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Dynamics of rigid bodies in fluid and limit eigenmodes; theoretical research

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1. Introduction

Fluid pressure forces, generated by a moving body, are exerted on the body itself and on other bodies, restricting the fluid domain. This interaction was widely investigated in a variety of aspects and under different assumptions by M. C. Junger, D. Feit, I. S. Sheinin: the fluid is compressible and incompressible, viscous and ideal [1, 2]. The whole building structure usually can be divided into several separate sections, assumed to be rigid, when dynamics of the structure is concerned [3]. So, solution can be divided in two stages: 1) forces, acting on any separate rigid body, 2) dynamics of the whole mechanical system. Pressure, exerted by the fluid on a plate can be relatively easy evaluated if fluid is assumed incompressible: an analytic function, mapping conformally the fluid flow domain to upper half complex plane, is determined by V. Kargaudas, M. Augonis [4]. Interaction of deformable plates and fluid basically is more complicated as presented by D. G. Crighton, R. A. Horn, C. R. Jonson [5, 6].

Our main concern in this paper is dynamics of several rigid bodies. Particular attention has been given to the case when these bodies are identical and their motion is independent in vacuum. So, these bodies can be related to each other only by the fluid. A very low fluid density case is discussed. The air can be an example of such fluid. The concept of limit eigenmodes is introduced. Eigenmodes can be important if forced vibration frequency is resonant.

2. Dynamics in vacuum

Lagrange's equations are convenient to apply in many cases of complicated systems. Two plates A_1A_2 and A_3A_4 are presented in Fig. 1. The remaining part of the wall is assumed to be motionless. If mass center of the plate is not at the midpoint of this plate and two supports of different stiffness are not at the end of the plate, then, for example, the kinetic energy T_1 and potential energy Π_1 of this plate can be expressed

$$2T_1 = m_{11}\dot{u}_1^2 + 2m_{12}\dot{u}_1\dot{u}_2 + m_{22}\dot{u}_2^2$$
$$2\Pi_1 = k_{11}u_1^2 + 2k_{12}u_1u_2 + k_{22}u_2^2$$

where m_{ij} are values depending on the plate mass and distances $l_1^{'}, l_1^{''}$; k_{ij} are values depending on the spring stiffness and distances $h_1^{'}, h_1^{''}$, i,j =1, 2; u_1, u_2 are displacements of the plate boundary points in x direction (Fig. 2).

Equations

$$\frac{d}{dt}\frac{\partial T_1}{\partial \dot{u}_j} - \frac{\partial T_1}{\partial u_j} + \frac{\partial \Pi_1}{\partial u_j} = 0, j = 1,2$$

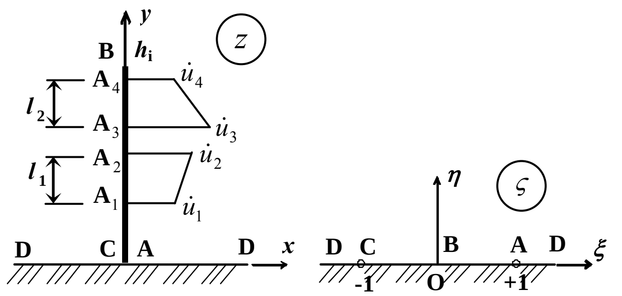

Fig. 1 Vertical plate AB and fluid flow domain in complex plane $z = x + iy$, conformally mapped to upper half-plane of auxiliary complex variable $\varsigma = \xi + i\eta$

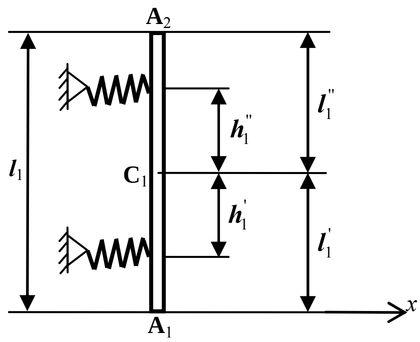

Fig. 2 The first plate, mounted on elastic springs, and the mass center C_1

describe motion of the plate if damping and excitation forces are not included. Dynamics of the second plate is presented in a similar way.

Matrixes can be applied for these systems of equations. If displacements of the first plate A_1A_2 are $\boldsymbol{u}_1 = \begin{Vmatrix} u_1 \\ u_2 \end{Vmatrix}$, then

$$\boldsymbol{A}_1\ddot{\boldsymbol{u}}_1 + \boldsymbol{C}_1\boldsymbol{u}_1 = 0 \quad (1)$$

where matrixes $\boldsymbol{A}_1 = \begin{Vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{Vmatrix}$, $\boldsymbol{C}_1 = \begin{Vmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{Vmatrix}$ and $m_{12} = m_{21}$, $k_{12} = k_{21}$. If vibrations are harmonic then $\boldsymbol{u}_1 = \boldsymbol{q}_1 e^{i\omega t}$, $\boldsymbol{q}_1 = \begin{Vmatrix} q_1 \\ q_2 \end{Vmatrix}$ and $\ddot{\boldsymbol{u}}_1 = -\omega^2 \boldsymbol{q}_1 e^{i\omega t}$.

Eq. (1) can be rewritten

$$\left(-\boldsymbol{A}_1 + \lambda \boldsymbol{C}_1\right)\boldsymbol{q}_1 = 0 \quad (2)$$

where $\lambda = \omega^{-2}$. Eq. (2) can be written in a form

$$(\boldsymbol{B}_1 - \lambda \boldsymbol{I})\boldsymbol{r}_1 = 0 \tag{3}$$

if a new variable $\boldsymbol{r}_1 = \boldsymbol{C}_1^{1/2}\boldsymbol{q}_1$ and identity $\boldsymbol{A}_1 - \lambda \boldsymbol{C}_1 = \boldsymbol{C}_1^{1/2}(\boldsymbol{B}_1 - \lambda \boldsymbol{I})\boldsymbol{C}_1^{1/2}$, where $\boldsymbol{B}_1 = \boldsymbol{C}_1^{-1/2}\boldsymbol{A}_1\boldsymbol{C}_1^{-1/2}$ are applied.

Nonzero solutions λ_j of Eq. (3) are eigenvalues of the symmetrical matrix $\boldsymbol{B}_1$. It follows from the symmetry of the matrix that λ_j are real numbers, j=1, 2. In these equalities $\boldsymbol{I}$ is identity matrix. If the matrix $\boldsymbol{T}_1 = \|\boldsymbol{r}_{11} \quad \boldsymbol{r}_{12}\|$ is formed by the eigenvectors columns $\boldsymbol{r}_{11}$, $\boldsymbol{r}_{12}$ of the matrix $\boldsymbol{B}_1$, then $\boldsymbol{T}_1'\boldsymbol{B}_1\boldsymbol{T}_1 = \begin{Vmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{Vmatrix} = \boldsymbol{\Lambda}_1$ [7]. Eigenvalues $\lambda_j = \omega_j^{-2}$ of the matrix $\boldsymbol{B}_1$ are in the principal diagonal of matrix $\boldsymbol{\Lambda}_1$. The motion of the second plate A_3A_4 can be presented by similar matrix equations after points A_1 and A_2 are substituted for A_3 and A_4. Then instead of (1)

$$\boldsymbol{A}_2\ddot{\boldsymbol{u}}_2 + \boldsymbol{C}_2\boldsymbol{u}_2 = 0 \tag{4}$$

Both matrix Eqs. (1) and (4) can be combined within a single equation $\boldsymbol{A}\ddot{\boldsymbol{u}} + \boldsymbol{C}\boldsymbol{u} = 0$, where

$$\boldsymbol{A} = \begin{Vmatrix} \boldsymbol{A}_1 & \boldsymbol{N} \\ \boldsymbol{N} & \boldsymbol{A}_2 \end{Vmatrix}, \quad \boldsymbol{C} = \begin{Vmatrix} \boldsymbol{C}_1 & \boldsymbol{N} \\ \boldsymbol{N} & \boldsymbol{C}_2 \end{Vmatrix} \tag{5}$$

and $\boldsymbol{N} = \begin{Vmatrix} 0 & 0 \\ 0 & 0 \end{Vmatrix}$ is zero matrix of the second order. This equation, like Eq (3), can be transformed to

$$(\boldsymbol{B} - \lambda \boldsymbol{I})\boldsymbol{r} = 0 \tag{6}$$

Eigenvalues of block matrix $\boldsymbol{B}$ are λ_j, and $$eigenvectors

$$\boldsymbol{r}_1 = \begin{Vmatrix} r_{11} \\ r_{12} \\ 0 \\ 0 \end{Vmatrix}, \quad \boldsymbol{r}_2 = \begin{Vmatrix} r_{21} \\ r_{22} \\ 0 \\ 0 \end{Vmatrix}, \quad \boldsymbol{r}_3 = \begin{Vmatrix} 0 \\ 0 \\ r_{33} \\ r_{34} \end{Vmatrix}, \quad \boldsymbol{r}_4 = \begin{Vmatrix} 0 \\ 0 \\ r_{43} \\ r_{44} \end{Vmatrix}$$

Diagonal matrix $\boldsymbol{TBT}' = \boldsymbol{\Lambda} = diag\{\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4\}$ can be obtained if matrix $\boldsymbol{T} = \|\boldsymbol{r}_1 \quad \boldsymbol{r}_2 \quad \boldsymbol{r}_3 \quad \boldsymbol{r}_4\|$ is formed by the eigenvectors columns. If the both plates are identical and identically supported then $\boldsymbol{A}_2 = \boldsymbol{A}_1$, $\boldsymbol{C}_2 = \boldsymbol{C}_1$ and $\boldsymbol{\Lambda} = diag\{\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4\}$, $r_{11} = r_{33}$, $r_{12} = r_{34}$, $r_{21} = r_{43}$, $r_{22} = r_{44}$, but $\boldsymbol{r}_1 \neq \boldsymbol{r}_3$ and $\boldsymbol{r}_2 \neq \boldsymbol{r}_4$. Although the eigenvalues are multiple the matrix $\boldsymbol{B}$ remains nondefective because all eigenvectors are linearly independent [7].

3. Fluid influence

If the fluid is ideal and incompressible then pressure in the fluid $p = -\rho\dot{\varphi}$, where ρ is density of undisturbed fluid, φ is velocity potential. The kinetic energy could be expressed from the first Green's formula

$$T_S = \frac{\rho}{2}\int_{(S)} \varphi \dot{u}_n dS \tag{7}$$

where S is closed surface in the fluid, u_n is fluid velocity component, normal to the surface. If the fluid flow is plane then S is the contour. Mechanical system with two vertical plates A_1A_2 and A_3A_4 is assumed in this investigation. The fluid is in the complex plane $z = x + iy$ and occupies any singly connected domain. Let analytical function $z = z(\varsigma)$ maps conformally a fluid occupied domain in z plane to the upper half-plane $Im\,\varsigma = \eta \geq 0$ (Fig. 1). Then the equality

$$Im\frac{dz}{d\varsigma} = \frac{dy}{d\xi} = C\frac{f(\xi)}{\sqrt{1-\xi^2}}, \ -1 \leq \xi \leq +1 \tag{8}$$

on the wall can be presented.

Two velocity diagrams of the plates in the wall AB of height h (Fig. 1) are displayed. Let's denote

$$u(y,t)f(\xi) = \sum_{k=0}^{\infty} a_k T_k(\xi) \tag{9}$$

where $u(y,t)$ are displacements of the wall AB particles as a function of ordinate y and time t, $T_k(\xi)$ is Chebyshev polynomials [4]. The function $y = y(\xi)$ is assumed to be deduced from Eq. (8) by integration

$$y = C\int_{+1}^{\xi} \frac{f(\xi)}{\sqrt{1-\xi^2}} d\xi = Cf_0(\xi) \tag{10}$$

Factors of the convergent Chebyshev series a_k are the time t functions and can be obtained applying othogonality of Chebyshev polynomials. Substitution $\xi = cos\,\tau$ is used for the formula

$$\frac{\pi}{2}\dot{a}_k = \int_0^{\pi} cos\,k\tau\,\dot{u}(y,t)f(cos\,\tau)d\tau \tag{11}$$

because $T_k(\xi) = T_k(cos\,\tau) = cos\,k\tau$. Thus from Eqs. (7) and (8) follows

$$T_S = \frac{\rho}{2}\int_{-1}^{+1} \varphi(\xi)\dot{u}(y,t)\frac{dy}{d\xi}d\xi =$$
$$= \frac{\rho}{2}C\int_0^{\pi} \varphi(cos\,\tau)\dot{u}(y,t)f(cos\,\tau)d\tau$$

Since fluid flow potential $\varphi = C\sum_{k=1}^{\infty} \dot{a}_k \frac{T_k(\xi)}{k}$ [4] kinetic energy of the fluid can be determined by applying Eq. (11) once again

$$T_S = \frac{\pi\rho}{4} C^2 \sum_{k=1}^{\infty} \frac{\dot{a}_k^{\,2}}{k} \tag{12}$$

Velocity of any point of the first plate when $y_1 \le y \le y_2$ is

$$\dot{u} = \alpha_1 + \beta_1 \frac{y}{C} = \alpha_1 + \beta_1 f_0(\xi) \tag{13}$$

where $\alpha_1 = \frac{y_2 \dot{u}_1 - y_1 \dot{u}_2}{y_2 - y_1}$; $\beta_1 = C \frac{\dot{u}_2 - \dot{u}_1}{y_2 - y_1}$.

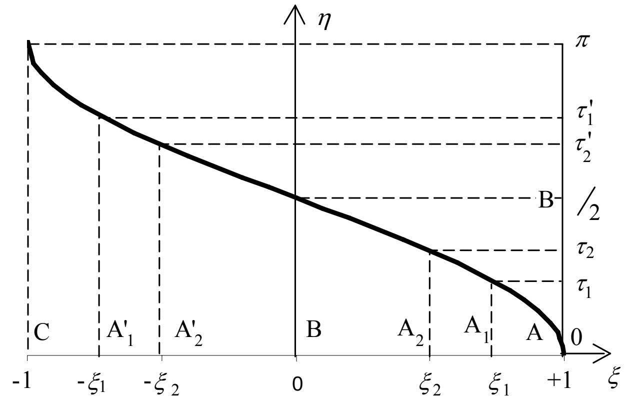

Fig. 3 Mapping of the interval [-1, +1] of real part ξ of the complex variable ξ to the interval [0, π] of variable $\tau = arccos\,\xi$. Mapping of both sides of only the first plate is depicted

Function $f_0(\xi) = y/C$ is expressed from Eq. (10), dimension of constant *C* normally is identical with *h*. In a similar manner velocity of the second plate can be expressed. Velocities on the both sides of plates are the same in most cases. Then velocities and diagrams as functions of ξ are the same in ranges $-1 \le \xi \le 0$ and $0 \le \xi \le +1$, but if the domain of fluid is not symmetric with respect to the *y* axis then $\varphi(\xi) \ne \varphi(-\xi)$. So, generally integrals $\dot{a}_k$ obtained from Eq. (11), in the intervals $[\tau_1, \tau_2]$ and $[\tau_2{}', \tau_1{}']$ are not equal (Fig. 3). If the values of velocities $\alpha_j + \beta_j f_0(cos\,\tau)$ from Eq. (13) and values α_j and β_j are substituted, then

$$\frac{\pi}{2} \dot{a}_k = \sum_{j=1}^{4} e_{kj} \dot{u}_j \tag{14}$$

can be evaluated integrating with respect to τ over the whole interval $[0, \pi]$.

For the first plate

$$e_{k1} = \frac{c_k^{(1)} y_2 - b_k^{(1)} C}{y_2 - y_1} \; ; \; e_{k2} = \frac{-c_k^{(1)} y_1 + b_k^{(1)} C}{y_2 - y_1}$$

$$c_k^{(1)} = \int_{\tau_1}^{\tau_2} cos\, k\tau f(cos\,\tau) d\tau + \int_{\tau'_2}^{\tau'_1} cos\, k\tau f(cos\,\tau) d\tau$$

$$b_k^{(1)} = \int_{\tau_1}^{\tau_2} cos\, k\tau f(cos\,\tau) f_0(cos\,\tau) d\tau + \int_{\tau'_2}^{\tau'_1} cos\, k\tau f(cos\,\tau) f_0(cos\,\tau) d\tau$$

In a similar way e_{k3}, e_{k4} can be expressed for the second plate. Substituting Eq. (14) into Eq. (12) one obtains

$$T_S = \frac{\rho}{2} \sum_{i=1}^{4} \sum_{j=1}^{4} \alpha_{ij} \dot{u}_i \dot{u}_j \tag{15}$$

where

$$\alpha_{ij} = \frac{2}{\pi} C^2 \sum_{k=1}^{\infty} \frac{e_{ki} e_{kj}}{k} \tag{16}$$

These formulae hold true if wall AB consists of any two plates located in the wall (Fig. 1). It is possible that A_1 coincides with A_1, A_2 with A_3, A_4 with B, but leakage between the plates is assumed impossible. Naturally the number of plates may be not two, but any number *n* as well. Then sums (14) and (15) will be not to 4, but to 2*n*. Inertial properties of a fluid in Lagrange's equations are determined by the derivative

$$\frac{d}{dt} \frac{\partial T_S}{\partial \dot{u}_j} = \rho \sum_{i=1}^{4} \alpha_{ji} \dot{u}_i \, , \; j = 1,2,3,4$$

Motion of the whole mechanical system, composed of two plates and the fluid, is described by matrix equation

$$(\boldsymbol{A} + \rho \boldsymbol{H}) \ddot{\boldsymbol{u}} + \boldsymbol{C} \boldsymbol{u} = 0 \tag{17}$$

where the influence of fluid is defined by the matrix

$$\boldsymbol{H} = \begin{Vmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \end{Vmatrix}$$

It can be seen from Eq. (16) that $\alpha_{ij} = \alpha_{ji}$, so, $\boldsymbol{H}$ is symmetrical matrix. Substitutions $\boldsymbol{u} = \boldsymbol{q} e^{iwt}$, $\boldsymbol{r} = \boldsymbol{C}^{1/2} \boldsymbol{q}$, can be used as in the case of the system in vacuum, so, from Eq. (17) follows

$$(\boldsymbol{B}_H - \lambda \boldsymbol{I}) \boldsymbol{r} = 0 \tag{18}$$

where $\boldsymbol{B}_H = \boldsymbol{C}^{-1/2} (\boldsymbol{A} + \rho \boldsymbol{H}) \boldsymbol{C}^{-1/2} = \boldsymbol{B} + \rho \boldsymbol{H}_B$.

Matrix $\boldsymbol{H}_B = \boldsymbol{C}^{-1/2} \boldsymbol{H} \boldsymbol{C}^{-1/2}$ remains symmetrical, and $\boldsymbol{B}$ is already determined block matrix.

4. Limit eigenvalues and eigenvectors

Matrix Eq. (6) of vibrations in vacuum is obtained if density of the fluid $\rho = 0$ is substituted into Eq. (18).

Variation of eigenvalues and eigenvectors of Eq. (18) when $\rho \to 0$, but $\rho \neq 0$, is investigated in this chapter.

Eqs. (6) and (18) differ in small perturbation matrix $\rho \boldsymbol{H}_B$. In [7] chapter 6, location of eigenvalues is considered too. If $\boldsymbol{r} = \boldsymbol{T}'\boldsymbol{b}$ is substituted to Eq. (18) and then multiplication on matrix $\boldsymbol{T}$, formed by eigenvectors columns of matrix $\boldsymbol{B}$, is performed, then $(\Lambda + \rho \boldsymbol{T}\boldsymbol{H}_B\boldsymbol{T}' - \lambda \boldsymbol{I})\boldsymbol{b} = 0$.

Eigenvalues of matrixes $\boldsymbol{B}$ and $\Lambda = diag\{\lambda_1, \lambda_2, ..., \lambda_n\}$ coincide, because $\boldsymbol{T}$ is nonsingular, so, the set of eigenvectors is linearly independent. The same can be stated about $\boldsymbol{B}_H$ and $\Lambda + \rho \boldsymbol{T}\boldsymbol{H}_B\boldsymbol{T}'$. Gershgorin circle theorem proves, that eigenvalues of $\boldsymbol{B}_H$ are contained within circles [7]

$$\left| z - \lambda_j \right| \leq \rho \sum_{i=1}^{n} \left| a_{ij} \right|, \; i \neq j, j = 1, 2, ..., n$$

where z is complex variable, a_{ij} are terms of the matrix $\boldsymbol{T}\boldsymbol{H}_B\boldsymbol{T}'$, n is the order of quadratic matrix. From the inequality it follows that the circles radii approach zero and eigenvalues of matrix $\boldsymbol{B}_H$ approach eigenvalues of matrix $\boldsymbol{B}$ when $\rho \to 0$. Eigenfrequencies ω_j and matrix eigenvalues are related $\lambda_j = \omega_j^{-2}$, so this proves

Corollary 1. All eigenfrequencies of any structure, submerged in fluid, approach eigenfrequencies of the same structure in vacuum, when density of the fluid approach zero.

It must be emphasized, that no similar general corollary can be proved about eigenvectors of the matrixes $\boldsymbol{B}$ and $\boldsymbol{B}_H$. It can be proved by particular example and calculations that under specific situation eigenvectors of matrix $\boldsymbol{B}_H$ do not approach eigenvectors of matrix $\boldsymbol{B}$ when $\rho \to 0$. In this paper general conditions, that should be satisfied for these matrixes to have different eigenvectors, are discussed.

If matrix $\boldsymbol{A}$ and $\boldsymbol{C}$ are block matrixes, defined by Eqs. (5), then $\boldsymbol{B} = \boldsymbol{C}^{-1/2}\boldsymbol{A}\boldsymbol{C}^{-1/2}$ is the block matrix too:

$$\boldsymbol{B} = \begin{Vmatrix} \boldsymbol{B}_1 & \boldsymbol{N} \\ \boldsymbol{N} & \boldsymbol{B}_2 \end{Vmatrix} \tag{19}$$

moreover, if $\boldsymbol{A}_1 = \boldsymbol{A}_2$ and $\boldsymbol{C}_1 = \boldsymbol{C}_2$ then it follows that $\boldsymbol{B}_1 = \boldsymbol{B}_2$.

If λ_j and $\boldsymbol{r}_j$ are eigenvalues and eigenvectors of matrix $\boldsymbol{B}$ then $\boldsymbol{B}\boldsymbol{r}_j = \lambda_j \boldsymbol{r}_j$. Eigenvectors of matrix $\boldsymbol{B}_H$ can be denoted by $\boldsymbol{r}_j + \boldsymbol{n}_j$, eigenvalues by $\lambda_j + \rho\mu_j$. Eigenvalues of $\boldsymbol{B}_H$ approach eigenvalues of $\boldsymbol{B}$ when $\rho \to 0$, as it was proved. But $\boldsymbol{r}_j + \boldsymbol{n}_j$ need not to approach $\boldsymbol{r}_j$ when $\rho \to 0$. Since

$$(\boldsymbol{B} + \rho \boldsymbol{H}_B)(\boldsymbol{r}_j + \boldsymbol{n}_j) = (\lambda_j + \rho\mu_j)(\boldsymbol{r}_j + \boldsymbol{n}_j)$$

then after rearrangement

$$(\boldsymbol{B} - \lambda_j \boldsymbol{I})\boldsymbol{n}_j + \rho \boldsymbol{H}_B(\boldsymbol{r}_j + \boldsymbol{n}_j) = \rho\mu_j(\boldsymbol{r}_j + \boldsymbol{n}_j) \tag{20}$$

Denote by $\boldsymbol{D}_1(\lambda)$ adjoint matrix of the matrix $\boldsymbol{B}_1 - \lambda \boldsymbol{I}$. For example if

$$\boldsymbol{B}_1 = \begin{Vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{Vmatrix} \text{ then } \boldsymbol{D}_1(\lambda) = \begin{Vmatrix} b_{22} - \lambda & -b_{21} \\ -b_{12} & b_{11} - \lambda \end{Vmatrix}$$

It has been known that $\boldsymbol{D}_1(\lambda)(\boldsymbol{B}_1 - \lambda \boldsymbol{I}) = \Delta(\lambda)\boldsymbol{I}$ [7] where $\Delta(\lambda)$ is characteristic polynomial $\Delta(\lambda) = det(\boldsymbol{B}_1 - \lambda \boldsymbol{I})$. In these equations λ is any number. If $\lambda = \lambda_j$, then $\boldsymbol{D}_1(\lambda)(\boldsymbol{B}_1 - \lambda \boldsymbol{I}) = \boldsymbol{N}$. Zero matrix is on the right side of the equation.

If two blocks in matrix (19) are equal $\boldsymbol{B}_1 = \boldsymbol{B}_2$, then matrix $\boldsymbol{D}(\lambda) = \begin{Vmatrix} \boldsymbol{D}_1(\lambda) & \boldsymbol{N} \\ \boldsymbol{N} & \boldsymbol{D}_1(\lambda) \end{Vmatrix}$ and if $\lambda = \lambda_j$ then

$$\boldsymbol{D}_1(\lambda)(\boldsymbol{B}_1 - \lambda \boldsymbol{I}) = \begin{Vmatrix} \boldsymbol{N} & \boldsymbol{N} \\ \boldsymbol{N} & \boldsymbol{N} \end{Vmatrix} \tag{21}$$

the factor ρ can be dropped if Eq. (20) is multiplied by matrix $\boldsymbol{D}(\lambda_j)$, where $\boldsymbol{B}_1 = \boldsymbol{B}_2$. Consequently, it follows

$$\boldsymbol{D}(\lambda_j)\boldsymbol{H}_B(\boldsymbol{r}_j + \boldsymbol{n}_j) = \mu_j \boldsymbol{D}(\lambda_j)(\boldsymbol{r}_j + \boldsymbol{n}_j) \tag{22}$$

Naturally, that eigenvector $\boldsymbol{r}_j + \boldsymbol{n}_j$ of matrix $\boldsymbol{D}(\lambda_j)\boldsymbol{H}_B$ does not depend on ρ because ρ is absent in Eq. (22). Number μ_i is not an eigenvalue of matrix $\boldsymbol{B}$, neither of matrix $\boldsymbol{B}_H$. Matrix $\boldsymbol{D}(\lambda_j)\boldsymbol{H}_B$ is singular and some eigenvalues are equal to zero because $det\, \boldsymbol{D}_1(\lambda_j) = 0$ and $det\, \boldsymbol{D}(\lambda_j) = 0$. But the eigenvectors of matrix $\boldsymbol{D}(\lambda_j)\boldsymbol{H}_B$, corresponding eigenvalues of which are not equal to zero, are limit eigenvectors of $\boldsymbol{B}_H$ when $\rho \to 0$. Different eigenvalues of matrix $\boldsymbol{D}(\lambda_j)\boldsymbol{H}_B$ can be obtained if different values of λ_j, $j = 1, 2, ..., n$, are used.

If $\boldsymbol{A}_1 \neq \boldsymbol{A}_2$ or $\boldsymbol{C}_1 \neq \boldsymbol{C}_2$ and consequently $\boldsymbol{B}_1 \neq \boldsymbol{B}_2$, then matrix

$$\boldsymbol{D}(\lambda) = \begin{Vmatrix} \boldsymbol{D}_1(\lambda) & \boldsymbol{N} \\ \boldsymbol{N} & \boldsymbol{D}_2(\lambda) \end{Vmatrix}$$

and identity is not possible for any $\lambda = \lambda_j$.

Corollary 2: If a structure is composed of several equal in size bodies with identical supports, then eigenmodes of the structure in fluid can totally differ from eigenmodes of the same structure in vacuum, even if density of the fluid is low. If the bodies are not equal in size or supports are not identical, and density of the fluid approaches zero, then eigenmodes of the structure in fluid approach eigenmodes of the same structure in vacuum.

5. Conclusions

1. If rigid plates are in ideal incompressible fluid and fluid flow can be assumed plane, then the fluid action on the plates can be determined through the use of fluid kinetic energy. Kinetic energy of the fluid can be expressed by infinite series. Any term of the series can be deduced if analytical function, comformally mapping the fluid flow domain to upper half-plane, is obtained.

2. Motion of mechanical system, composed of several rigid plates, can be more easily determined if kinetic energy of the fluid, dependent on generalized velocities, is obtained: Lagrange's equations of the whole mechanical system can be applied instead of differential equations for every rigid plate.

3. If mechanical system is composed of several completely identical rigid plates, independently moving in vacuum, then eigenmodes of the whole system, submerged into fluid, do not approach eigenmodes of the system in vacuum when density of the fluid approaches zero. It can be particularly important when forced oscillations are in proximity to resonance.

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KIETŲJŲ KŪNŲ DINAMIKA SKYSTYJE IR RIBINĖS SAVOSIOS FORMOS; TEORINIAI TYRIMAI

R e z i u m ė

Straipsnyje tiriama idealaus nesuspaudžiamo skysčio ir panardintų jame kietųjų kūnų tarpusavio sąveika. Jei kieti kūnai nesujungti tarpusavyje, tai jie vienas kitą veikia tik per skystį. Tiriamas atvejis, kai keli kūnai yra vienodai atremti, todėl jų savieji dažniai vakuume sutampa. Kai skysčio tankis artėja prie nulio, tai visos konstrukcijos savieji dažniai skystyje artėja prie kūnų savųjų dažnių vakuume. Šiame straipsnyje įrodyta, kad savosios formos skystyje gali visiškai skirtis nuo savųjų formų vakuume ir neartėti prie jų. Aiškinama tokio paradoksalaus skirtumo tarp savųjų dažnių ir savųjų formų priežastis. Parodoma, kad ribinės savosios formos nesutampa su savosiomis formomis vakuume kaip tik tada, kai skirtingų kūnų savieji dažniai vakuume yra vienodi. Šis tyrimas gali būti reikšmingas, jei tokios konstrukcijos priverstinių svyravimų dažniai yra rezonansiniai.

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DYNAMICS OF RIGID BODIES IN FLUID AND LIMIT EIGENMODES; THEORETICAL RESEARCH

S u m m a r y

Interaction between ideal incompressible fluid and a rigid body, submerged into the fluid, is investigated. If rigid bodies are not fastened together by mechanical connections the interaction of the bodies is possible only if the fluid is present. The case when several bodies are identical and their supports are the same is investigated: eigenfrequencies of these bodies in vacuum coincide. If density of the fluid approaches zero then all eigenfrequencies of the structures in the fluid approaches eigenfrequencies of the same structures in vacuum. In the paper it is given a proof that eigenmodes in the fluid can be completely different from eigenmodes in vacuum and do not approach eigenmodes in vacuum. The reason of this paradoxical distinction between the eigenfrequencies and the eigenmodes is presented. It is revealed, that the limit eigenmodes and the vacuum eigenmodes are different when eigenfrequencies of several bodies in vacuum coincide. This investigation can be significant if forced vibrations frequency of such mechanical system is resonant.

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ДИНАМИКА ТВЕРДЫХ ТЕЛ В ЖИДКОСТИ И ПРЕДЕЛЬНЫЕ СОБСТВЕННЫЕ ФОРМЫ; ТЕОРЕТИЧЕСКОЕ ИССЛЕДОВАНИЕ

Р е з ю м е

В статье исследуется взаимодействие идеальной несжимаемой жидкости с твердым телом в ней. Если твердые тела не соединены между собой, то их взаимодействие осуществляется только через жидкость. Исследуется случай, когда несколько тел одинаковы и их опоры одинаковы, поэтому их собственные частоты в вакууме совпадают. В этой статье доказано, что собственные формы в жидкости могут совсем не совпадать с собственными формами в вакууме и не приближаются к ним. Объясняется причина такого парадоксального отличия собственных частот от собственных форм. Показывается, что предельные собственные формы не совпадают с собственными формами в вакууме именно тогда, когда собственные частоты тел в вакууме совпадают. Эти исследования могут быть важными, если частоты вынужденных колебаний резонансные.

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