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Investigation of radial misalignment influence on dynamics of precise rotor system

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1. Introduction

The precision of connection between the drive and the executive mechanism has an significant influence on reliability of mechanical system. The classification of vibration damping devices for rotation systems developed in Vilnius Gediminas technical university is presented in the paper.

The problems connected with dynamics of mechanical system at certain radial misalignment of shafts are investigated in the offered work. As an object of research is the system of two shafts connected by elastic centrifugal ring coupling.

It is established, that in the coupling, connecting radially misaligned shafts, an internal moment of resistance to rotation is arising. Using the principle of d'Alamber for rotary movement, the differential equation describing rotation of the second shaft is written. It is shown, that having executed certain actions and entering a new variable, this equation can be reduced to the expression describing free fluctuations of a mathematical pendulum. As radial misalignment of the shafts is small in comparison with other dimensions, the method of small parameter is applied for the solution of this equation. The received solutions show, that rotational fluctuations arise in rotating misaligned system. The frequency of fluctuations is double the rotation frequency.

Application restrictions of the method of small parameter are investigated and units of its application are established. Results of the investigation were applied for reducing vibrations of mechatronic systems.

The rotor system consists of many synchronously rotating links. Due to shafts misalignment, nonbalanced parts, manufacture and assembly errors and variation of power supply, the links rotate irregularly. The above factors cause an increased dynamic loads in machines and mechanisms, which intensify rotary vibrations. When summed, these factors reach rather high values. For example, in powerful pumping stations of oil fields, a gear shaft misalignment reaches 2-4 mm in radial direction and up to 10 mm in axial direction.

The research was carried out with the aim of decreasing rotary vibrations and the forces which generate it. One of the most effective ways of decreasing vibrations is an improvement of structural elements of machines and their replacement with structures resistant to vibrations. For this purpose, effective RMTSD (Rotary motion transmission and stabilisation devices) in the form of various clutches and vibration dampers can be used. Vibration protection in coupling devices manifests itself as suppression of vibration of constituent links of the elements [1–3].

Lately, two ways have been used to avoid undesirable harmful vibrations, i.e. the development of devices with low activity vibrations and installation of special structural units which suppress and absorb vibrations in machines.

In rotor systems, flexible clutches with non linear characteristics are commonly used. The clutches used up to now do not completely match the vibration protection requirements. They poorly suppress vibration and impacts. At present, new clutch structures, the action of which is based on interaction between the forces of rotating masses and those of flexible elements, are worth special attention. Due to this interaction, radial stiffness of the shaft and coupling device (clutch) decreases. Thus, the development of new rotary motion transmission and stabilization devices, their metrical synthesis and optimisation of flexible element parameters through optimal location and orientation of their elements in constructions, enable us to obtain devices with good operational characteristics.

The RMTSD are used because the clutches of rotary moment transfer do not meet ever increasing requirements of industry. With smaller dimensions of machines, as well as higher speeds and productivity, dynamic loads of some parts of machines have grown considerably. For obtaining more precise and reliable machines, we must diminish the harmful vibrations generated. Increasing speeds of machines impose heavy demands on RMTSD. They have to compensate radial and angular misalignments of coupling shaft axes, to suppress impacts and interferences, to isolate more efficiently vibrations, to avoid resonance or to remove them from the zone of machine working speeds, to suppress dynamically rotary vibrations, and to stabilize the rotary motion on the principle of sequence, etc. Besides, the RMTSD must possess good operational characteristics at high temperatures and in the presence of dangerous chemical substances. They also must be easily assembled, dismantled and repaired, as well as being steady, firm and reliable.

2. Classification of the new RMTSD developed

The authors have developed more than two hundred design schemes of new devices which were copyrighted in USSR and covered by the patents of the USA, England and Germany. More than 170 design schemes belong to RMTSD, clutches and rotary vibration dampers. According to their functions (Fig. 1), RMTSD are classified as [1]:

- clutches for transmission and stabilization of ro-

tary motion;
- vibration dampers for stabilization of rotary motion. According to the type of elastically creeping elements (ECE), the clutches and vibration dampers are classified as clutches with homogeneous flexible elements (HFE), clutches with composite flexible elements (CFE), clutches with cord elements and clutches with continuum ECE.

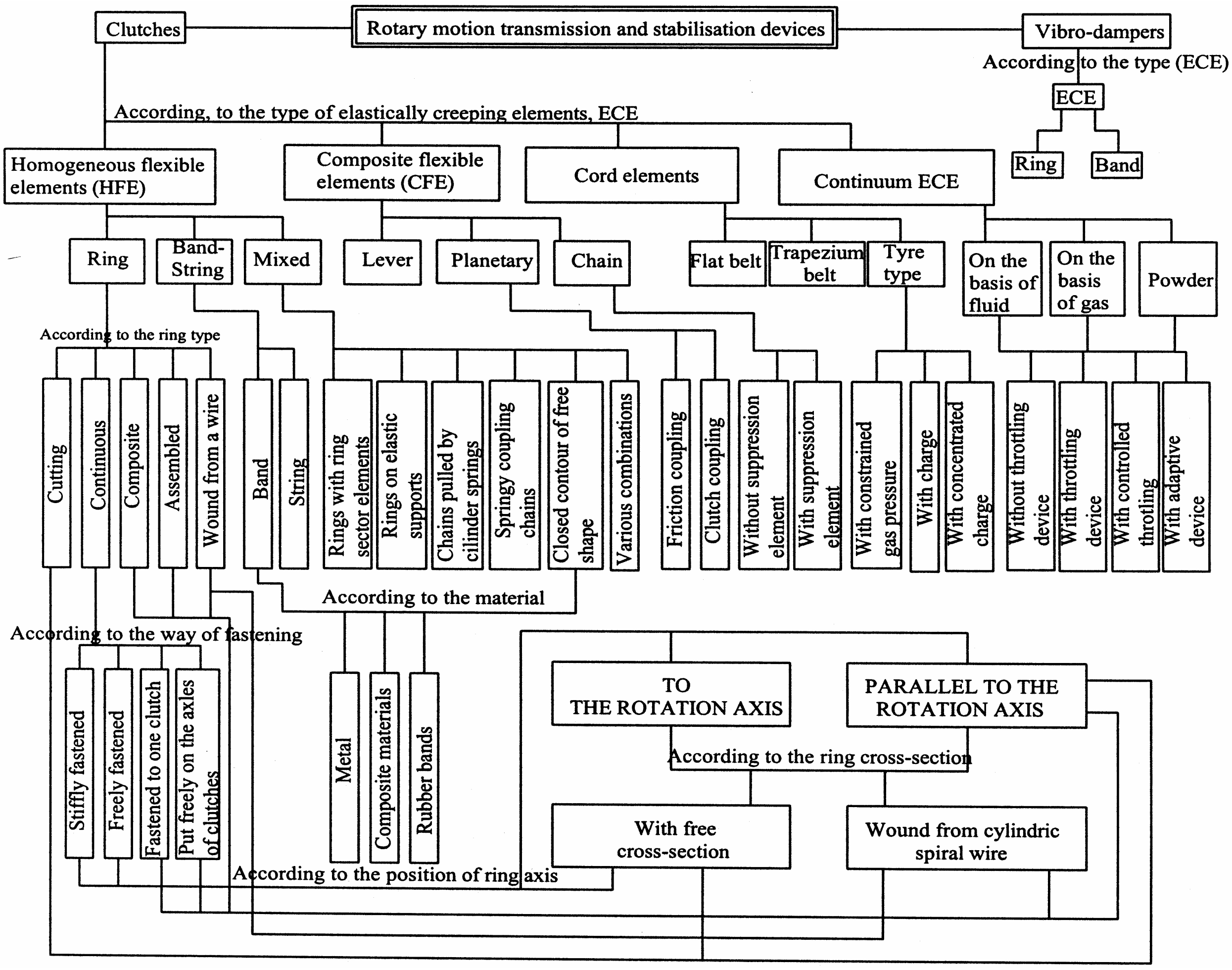

Fig. 1 Classification of rotary motion transmission

3. The object of the investigation

One of the methods of an unevenness reduction of rotational movement is based on the usage of elastic centrifugal ring couplings in the cardan drive. It was already described in [4]. Herein, we will discuss the dynamic processes in such drive, when shafts being misaligned in radial direction are connected using the above-mentioned couplings.

The construction of the simplest elastic centrifugal ring coupling is presented in Fig. 2.

The coupling consists of the driving half-coupling *1* and the driven half-coupling *2*. The terminals *3* and *4* of the half-couplings are connected with the elastic steel ring *5*, usually made of a wire wound into a circle. When the driving half-coupling is loaded with of torque, a slight angular shifting of the half-couplings in respect of each other takes place at their rotation because of elastic deformations of the ring *5*, and the of the ring shape becomes to an ellipsis. On rotation of the system, the elasticity close of the ring and the centrifugal forces of its distributed mass seek to restore the initial round shape of the ring and simultaneously reduce the coupling deformation. So, the links of two types (elastic and centrifugal) participate in transmission of a rotary motion [5].

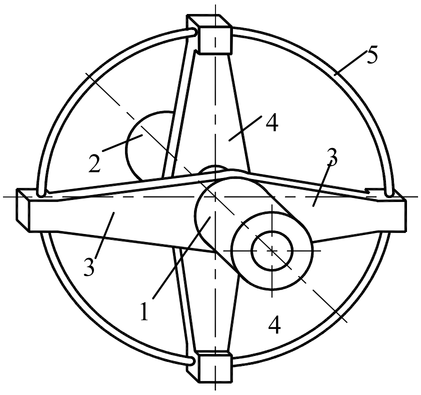

Fig. 2 Structure of the simplest elastic centrifugal ring coupling

4. Connection of shafts misaligned in radial direction

The above-described situation is characteristic for a connection of ideally aligned shafts. In most cases, the axes of the shafts do not coincide, for example, because of vibrations of the body in respect to driving wheels that appear in of the vehicle transmission.

In the case of radial misalignment of connected shafts, the shape of deformed elastic ring will be different (Fig. 3); the reactions F_x and F_y appear in fitting points of the half-couplings terminals.

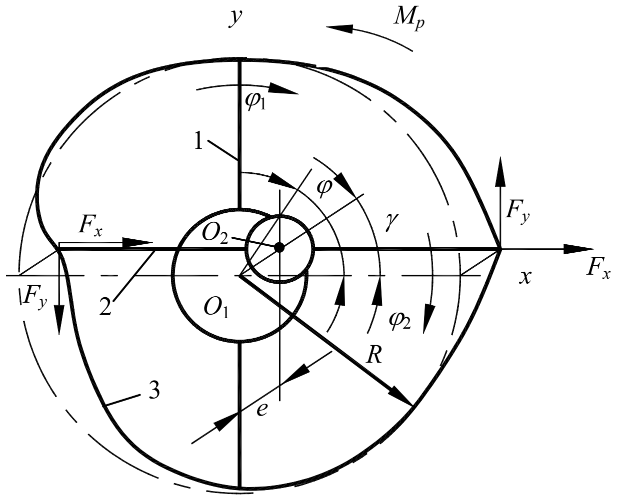

Fig. 3 Elastic ring deformation of the coupling in the case of radial misalignment

In this case, potential energy of the deformed elastic ring is found from the following expression

$$\Pi = \Sigma_n \int_0^{\frac{2\pi}{n}} \frac{M_l R d\varphi}{2EI} + \Sigma_n \int_0^{\frac{2\pi}{n}} \frac{N_a R d\varphi}{2EA} \tag{1}$$

where M_l is bending moment in the ring; N_a is axial force in it; R is bend radius of the ring; EI, EA – bending and stretching stiffness of the ring; φ is angular coordinate; n is the number of ring segments between half-couplings terminals.

Taking into consideration that

$$\left.\begin{aligned} \frac{\partial \Pi}{\partial F_x} &= e\cos\gamma \\ \frac{\partial \Pi}{\partial F_y} &= e\sin\gamma \end{aligned}\right\} \tag{2}$$

we define reactions F_x and F_y as functions of radial misalignment e

$$\left.\begin{aligned} F_x &= \frac{2EI}{R}\left(a\cos\gamma + b\sin\gamma\right) \\ F_y &= \frac{2EI}{R}\left(a\sin\gamma + b\cos\gamma\right) \end{aligned}\right\} \tag{3}$$

where $a = \frac{\pi^2 - 8}{4\pi}$, $b = \frac{\pi - 4}{2\pi}$.

On rotation of the system, an internal moment of resistance to rotation M_p appears in the coupling. It is found from the equation of moments about the point O_1

$$M_p = 2e\left(F_y\cos\gamma - F_x\sin\gamma\right) \tag{4}$$

Using the expressions of reactions F_x and F_y from (3), we obtain

$$M_p = 157\frac{EI}{R^3}e^2\cos 2\gamma \tag{5}$$

In the case under discussion, we may examine the rotating system as two-mass system with two degrees of freedom, considering that its generalized coordinates are rotation angle of the first shaft with the half-coupling *1* φ_1 and rotation angle of the second shaft with the half-coupling *2* φ_2 (Fig. 2).

Taking into account that rotation angle γ of the coupling in respect of misalignment direction may be interpreted as

$$\gamma = \frac{\varphi_1 + \varphi_2}{2} \tag{6}$$

the expression of internal moment of resistance to rotation becomes the following

$$M_p = 157\frac{EI}{R^3}e^2\cos\left(\varphi_1 + \varphi_2\right) \tag{7}$$

On an investigation of the impact of shafts misalignment upon rotating system dynamics, we suppose that rotation of the first shaft is uniform. In such case, the generalized coordinates that describe movement of the system will be the following

$$\left.\begin{aligned} \varphi_1 &= \alpha_1 + \omega t \\ \varphi_2 &= \alpha_2 + \omega t + x_e \end{aligned}\right\} \tag{8}$$

where α_1, α_2 are initial angles of rotation, ω is the angular speed, t is time, x_e is the value (angle) that characterizes rotation unevenness of the second shaft.

According to d'Alamber's principle, rotation of the second shaft is described by the following equation

$$I_2\ddot{\varphi}_2 - M_p = 0 \tag{9}$$

where: I_2 is the moment of inertia of the second shaft.

From the Eqs. (7), (8) and (9), we define

$$\ddot{x}_e - 157\frac{e^2}{I_2}\frac{EI}{R^3}\cos\left(\alpha_1 + \alpha_2 + 2\omega t + x_e\right) = 0 \tag{10}$$

We introduce a new variable

$$y_e = \frac{\pi}{2} - \left(\alpha_1 + \alpha_2 + 2\omega t + x_e\right) \tag{11}$$

and find its second derivative in time and obtain the equation that virtually describes free oscillations of mathematical pendulum

$$\ddot{y}_e + A_e\sin y_e = 0 \tag{12}$$

where $A_e = 157 \dfrac{e^2}{I_2} \dfrac{EI}{R^3}$.

If we suppose that at the moment t_0 $y_e(t_0) = \alpha_y$, $\dot{y}_e(t_0) = \beta_y$, we find

$$\dot{y}_e = \pm\sqrt{2A_e \cos y_e + \beta_y^2 - 2A_e \cos \alpha_y} \tag{13}$$

Then after the corresponding transformations

$$t - t_0 = \int\limits_{\alpha_y}^{y_e} \frac{d\, y_e}{\sqrt{2A_e\left(\cos y_e - \cos \alpha_y\right) + \beta_y^2}} \tag{14}$$

Then introducing of the following designations

$$\sin \frac{1}{2} y_e = k_1 u_1 , \quad k_1^2 = \sin^2 \frac{1}{2}\alpha_y + \frac{\beta_y^2}{4A_e} \tag{15}$$

the expression (14) turns into elliptic integral of the first degree

$$\sqrt{A_e}\left(t - t_0\right) = \int\limits_{\frac{1}{k_1}\sin\frac{\alpha_y}{2}}^{u_1} \frac{d\, u_1}{\sqrt{\left(1 - u_1^2\right)\left(1 - k_1^2 u_1^2\right)}} \tag{16}$$

that may be solved in respect of x_e.

5. Solution using the small parameter method

Bearing in mind that misalignment e is small in comparison with other dimensions, and, in addition, it is raised to the second power in the expression, we will try to find a solution of the expression (12) using the small parameter method

$$y_e = y_{e_0} + \varepsilon\, y_{e_1} + \varepsilon^2 y_{e_2} + \ldots + \varepsilon^n y_{e_n} + \ldots \tag{17}$$

where ε is a small positive parameter.

If we limit ourselves with two first members, we obtain

$$\begin{cases} y_{e0} = 0 \\ \ddot{y}_{e_1} + A_e \sin y_0 = 0 \end{cases} \tag{18}$$

Taking into account the initial conditions from the expression (11), we define the generalized coordinate φ_2, describing rotation of the second shaft will be equal to

$$\varphi_2 = \alpha_2 + \omega t + \frac{39.25\, e^2\, EI}{\omega^2 I_2 R^3} \sin\left(2\omega t + \delta_0\right) \tag{19}$$

where δ_0 is constant.

The last member of the expression (19) indicates that rotation of the second shaft is uneven, its angle of rotation is supplemented with a periodic component of the double frequency of rotation.

Another very important circumstance: the amplitude of this periodical part decreases upon increasing the angular speed (ω^2 is included into the denominator). This indicates evenness increase of the movement.

6. Restrictions of an applications of the small parameter method

The presence of angular speed in the denominator of the of amplitude expression indicates that the small parameter method cannot be applied for the solution of the equation (12) in all cases (when $\omega \to 0$, the amplitude is growing up to infinity).

The application problem of the expression (19) may be settled on a base of the following considerations. The positive shifting angle of the half-couplings in respect to each other upon an impact of the moment of resistance

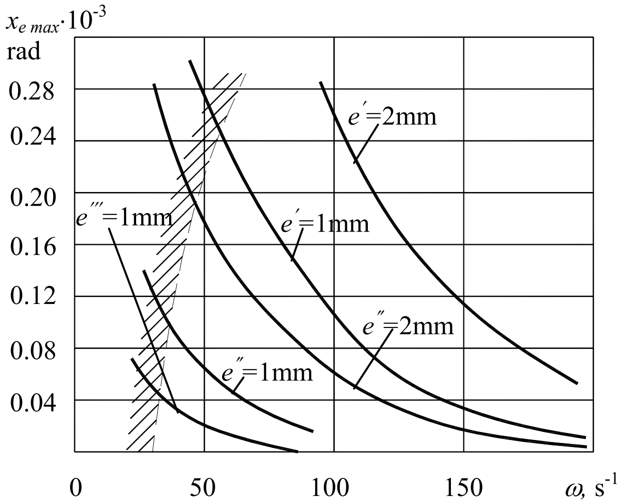

Fig. 4 Dependence of the amplitude of rotational vibrations of the second shaft on angular rotational speed, when wire diameter of the ring d = 3 mm, $'$ - R = =0.1 m, $''$ - R = 0.2 m, $'''$ - R = 0.3 m

to rotation M_p may be found from the following expression

$$x_e = \frac{M_p}{c_s} \tag{20}$$

where c_s is the rotational stiffness of the coupling.

Knowing that $c_s = 87\dfrac{EI}{R^3}$ [6], we find

$$x_{e_{max}} = 1.8\left(\frac{e}{R}\right)^2 \tag{21}$$

So, by comparing expressions (21) and (19) according to the condition of nonexceeding x_{emax}, we define the lower limit value of the angular speed applicable to the expression (19) (the hatched zone in Fig. 4).

7. Conclusions

1. Elastic centrifugal ring couplings may be successfully used for connection of misaligned shafts.

2. Rotation of rotating system misaligned in radial direction may be described by an equation that virtually coincides with the equation describing free oscillations of a

mathematical pendulum.

3. The frequency of the additional periodical component of the coordinate of rotation unevenness is equal to the double rotation frequency of the system.

4. An application of the small parameter system is not suitable in all cases. It is necessary to take into consideration specific features of the coupling construction.

5. If two misaligned shafts are connected by an elastic centrifugal ring coupling, rotation evenness increases with an increase of rotational speed.

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RADIALINIO NEAŠIŠKUMO ĮTAKOS PRECIZINĖS ROTORINĖS SISTEMOS DINAMIKAI TYRIMAS

R e z i u m ė

Remiantis Vilniaus Gedimino technikos universitete sukurtų sukamųjų virpesių slopinimo įrenginių klasifikacija, tiriama rotorinės sistemos su nesutampančiomis ašimis dinamikos problema. Sistemą sudaro du velenai, sujungti tampria išcentrine žiedine mova. Nustatyta, kad movoje, jungiančioje velenus, kurių ašys nesutampa radialine kryptimi, kyla vidinis pasipriešinimo momentas. Taikant rotorinei sistemai d'Alambero principą, sudaryta diferencialinė lygtis, aprašanti antrojo veleno sukimąsi. Gauti sprendiniai parodė, kad sistemoje su nesutampančiomis ašimis kyla sukamieji virpesiai, kurių dažnis du kartus didesnis už sistemos sukimosi dažnį. Tyrimų rezultatai panaudoti precizinių rotorinių sistemų sukamiesiems virpesiams mažinti.

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INVESTIGATION OF RADIAL MISALIGNMENT INFLUENCE TO DYNAMICS OF PRECISE ROTOR SYSTEM

S u m m a r y

After presentation of the classification of vibration damping devices for rotation systems developed in Vilnius Gediminas technical university, the problems connected with dynamics of rotor system at certain radial misalignment of shafts are investigated in the offered work. As object of research is the system of two shafts connected by elastic centrifugal ring coupling. It is established, that in the coupling, connecting radially misaligned shafts, an internal resistance moment to rotation is arising. Using the principle of d'Alamber for rotary movement, the differential equation describing rotation of the second shaft is worked out. The received solutions show, that rotational fluctuations arise in rotating misaligned system. The frequency of fluctuations is double the rotation frequency. Results of investigation were applied for rotational vibrations reducing of rotor systems.

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ИССЛЕДОВАНИЕ ВЛИЯНИЯ РАДИАЛЬНОЙ НЕСООСНОСТИ НА ДИНАМИКУ ПРЕЦИЗИОННЫХ РОТОРНЫХ СИСТЕМ

Р е з ю м е

Представив классификацию разработанных в Вильнюсском техническом университете им. Гедиминаса приспособлений для гашения крутильных колебаний роторных систем, в данной работе исследуется вопрос динамики несоосной роторной системы. Объектом исследования является система двух валов, соединенных при помощи упругой кольцевой центробежной муфты. Установлено, что в муфте, соединяющей несоосные валы, возникает внутренний момент сопротивления. Воспользовавшись принципом д'Аламбера, составлено дифференциальное уравнение, описывающее вращение второго вала. Полученные решения показывают, что в системе с несоосными валами возникают крутильные колебания с частотой, двукратной частоте вращения. Результаты исследований использованы для снижения колебаний роторных систем.

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