experimental results and discussions

The influence of elastic features of contact link elements mechanical system on transient function are investigated by the mechanical impedance methods presented as “bush–stick (pin) – bush“ systems. The sample was divided in to two elements – double steel cylinder, where the amplitude frequency characteristics were presented in Fig. 3, a. The experimental results of transient mechanical impedance are given in the form of diagram $|Z_{12}| = F(f)$ in Fig. 5.

The cylinder is butted by the joint of rigid elements „ bush-stick (pin)-bush“–mechanical system. In the transient mechanical impedance frequency spectra (see Fig. 5 curves *1* and *2*), the change of resonance frequencies $f_{0.1}^{(1)}; f_{0.2}^{(1)}$ are observed at lower frequency range: $f_0^{(1)} > f_{0.1}^{(1)}$ and $f_{0.2}^{(1)} < f_0$, see Figs. 3 and 5. The separation of solid mechanical system into several (in this case three parts) mechanical elements maximizes the active mechanical resistance part ReZ_{12} of the solid system. The elastic – thread turn element has significant influence in the joint by pin (thread joint) without output surface contact ($S_{K,pav} \Rightarrow 0$, see Fig. 5). The transient mechanical impedance is significantly reduced in upper range of medium frequency

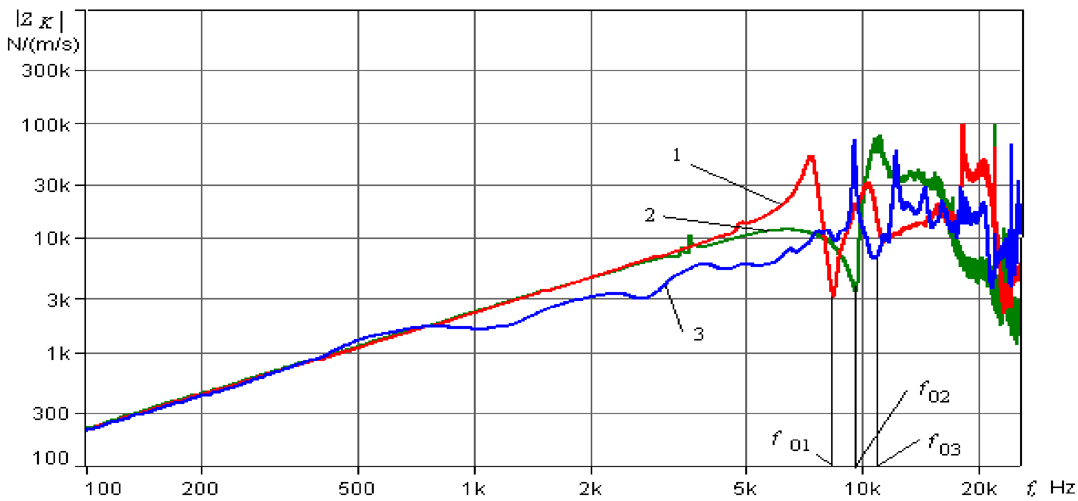


Fig. 5 Transient mechanical impedance frequency spectrum $|Z_{12}| = F(f)$ of mechanical system “bush–stick (pin)–bush“: *1* - rigid bush joint by metal stick ø14; *2* - rigid collet joint M14x1.75–pin; *3* - bush joint M14x1.75 pin (stick)

($f \geq 500$ Hz, see Fig. 5, curve 3).That part may be compensated by the oil layer at medium surfaces or joint plasticity of the details material (pins, male screw, see results in [6]).

4. Conclusions

The amplitude frequency characteristics of link inter-element contact layers of the mechanical system are defined by the transient elastic mechanical impedance.

The resonance phenomenon of mechanical detail thread join (thread, male screw) is wider in the low frequency range and could have a profound effect in the sound vibration system process.

The influence of elastic properties of details contact layers on the system (component, joint) vibroconduction can be eliminated by the use of superior layers (microroughness) of higher plasticity material for the connective elements (male screw, seal).

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MECHANINĖS SISTEMOS GRANDŽIŲ TARPELEMENTINIO SĄLYČIO TAMPRIŲJŲ IR SLOPINAMŲJŲ SAVYBIŲ ĮTAKA JOS AMPLITUDĖS DAŽNINĖMS CHARAKTERISTIKOMS

R e z i u m ė

Numatant mechaninių sistemų kompleksinę dažninę charakteristiką vertintinos ir tarpelementinio sąlyčio sluoksnių tamprio ir sklaidos savybės, galinčios daryti įtaką perdavimo funkcijai. Detalių kontaktinio sąlyčio savybėms tirti pasitelkti pereinamojo mechaninio impedanso metodai. Teoriniuose tyrimuose nagrinėtos detalių kontaktinių paviršių mikronelygumų ir srieginio jungimo elementų kompleksinės mechaninės varžos raiškos pobūdis. Šių kontaktinių elementų amplitudės dažninei charakteristikai apibūdinti tinka tamprio mechaninio impedanso savybės. Eksperimentiniais tyrimais nustatyta detalių sriegines jungties įtaka tiriamos mechaninės sistemos vibrolaidumui ir dažninės jos raiškos savybės sritys.

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EFFEECTS OF ELASTIC AND DAMPING FEATURES OF MECHANICAL SYSTEMS ON AMPLITUDE FREQUENCY CHARACTERISTICS

S u m m a r y

In determining the complex frequency characteristics of mechanical systems, the elastic and radiation features of the inter–element contact layers need to be considered that will have on influence an the transfer function. To investigate the characteristics of details contact the methods of transient mechanical impedance was used. Theoretical study is presented to assess the significance of microcontact surface roughness and of the threaded contact elements on the character of complex mechanical resistance. For the described contact elements, amplitude frequency characteristic is used for the elastic features of mechanical impedance. Using the experimental results, the influence of threaded joint on transfer system function and its frequency characteristics are determined.

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ВЛИЯНИЯ УПРУГИХ И ДИССИПАТИВНЫХ СВОЙСТВ СЛОЯ СОПРЯЖЕНИЯ ДЕТАЛЕЙ НА АМПЛИТУДНО-ЧАСТОТНЫЕ ХАРАКТЕРИСТИКИ МЕХАНИЧЕСКОЙ СИСТЕМЫ

Р е з ю м е

Предвидя комплексно-частотную характеристику механических систем, необходимо учесть свойства контактных слоев межэлементарной упругости и распространения, которых могут определить передающую функцию. Для исследования свойств контактного влияния применены методы переходного механического импеданса. В теоретических работах исследован характер михронеровности контактных поверхностей деталей и комплексное механическое сопротивление винтового соединения. Для описания амплитудно-частотной характеристики контактных элементов применяется характеристики упругости механического импеданса. Эхсперементальными исследованиями установлено влияние винтового соединения на вибропроводимость исследованной механической системы и область ее частот для определения свойств.

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