

sure in the contact, in physical sense (as, for example, Brinell's hardness HB), and represents the module of material hardening at spherical indenter – intrusion into it the contact module of hardening. Besides plastic hardness $H\bar{H}$ allows to create a uniform scale of hardness for all materials.

The use of the concept of plastic hardness, appeared productive not only for the definition parameters of contact (Eqs. (1)-(3)), but also for the solution of a number of practical problems [2, 4]. In particular, engineering solution of a problem on elasto-plastic flattening of the sphere [5] is executed, with use of this concept. At flattening material of the sphere is deformed elasto-plastically, and the detail material is deformed elastically. Calculation dependences for such kind of contact are used at hardness control of small machine details (for example, shot, ball, etc.) by flattening [6], and the definition of contact parameters of rough surfaces [7].

Obviously full approach of contacting details, for elasto-plastic flattening of the sphere [5], is determined by the formula similar to Eq. (1)

$$\alpha_c = \alpha_{y,c} + h_c \quad (4)$$

The elastic part $\alpha_{y,c}$ of full approach can be calculated according to the H. Hertz's formula for the case of only elastic contact of flat surface with a convex curvilinear surface which radius of curvature is equal to the radius $R_{b,c}$ of curvature of residual print on the sphere (Fig. 2)

$$\alpha_{y,c} = \sqrt[3]{\frac{9\pi^2 F^2 (k_1 + k_2)^2}{16R \left(1 + \frac{2h_c}{\alpha_{y,c}}\right)}} \quad (5)$$

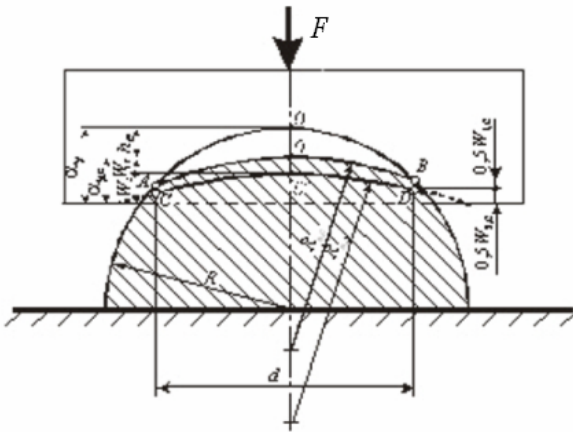


Fig. 2 The scheme of elasto-plastic flattening of spherical segment (sphere): R , $R_{H,c}$, $R_{B,c}$ are accordingly initial radius of the sphere and radii of curvature of contact surface under loading and the surfaces of residual print; $W_{1,c}$ and $W_{2,c}$ is elastic restoration of the surfaces of spherical segment and the detail in the centre of their contact after unloading; α_c , h_c , $\alpha_{y,c}$ are full, residual and convertible elastic approaches of contacting details; d_c is diameter of residual print; F is contact loading

It is necessary to note, that the last formula is

identical to the Eq. (2) for the case of intrusion of elastic sphere in to the detail surface; however the approaches which were calculated by Eqs. (2) and (5) do not coincide, as at identical loadings F residual flattening h_c is not equal to depth h of residual print at intrusion.

Residual flattening h_c determined by the formula [5] is received with use of the method of the theory of dimension

$$h_c = 0.331 R \left(\frac{F - F_0}{H\bar{H} R^2} \right)^{1,23} \quad (6)$$

where h_c is residual flattening of the sphere; R is radius of sphere's curvature.

As for the methods of calculated definition parameters of contact (depth and diameter of a residual print) in conditions when the sphere and detail have commensurable (close) hardness such methods are not found in the literature known to us.

3. Theoretical and experimental research

We shall consider the diagram of elasto-plastic contact (Fig. 3) for a sphere and a detail in the specified conditions. The initial contact of details occurs in a point. With the growth of loading F materials of the sphere and the detail are deformed in the beginning only elastically, and then elasto-plastically. The sphere takes root into the surface of the detail due to more rigid form. Thus radius R_s

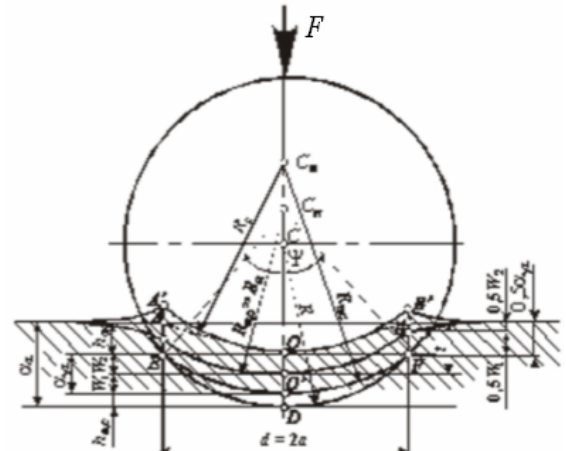


Fig. 3 The scheme of contact of a sphere and a detail at commensurable hardness of their materials: R is radius of the sphere; $R_{H,c}$ is radius of curvature of the contact surface under loading; $R_{B,c}$ is radius of curvature of restored (after unloading) print on the surface of plastically flattened sphere; R_s is radius of curvature of restored print on the detail surface; h_{ϕ} is residual print depth (residual approach) of the detail; $h_{\phi,c}$ is flattening (residual approach) of the sphere; W_1 and W_2 is elastic deformation of the sphere under loading F and elastic restoration of the detail in the centre of contact after unloading; t is full depth of the print under loading; $\alpha_{y,c}$ and α_c is a convertible elastic part and full approach in the contact; C , C_H and C_B are centers of curvature of not deformed initial surface of the sphere, of the dent surface under loading and of the surface of restored dent; d is diameter of the print

under loading of the deformed surface of the sphere is more than its initial radius R . The maximal moving in the zone of contact is observed at the greatest contact pressure, which is on the axes of action of loading F . After removal of loading there is elastic restoration in the contact as a result of which the radius of curvature of the sphere is restored up to $R_{e,c}$, and the radius of curvature of the surface of the print increases up to R_e .

Full approach of contacting details consists of three parts [8]: elastic approach $\alpha_{y,\Sigma}$, actual depth of residual print on the detail surface h_ϕ and actual size of residual flattening of the sphere $h_{c,\phi}$ (Fig. 3), that is

$$\alpha_\Sigma = \alpha_{y,\Sigma} + h_\phi + h_{c,\phi} \quad (7)$$

The elastic part $\alpha_{y,\Sigma}$ of full approach in this case can be calculated according to H. Hertz's [9] formula in which similarly to the offer of Tabor [10] for the case of sphere intrusion into the detail surface the calculation radius of curvature R_p is necessary to determine according to the radius of curvature of the surface of restored print. With the reference to considered (an examined) case (Fig. 3), calculation radius R_p is determine by the formula

$$\frac{1}{R_p} = \frac{1}{R_{c,e}} \pm \frac{1}{R_e} \quad (8)$$

in which "+" corresponds to positive curvature of the surface (a convex surface), and "-" to negative curvature of the surface (a concave surface).

For the definition of radii $R_{e,c}$ and R_e we shall consider the diagram of power elasto-plastic contact of a sphere and a flat surface of the detail (see Fig. 3). We shall accept the following :

1) with removal of loading F the print diameter d on the sphere surface and on the detail surface does not change;

2) surfaces of restored prints on the sphere surface (line $EO'F$) and on the detail surface (line AOB) have spherical form;

3) the surface of the detail outside of contact remains flat, and the surface of the sphere outside of contact remains spherical and has radius R , that is the material outside of the sphere and its general deformation is not consideration.

The first assumption is based that after loading removal there is intensive restoration in the center of the print, and its diameter changes a little bit. Our experiments in which the compared diameters of residual prints on surfaces of the flattened sphere and the diameters of contact under loading (for measurement the detail surface before loading was covered with a thin layer of coal), have shown, that the difference of these diameters does not exceed 3%. This result is not unexpected as it is known [11], that even in case of elastic sphere intrusion into the surface of a detail (when residual dent is surrounded with elastic material) diameter reduction of the print does not exceed 10% with loading removal even for steels of high hardness. Also it is necessary to note, that around of a dent on the detail surface the excrescence a little bit towering above initial surface level so is, as a rule the restored dent looks like curve $A'OB'$; as diameters of the dents usually are meas-

ured at comb level of excrescence they even come more nearer to the diameters of not restored prints.

Practical reliability of the second assumption for the surface of restored print (dent) on a detail is confirmed by the data of works [11, 12], and for a the surface of restored print on the sphere – the data of work [5].

The third assumption for a detail surface outside of contact corresponds to a postulate accepted in the theory of plasticity and used by A.Y. Ishlinsky in his work [1]. What about the sphere surface outside contact, according to experimental data of work [13] more or less noticeable metal movement in contact zone is observed only at low residual flattening $h_c < 0.02R$.

At these assumptions we shall determine radii of curvature $R_{e,c}$ and R_e : from equality condition of diameter of the base (diameter d on Fig. 3) segments AOB and EDF we shall receive (without accounting low value members of the second order)

$$d = 2\sqrt{2R_e h_\phi} = 2\sqrt{2R[\alpha_\Sigma - 0.5(W_1 + W_2)]} \quad (9)$$

when with accounting of the Eq. (1)

$$R_e = R \frac{\alpha_\Sigma - 0.5(W_1 + W_2)}{h_\phi} = R \frac{h_\phi + h_{c,\phi} + 0.5\alpha_{y,\Sigma}}{h_\phi} \quad (10)$$

Similarly from diameter equality condition of the bases of segments $EO'F$ and EDF we shall receive

$$\begin{aligned} d &= 2\sqrt{2R_{c,e} [h_\phi + \alpha_{y,\Sigma} - 0.5(W_1 + W_2)]} = \\ &= 2\sqrt{2R[\alpha_\Sigma - 0.5(W_1 + W_2)]} \end{aligned}$$

when

$$R_{c,e} = R \frac{h_\phi + h_{c,\phi} + 0.5\alpha_{y,\Sigma}}{h_\phi + 0.5\alpha_{y,\Sigma}} \quad (11)$$

Substituting expressions for R_e and $R_{c,e}$ into the Eq. (8), after transformations we shall receive an expression for the definition of calculation radius of curvature R_p

$$R_p = R \left(1 + \frac{2(h_\phi + h_{c,\phi})}{\alpha_{y,\Sigma}} \right) \quad (12)$$

In physical sense R_p corresponds to the radius of such a sphere at which power contact to a flat surface the diameter of residual print and elastic part of approach appear to be the same, as in real conditions of contact interaction of a sphere and a detail.

With accounting of the last expression of H. Hertz's formula [14] for the definition of elastic part $\alpha_{y,\Sigma}$ full approach and diameter d_ϕ of residual print for the considered case with the introduction of $\left(1 + \frac{2(h_\phi + h_{c,\phi})}{\alpha_{y,\Sigma}} \right)$ which with the consideration of plastic deformation in contact, will become

$$\alpha_{y,\Sigma} = \sqrt[3]{\frac{9\pi^2 F^2 (k_1 + k_2)^2}{16R \left(1 + \frac{2(h_\phi + h_{c,\phi})}{\alpha_{y,\Sigma}}\right)}} \quad (13)$$

$$d_\phi = 2 \sqrt[3]{\frac{3\pi}{4} (k_1 + k_2) FR \left[1 + \frac{2(h_\phi + h_{c,\phi})}{\alpha_{y,\Sigma}}\right]} \quad (14)$$

Apparently from the Eq. (13) if flattening of the sphere is completely absent, that is $h_{c,\phi} = 0$, and $h_\phi = h$, the Eq. (13) will be transformed to the known Eq. (2) [2] for the case of «pure intrusion»; if residual print on the surface of the detail does not arise, that is $h_\phi = 0$, and $h_{c,\phi} = h_c$ the Eq. (13) will be transformed [5] to the Eq. (5) for the case of “pure flattening”. The Eq. (15) will similarly be transformed also.

It is necessary to emphasize, that as elastic properties of (E and ν) metals do not depend on deformation speed [15] Eqs (13) and (14) will be valid at shock loading also which is typical, for example, for shot-blasting process.

In the experimental research of contact laws in the specified conditions we defined full and residual approach, on the device which basic circuit is similar to the described in work [17]. On a surface of the basis symmetrically are placed three steel spherical segments (made of grade ШХ 15 steel and heat treatment for hardness $H_{D_{100}}$ 2430 and 3680 MPa; a segment height was equal to the sphere radius $R = 2.5$ mm). Hardness of a steel detail varied in the range $H_{D_{100}}$ 1600 - 9500 MPa (to provide various value of ratio $H_{D_{100}}/H_{D_{100}}$). Loading was made with the press of Brinel' ТИИ-2 ($F = 613 \dots 9810$ N). Full approach $\alpha_{y,\Sigma}$ under loading was measured with dial indicator (scale unit value). For reduction of the mistake brought by approach in contact of basic plane of spherical segment with the basis of the adaptation, their surface was carefully polished and ground in to each other. Depth Σh_ϕ was registered after removal of loading by the same indicator. Time of endurance under loading in all experiences made 10.

Calculation under the Eq. (13) was compared with the experimental data presented in Table 1: the greatest divergence of the results of experiment and calculation makes 7% (a confidential interval of 95%), and for diameter d_ϕ a residual print (Tab. 2) – 12% (in the same interval of variation).

The method of calculation definition of the depth of residual print on the surfaces of the detail h_ϕ and residual flattening of the sphere $h_{c,\phi}$ is described in work [18].

Dependence of relative depth of residual print h_ϕ/h on the surface of a detail from $H_{D_{100}}/H_{D_{100}}$ is described by the equation (in range $H_{D_{100}}/H_{D_{100}} = 0.57 - 1.86$)

$$h_\phi/h = \sqrt{(H_{D_{100}}/H_{D_{100}})^{0.9} - 0.6 - 0.074} \quad (15)$$

and relative residual flattening $h_{c,\phi}/h_c$ of the sphere from $H_{D_{100}}/H_{D_{100}}$ – by the equation

$$h_{c,\phi}/h_c = 0.926(H_{D_{100}}/H_{D_{100}})^{-0.9} - 0.528 \quad (16)$$

Table 1

Comparison of the settlement $\alpha_{y,\Sigma}$ and experimental $\alpha_{y,\Sigma}^{\text{exp}}$ values of total elastic approach, in contact of the sphere (radius $R = 2.5$ mm) and details at various hardness ratio of their materials

№	Hardness of detail $H_{D_{100}}$, MPa	Hardness of sphere $H_{D_{100}}$, MPa	$\frac{H_{D_{100}}}{H_{D_{100}}}$	Working loading F , kN	Total elastic rapprochement, μm		Δ , %
					$\alpha_{y,\Sigma}^{\text{exp}}$	$\alpha_{y,\Sigma}$ Eq. (13)	
1	5366	3679	0.69	1635	0.023	0.024	-3.24
				3270	0.033	0.033	-1.24
				6540	0.0415	0.045	-7.87
				9810	0.045	0.052	-15.9
2	4817	3679	0.76	1635	0.024	0.023	4.82
				3270	0.033	0.032	2.23
				4905	0.041	0.043	-5.83
				8175	0.043	0.047	-9.73
3	2904	2433	0.84	2453	0.019	0.018	4.55
				4905	0.024	0.023	6.06
				6540	0.033	0.032	2.61
				9810	0.041	0.041	-0.35
4	2256	2433	1.08	4905	0.017	0.017	-2.85
				3270	0.026	0.025	2.21
				6540	0.036	0.036	-0.76
				9810	0.043	0.044	-2.54
5	2904	3679	1.27	1635	0.023	0.022	3.46
				3270	0.034	0.032	6.55
				8175	0.039	0.039	0.10
				9810	0.048	0.049	-1.51
6	1605	2433	1.51	1635	0.017	0.018	-6.03
				2453	0.026	0.026	-0.36
				4905	0.037	0.037	-0.50
				9810	0.044	0.045	-2.20

Table 2

Change of the diameter of a residual print on the surface of a detail from grade 25ХГТ steel ($H_{D_{100}}$ 8840 MPa) depending on hardness ratio of materials of the sphere and the detail

№	Material of sphere and its hardness $H_{D_{100}}$, MPa	$\frac{H_{D_{100}}}{H_{D_{100}}}$	D , mm	P , kN	$d_{\phi, \text{exp}}$, mm	d_ϕ , Eq.(14)	Δd_ϕ
1	ШХ 15, $H_{D_{100}}=12630$	1.43	10	4.9	1.22	1.11	24.1
				9.8	1.60	1.35	15.7
				14.7	1.92	1.68	12.1
				19.6	2.19	1.96	10.7
				29.4	2.67	2.51	5.99
2	ШХ15, issue 800°C, $H_{D_{100}}=8735$	0.99	9.95	4.9	1.25	0.920	26.4
				9.8	1.71	1.386	18.0
				19.6	2.25	2.137	5.01
				29.4	2.79	2.660	4.63
				3	ШХ15, issue 600°C, $H_{D_{100}}=5770$	0.65	9.95
9.8	1.92	1.74	9.3				
19.6	2.67	2.73	-2.3				
29.4	3.30	3.51	-6.5				

In Fig. 4 the diagram of dependence of the attitude $\frac{\sum h_\phi}{h+h_c}$ of actual total residual approach to theoretical total residual approach (designed for pure introduction or flattening) on hardness ratio $H\mathcal{D}_w/H\mathcal{D}$ of the materials of spherical segment and detail is shown. In interval $0.57 \leq H\mathcal{D}_w/H\mathcal{D} \leq 1.86$ the attitude $\frac{\sum h_\phi}{h+h_c}$ changes from 0.6 up to 1, and in conditions of pure introduction or flattening is equal 1. It can be seen, that in interval $0.9 \leq H\mathcal{D}_w/H\mathcal{D} \leq 1.5$, irrespective to the ratio $H\mathcal{D}_w/H\mathcal{D}$ actual total residual approach $\sum h_\phi$ has no influence on the attitude $\frac{\sum h_\phi}{h+h_c}$, that is the depth increase of residual print with the growth $H\mathcal{D}_w/H\mathcal{D}$ in the specified interval is compensated by size reduction of residual flattening.

The line in Fig. 4 can be described by the equation

$$\frac{\sum h_\phi}{h+h_c} = 2.5 \left(\frac{H\mathcal{D}_w}{H\mathcal{D}} - 1.215 \right)^4 + 0.6 \quad (17)$$

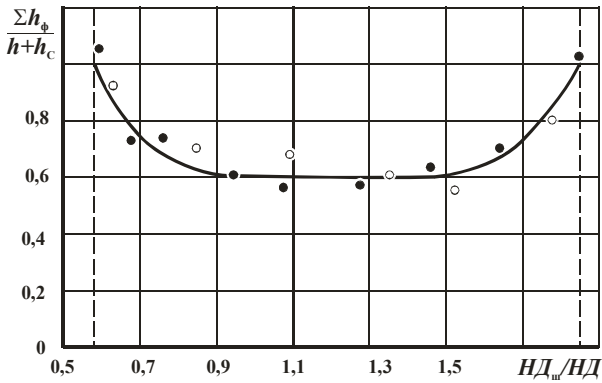


Fig. 4 The diagram of dependence of the attitude $\frac{\sum h_\phi}{h+h_c}$

of actual total residual approach to theoretical total residual approach (designed for pure introduction and flattening) on hardness ratio $H\mathcal{D}_w/H\mathcal{D}$ of materials of the sphere and detail: line is calculation by Eq. (11), light points-experimental data at $H\mathcal{D}_w$ 2430 MPa, and dark – at 3680 MPa

It is necessary to note, that the dependences given above describing parameters of elasto-plastic contact of the sphere with a surface of the detail in the considered case can be applicable and then when instead of the sphere not a spherical detail of double curvature is used, and also for a case of contact of two details of double curvature. Thus in all given dependences instead of radius R of the sphere it is necessary to take into account the given radius of curvature R_{np} of [2] contacting details, and under R_{np} to understand such radius of equivalent sphere at which convertible elastic and residual approach in its power contact with a flat plate will be the same, as in real contact of details of double curvature if in both cases working loadings and materials of contacting details are identical.

4. Conclusions

1. It is experimentally proved, that at power contact of a sphere and a detail which materials have commensurable hardness the elastic part $\alpha_{y,\Sigma}$, full approach in contact and diameter d_ϕ the residual print it is possible to define according to H. Hertz formulas in which the influence of plastic deformation on the surface of a detail and the spheres is simultaneously taken into account.

2. The error of definition of elastic part of full approach $\alpha_{y,\Sigma}$ in the offered dependence makes 7% (in confidence interval 0.95), and diameter d_ϕ of residual print – 12% (in confidential interval 0.95)

3. The results received in the work describing laws of contact interaction of a sphere and a flat detail at commensurable hardness of their materials, can be used for the definition: of loading ability at contact of two details of double curvature (for example rolling bearings,); the areas of contact of rough surfaces; modes of the combined strengthening technologies being the most effective for the increase of fatigue durability of machines parts.

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DETALIŲ TAMPRIAI PLASTINIO KONTAKTO PARAMETRŲ SLAIČIAVIMAS, ESANT SKIRTINGAM MEDŽIAGŲ IR JŲ KIETUMŲ SANTYKIUI

Re z i u m ė

Straipsnyje aprašyta kontakto parametrų (tamprijo ir visiškojo kontakto suartėjimo, liekamojo atspaudos matmenys) skaičiavimo metodika, esant kontaktuojančių paviršių abipusiškai tampriajai plastinei deformacijai pradinio taškinio kontakto sąlygomis.

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CALCULATION OF PARAMETERS AT ELASTO-PLASTIC CONTACT OF DETAILS WITH DIFFERENT RATIO OF THEIR MATERIALS HARDNESS

S u m m a r y

The definition technique of the parameters of contact (elastic and full approach in contact, the values of residual print) under conditions of mutual elasto-plastic deformations of contacting surfaces in originally point power contact is described.

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РАСЧЕТ ПАРАМЕТРОВ ПРУГОПЛАСТИЧЕСКОГО КОНТАКТА ДЕТАЛЕЙ ПРИ РАЗЛИЧНОМ СООТНОШЕНИИ ТВЕРДОСТЕЙ ИХ МАТЕРИАЛОВ

Р е з ю м е

Описана методика расчетного определения параметров контакта (упругое и полное сближение в контакте, размеры остаточного отпечатка) при условии обоюдной упругопластической деформации контактирующих поверхностей в условиях первоначально точечного силового контакта.

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