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Surface finish quality characterisation of machined workpieces using fractal analysis

B. Alabi*, T.A.O. Salau, S.A. Oke*****
**Department of Mechanical Engineering, University of Ibadan, Nigeria*
***Department of Mechanical Engineering, University of Ibadan, Nigeria*
****Department of Mechanical Engineering, University of Lagos, Nigeria, E-mail: sa_oke@yahoo.com*

1. Introduction

Characterisation of surface finishes of machined components and parts in machining operations is becoming more important as the world tends towards globalisation and competitiveness. Companies are choosing new technologies to improve on the surface finish of components and parts. A new method of fractal characterisation of the spectral trace of machined surfaces is investigated in this work. Fractals have been used to describe and quantify irregular fragments or complex shapes of materials such as shore-line, clouds, plants, brain cells, gold colloids, and sponge iron [1, 2]. Fractal analysis has also been used to study structural and mechanical attributes of some food products [3]. Kerdpiboon et al. [3] use artificial neural network analysis to predict shrinkage and rehydration of dried carrots, based on the inputs of moisture content and normalised fractal dimension analysis of the cell wall structure. Measured values of shrinkage and rehydration were predicted with an $R^2 > 0.95$ for the entire test samples.

Biancolin et al. [4] investigate an acoustic emission diagnosis technique for the study of fatigue cracks nucleation and propagation on steel fractal dimension (D) that evolve with the number of fatigue cycles (N) of the specimen. This was found useful to identify the condition of incipient collapse due to the nucleation and propagation of fatigue cracks on steel. The results suggest that it is possible to anticipate the detection of crack beginning relating to the other theoretical or experimental techniques. El-Sayed and Gaber [5] use the domain decomposition method to find the explicit and numerical solutions of the time fractional partial differential equations. Gaite [6] proves the relation between the box dimension of the fractal set (for $d \leq 3$) and the exponent of the Zipf law for convex voids. The author forbids the appearance of degenerate void shapes. Hans and Hu [7] utilise a revised fractal contact model followed Ahu to analyse the contact behaviours of electrode/workpiece and workpiece/workpiece interfaces for aluminum alloy and low carbon steel, respectively. A comparison with the predicted and the experimental curve illustrates that the predicted curve is in better agreement with the measured and the analysis results are right (see[8]).

Tatlier [9] applied fractal analysis to the patterns formed on the dance floor by footwork while performing various dance figures. The magnitude of fractal dimension is mainly dependent on simplicity/complexity of the dance figures and the characteristic rhythm of the music dictating the basic footwork and figures performed. He [10] reveals possible scenarios for predicting 69 particles at different energy scales in $11 + \phi^3$ fractal dimensions of a fractal M theory, where $\phi = (\sqrt{5} - 1)/2$. Zheng et al. [11] present a method to analyse the fractal properties of the 4-point binary and the 3-point ternary interpolatory subdivision schemes.

The results presented in this paper offer a direct means for fast generation of fractals. Aniszewska and Rubaczuk [12] present the results of physical stability calculations based on single fractal approximation. Numerical examination of physical stability proves that the new model is stable. Park et al. [13] develop a theoretical model to predict the porosity and specific resistance of cake layer based on fractal theory. The specific cake resistance decreases upon increasing the floc size and decreasing the fractal dimension. Davila and Pares [14] use fractal and lacunarity analysis to examine the structure of heat-induced gels of plasma proteins at pH 5.0, 6.0 and 7.0. It was found that the heterogeneity of cavities distribution pattern increased as pH decreased.

A closely related study to the current work is due to Olaosebikan [15] on the possibility of defining “surface finish” by means of certain parameters obtained from spectral analysis of the wave-form recorded for a given surface profile. The study involves the examination of five differently machined surfaces, and from quantitative analysis of their spectra, a new index is proposed for assessing “surface finish” which overcomes some shortcomings of the traditional centerline-average (CLA) value. The results obtained with the CLA method give 5 1 2 4 3, while Olaosebikan’s method yielded 5 1 2 4 3. The implication was that surface No.5 is most rough while surface No.3 is most smooth.

From the various studies presented above, it seems that improvement in the characterisation of machined surfaces with respect to operations characterisation with fractals has not been addressed, which is the subject of the current paper. The divisions of the rest of this paper are as follow. The next section considers the mathematical model, which is the foundation of this paper. Section 3 considers fractal characterisation in relationship with the trace spectra concept considered in this work. Finally, concluding remarks are given.

2. Mathematical model

The mathematical model that forms the foundation for the current study is based on three sets of equations and four variables. These equations and variables (X, Y, d and C) are related in accordance to power law. The concept of disk count Monte Carlo is linked to these variables and they are so interpreted. The first equation is stated as

$$Y = CX^d \quad (1)$$

here, Y is equal to the minimum number of disks of known size required to cover wholly a fractal image lying on a 2-D Euchidean plane. C in the expression related to the constant of proportionality. X represents the minimum number of disks of known size required to cover the datum length. Term d in Eq. (1) is fractal disk dimension of the fractal image under investigation. The formulation of Eq. (2) is made possible by finding the logarithm of both sides

$$Log\ (Y) = Log\ (C) + d\ Log\ (X) \quad (2)$$

The third equation is a re-written form of Eq. (2)

$$y = k + dx \quad (3)$$

The spectral trace was obtained in analogy form (graphical) from Olaosebikan [15]. The present fractal analysis required the spectral to be in digital form. For this reason five spectral traces (enlarged copies) were digitised using standard graph paper of 2mm by 2mm grid size. This seems to be the best digital resolution manually practicable. Modern digital system (digitising machine) was not within the reach for researchers of this study since its cost is prohibitive. The outputs of manually digitised spectral traces are shown in the next five figures (Figs. 1 to 5). The approach presented in this work is christened Fractal Spec TAO, named after one of the authors of this work for his contribution in the development of the methodology, which forms the thesis of this work.

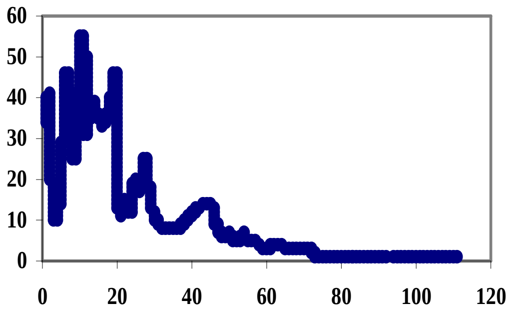

Fig. 1 Output of digitised spectral trace 1

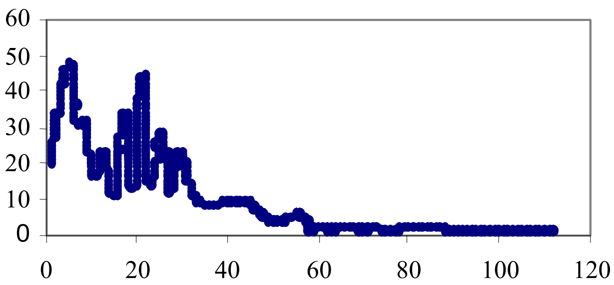

Fig. 2 Output of digitised spectral trace 2

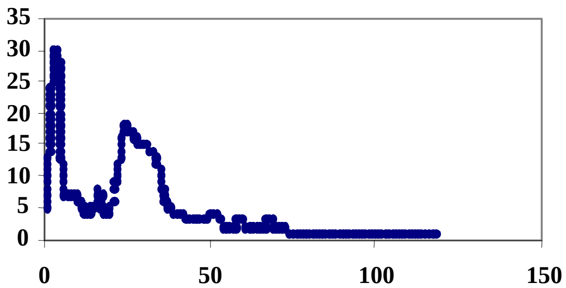

Fig. 3 Output of digitised spectral trace 3

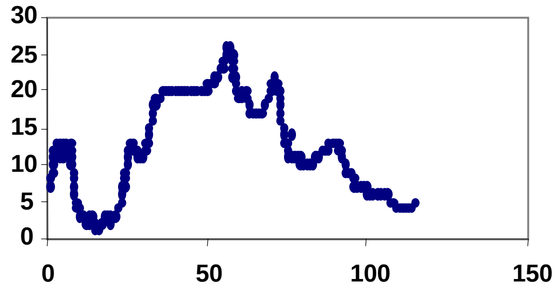

Fig. 4 Output of digitised spectral trace 4

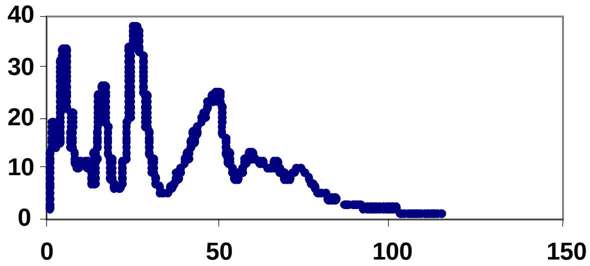

Fig. 5 Output of digitised spectral trace 5

3. Fractal experimentation

Five fractal images were analysed for their respective (d) as in Eq. (3). Here, it is observed that d represents the slope of the best line through log-log plot of pair of (X, Y). The computation of the fractal images was done using iterated functions system (IFS). Images were represented by cluster of 8000 solution points obtained after a trade off of the first 1000 solution points from a common starting point of (1, 2) that was chosen arbitrarily. The estimation of Y for corresponding X follows a Monte Carlo procedure. The random number generator seed value used for all case was 5678. For fixed X multiple estimate of Y is made and the least Y value from the set is recorded for the purpose of analyses for the d. For more details, see the tables and figures for respective fractal images. Notably, Fortran programme was used for the implementation of both IFS and the Monte Carlo procedures.

In particular, fractal analysis was applied to the spectral trace presented by Olaosebikan [15]. This spectral trace, obtained in analogue form (graphical), was digitised. The five spectral traces (enlarged copies) were digitised using standard graph paper of 2mm by 2mm grid size. The digital resolution obtained is the best manually practicable in the view of resources available to the authors. Although modern digital systems are available, they are rather too expensive, and hence are limitation of the study. The outputs of the manually digitised spectral trace are shown in Figs. 1 to 5.

Now, applying the fractal concept, the log-log plot for all the spectral traces (1 to 5) is made, and shown in Figs. 6 - 11. A Fortran programme was developed and tested based on some running parameters which are common to all the five cases. The seed, Iseed = 6789, the number of observation scales is 20, the number of iterations is

20. Noticeably, for all the cases considered computational time was found to be below 1 minute. By considering the results of the log-log plot for the spectral traces 1-5, fractal dimension by the disk count Monte Carlo approach is summarised in Table 1.

Theoretically, a curve is bound to have dimension of at least unity. Thus, the acceptability of this result depends on inherent human and computational errors. In Table 1, rank results show that the first machined surface is the roughest while the fourth surface is the smoothest. Thus, the five machined surfaces can be put simply as "15234" as against "51234" ranking obtained by CLA and spectral index methods. It is interesting that visual assessment of the digitised spectral traces for roughness agreed with the "15234" ranking in the present study.

Table 1

Comparison of present results with literature results

Fractal	Literature dimension	Present disk dimension	% relative absolute difference
Koch	1.2619	1.0972	13.1
Triangle	1.5850	1.5042	5.1
Xmas Tree	1.4650	1.3459	8.1
Fractal J	NA	1.4541	NA
Fractal W	NA	1.4514	NA
Fractal T	1.5850	1.4417	9.0

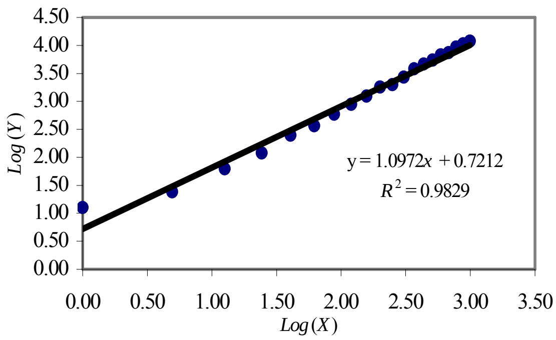

Fig. 6 Log-log plot for KOCH

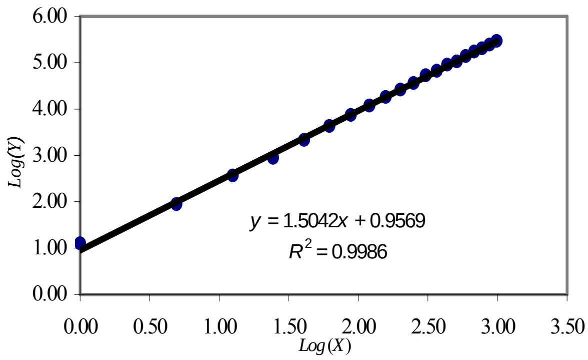

Fig. 7 Log-log plot for Triangle

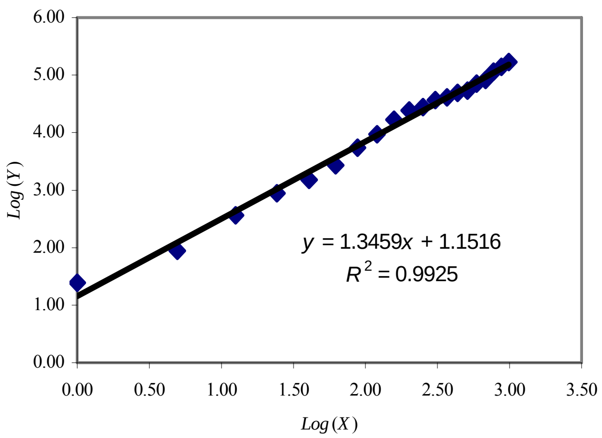

Fig. 8 Log-log plot for Xmas tree

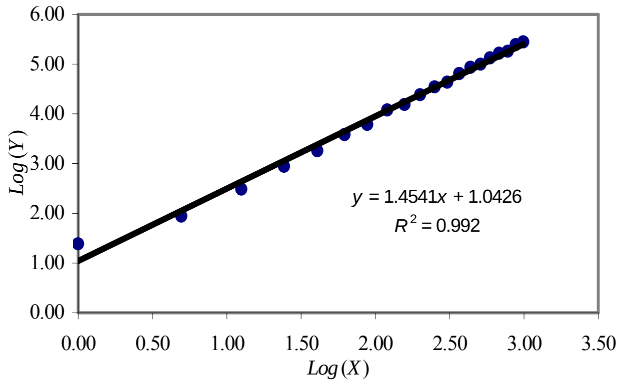

Fig. 9 Log-log plot for Fractal J

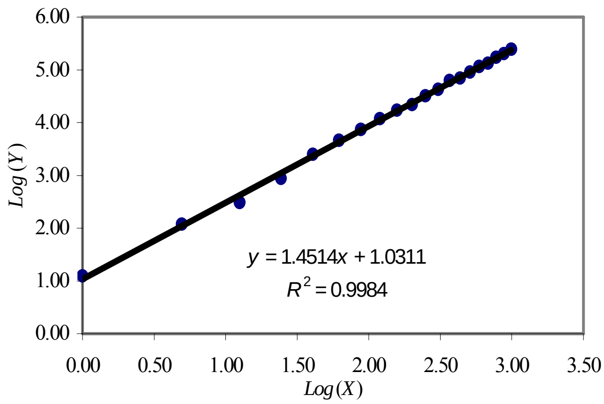

Fig. 10 Log-log plot for Fractal W

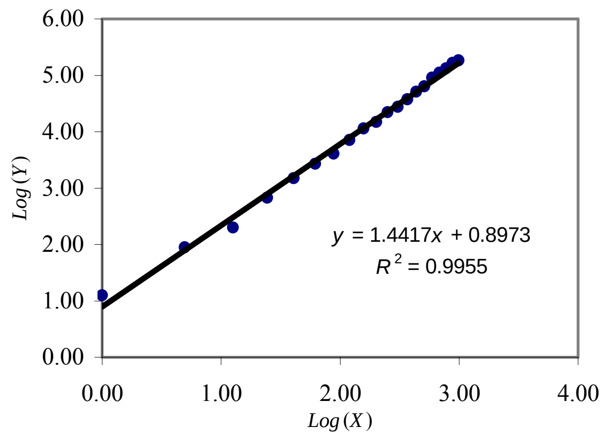

Fig. 11 Log-log plot for Fractal T

Referring to the graph of Disk Dimension Variation for fractal W shown above (Fig. 12), the Disk Dimension appears to tend toward a finite value with a gradual increase in the number of observation scale. Referring to Fig. 13, the percentage absolute relative difference range from 5.1% to 13.1% for all cases studied except for Fractal J and Fractal W that does not have literature dimension. The computation involved in each of the studied six cases lasted under three (3) minutes.

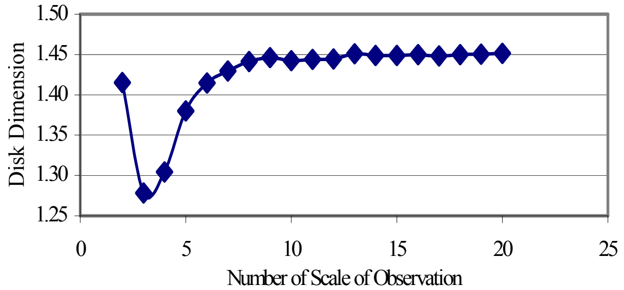

Fig. 12 Disk dimension variation for Fractal W

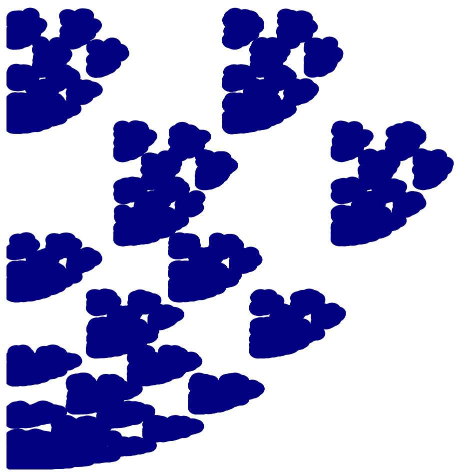

Fig. 13 Scatter diagram of fractal W (8000 points)

Referring to the log-log plot for spectral trace 1 (Fig. 14) fractal dimension by the disk count Monte Carlo approach is 1.1007 to four decimal. Referring to the log-log plot for spectral trace 2 shown above fractal dimension by the disk count Monte Carlo approach is 1.0864 to four decimal. Referring to the log-log plot for the spectral trace 3 (Fig. 16) the fractal dimension by the disk count Monte

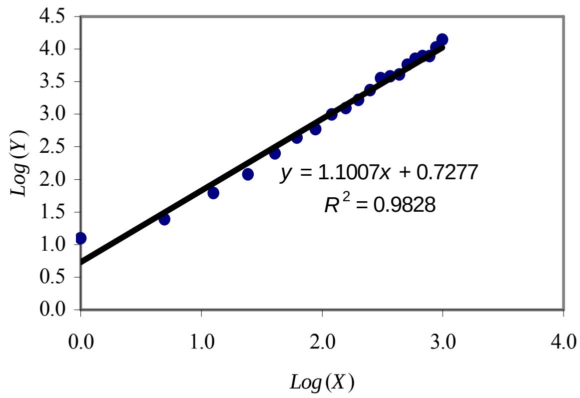

Fig. 14 Log-log plot for spectral trace 1

Carlo approach is 0.9355 to four decimal. Theoretically a curve is bound to have dimension of at least a UNITY! Thus the acceptability of this result is solely on inherent human and computational errors. Referring to the log-log plot for spectral trace 4 (Fig. 17) the acceptability of dimension of 0.9113 is as argued for spectral trace 3 with dimension of 0.9355.

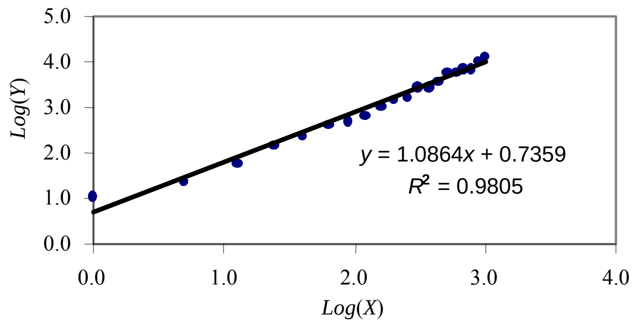

Fig. 15 Log-log plot for spectral trace 2

Referring to the log-log plot for spectral trace 4 (Fig. 17) fractal dimension by the disk count Monte Carlo approach is 1.0940 to four decimal. For illustration purpose, we report the procedure in transforming original data on the application of Monte Carlo approach in the search for minimum number of disk (known size) required to cover a particular spectral trace. Here, the example of Spectral trace 5 is given. Spectral trace 5 had the highest value of product of (*d* & *C*). *d* & *C* characterise the best line of fit to the data point on the two planes (circle size number versus corresponding measure). *d* is slope and *C* is intercept on the log-log 2-dimensional plane. Table 2 is used to obtain Table 3. To obtain Table 3, maintain column one of Table 2, search for minimum entry among (*Y*1 to *Y*20) of Table 2 and the results of this search form the entries for column two of Table 2 (from row 1 to row 20). Table 3 shows the rank of spectral trace dimension. From

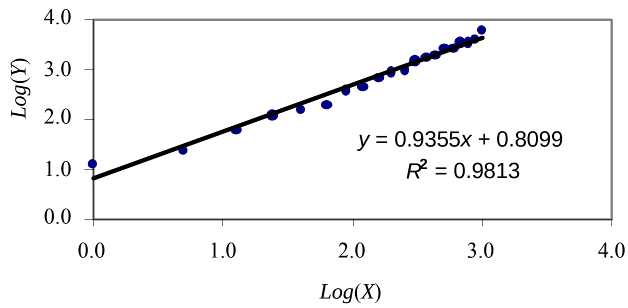

Fig. 16 Log-log plot for spectral trace 3

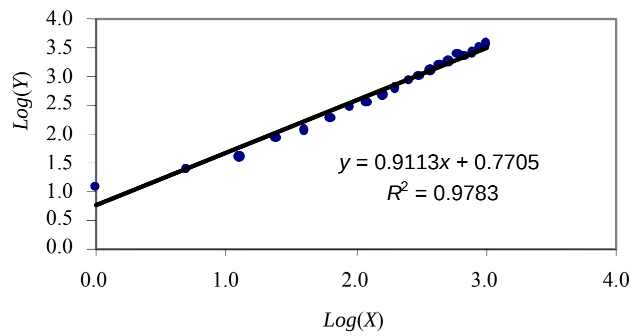

Fig. 17 Log-log plot for spectral trace 4

Table 2

Results of twenty iterations by Monte Carlo approach in search of minimum number of disk (known size) required to cover spectral trace (5)

Disk Number Req. to Cover Datum (*X*)	Iterations by Monte Carlo approach																			
	*Y*1	*Y*2	*Y*3	*Y*4	*Y*5	*Y*6	*Y*7	*Y*8	*Y*9	*Y*10	*Y*11	*Y*12	*Y*13	*Y*14	*Y*15	*Y*16	*Y*17	*Y*18	*Y*19	*Y*20
1	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
2	5	5	5	5	5	5	5	5	5	5	4	6	5	4	5	6	5	4	5	5
3	8	7	9	9	8	8	7	8	8	9	8	8	7	8	8	8	9	9	8	8
4	12	11	10	9	10	11	10	11	11	13	11	11	11	10	10	10	12	10	10	10
5	13	15	15	12	13	12	11	14	13	13	15	15	14	12	13	14	13	15	13	13
6	16	17	16	17	19	16	15	16	17	15	17	18	16	17	15	16	16	17	16	17
7	19	20	19	19	21	19	20	20	20	21	18	20	20	19	19	20	19	20	20	21
8	23	24	25	20	22	24	22	22	23	25	25	26	21	23	25	24	22	21	22	23
9	25	27	25	27	25	28	26	26	26	25	25	25	26	25	26	25	25	27	26	26
10	31	31	31	31	32	30	30	29	31	29	30	30	29	29	32	30	30	30	30	30
11	36	32	34	31	33	34	31	33	32	30	31	31	32	31	32	31	33	33	36	31
12	36	40	38	38	40	35	39	39	40	39	38	36	38	35	40	38	39	36	37	33
13	43	41	42	39	41	40	42	39	38	41	41	37	38	41	40	42	39	40	43	39
14	49	44	45	44	48	44	45	43	44	44	42	42	45	44	43	45	44	42	45	45
15	44	46	48	49	50	49	48	47	46	49	49	48	47	51	48	47	50	47	51	48
16	51	55	55	54	50	52	55	53	54	51	53	50	52	53	50	50	50	52	55	52
17	51	52	53	52	54	54	54	51	52	53	55	50	52	53	53	52	54	53	54	51
18	53	52	53	50	54	53	53	50	51	50	56	53	53	54	53	51	52	50	52	53
19	61	59	61	57	56	62	61	60	58	61	59	62	62	57	59	61	57	60	61	63
20	71	71	69	64	72	70	68	67	66	66	68	66	68	68	65	65	67	71	70	68

Table 3, the characterising index is defined as a product of two parameters (*d* & *C*) instead of just (*d*). The justification is that to uniquely define a straight line on a graph we need only two things (the slope and intercept). This is what was done to uniquely differentiate one surface from another. There is a possibility of two surfaces having the same fractal disk dimension.

Table 3

Ranked results of spectral trace dimension

Spectral traces	Disk dimension
Trace 1	1.1007
Trace 5	1.0940
Trace 2	1.0864
Trace 3	0.9355
Trace 4	0.9113

Referring to Table 3, the ranked results show that the first machined surface had the highest fractal disk dimension while the fourth machined surface had the smallest. Disk dimensions for the five machined surfaces are different. For the purpose of machined surface roughness characterisation a well-defined unique parameter is required. One such parameter is the product of (*d* & *C*) defined under Eq. (1). Once (*d* & *C*) are defined Eq. (1) is uniquely defined and the characterisation of five or more machined surfaces as in the present study becomes a possibility (See Table 4).

Referring to Table 4, the product of *d* & *C* is the highest for machined surface five (2.415342) and the smal-

Table 4

Ranked product of (*d* & *C*) for the five machined surfaces

Machined Surface by Number	Disk Dimension (*d*)	C_o = =*Log*(*C*)	*C* = =*Exp*(C_0)	Product of (*d*&*C*)
5	1.094	0.792	2.207808	2.415342
1	1.1007	0.7277	2.070313	2.278794
2	1.0864	0.7359	2.08736	2.267708
3	0.9355	0.8099	2.247683	2.102708
4	0.9113	0.7705	2.160846	1.969179

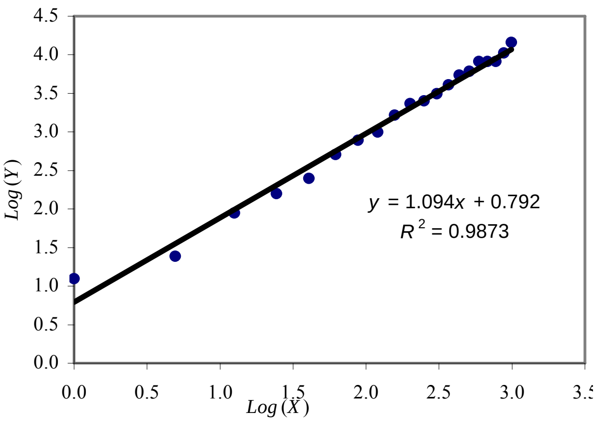

Fig. 18 Log-log plot for spectra trace 5

lest for machined surface four (i.e. 1.969179). Thus, the machined surface five is the roughest while machined surface four is the smoothes. The product of (*d* & *C*) for the five machined surfaces are different. Roughness of the five machined surfaces ranked as “51234” as against “51243” ranking obtained by CLA and Spectral Index methods (see Fig. 18). This amount is about 60% of agreement assumingno error was made. The deviation from perfect agreement may be due to human error at digitisation level. Or possibly the present results may be a manifestation of the superiority of the present method over both CLA and Spectral Index method. This definitely will require further research for clarification.

4. Conclusions

The current study shows a new method, christened Fractal Spec TAO, which seems promising in estimating the disk dimension of fractal image. Computational test is carried out using the method on five machined workpiece surfaces. The results obtained are ranked and compared with CLA method and the spectral index method. The product of (*d* & *C*) for the five machined surface is different, establishing uniqueness of the proposed method. However, the ranked results of product of (*d* & *C*) partially agreed with both CLA and Spectral index methods. The study shows the possibility of fractal disk dimension characterisation of machined surfaces provided human error at digitisation level can be eliminated. It is also concluded that the complexity of the spectral trace implies the relative roughness of machined surface. With availability of digitiser this new method can be integrated to a machine components producing machine as a quality control unit.

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PAVIRŠIAUS APDOROJIMO KOKYBĖS NUSTATYMAS FRAKTALINE ANALIZE

R e z i u m ė

Straipsnyje siūlomas naujas metodas mechaniškai apdirbtų paviršių kokybei nustatyti fraktaline analize. Tai galėtų būti Olaosebikaineno spektrinės analizės metodo, skirto paviršiaus apdirbimo kokybei nustatyti, patobulinimas. Matematinis modelis paremtas Monte Karlo žiedų skaičiavimu priartėjimo metodu. Jis buvo išplėtotas ir patikrintas modeliuojant gautus rezultatus Fortrano programavimo kalba parengta programa. Tyrinėti penki apdirbti paviršiai, kurie buvo suklasifikuoti atsižvelgiant į fraktalinius matmenis, nustatytus atitinkamam paviršiui pagal jo spektro juostą. Ruošiniai, apdirbti skirtingomis operacijomis, turi savą paviršiaus ypatybę, priklausomą nuo pasirinkto apdirbimo būdo (frezavimo, šlifavimo ir t. t.). Pagal turimus šešis (A, B, C, D, E ir F) fraktalinius atvaizdus buvo nustatyti atitinkami spektriniai fraktaliniai matmenys. Spėjama, kad šie įvertinamieji rodikliai sutaps su įvertinamaisiais rodikliais, gautais CLA ir spektrinio indekso metodais. Priešingai, įvertinamieji rezultatai skiriasi nuo rezultatų, gautų CLA ir spektrinio indekso metodais. Naujas metodas, atrodo, yra geresnės kokybės palyginti su CLA ir spektrinio indekso metodais, kadangi yra tikslesnis, gerokai sutrumpėja apskaičiavimo trukmė. Absoliutus minėtų metodų rezultatų tarpusavio skirtumas sudaro 13.1%. Skaičiavimai trunka tik 3 minutes.

B. Alabi, T.A.O. Salau, S.A. Oke

SURFACE FINISH QUALITY CHARACTERISATION OF MACHINED WORKPIECES USING FRACTAL ANALYSIS

S u m m a r y

A new method on machined surface finish quality characterization using fractal analysis is proposed. This seems to be an improvement on Olaosebikan's spectral analysis index method for surface finish assessment. Mathematical model based on disk count Monte Carlo approach is developed and tested with simulated results from computer programme written in Fortran. Test cases involve five-finished machine surfaces (work pieces) that are ranked based on fractal dimensions obtained for the respective machined surface spectral trace. The work pieces, made using different machining operations (milling, grinding, etc.), have their quality of finishing described as a function of the machine operation that each workpiece passes through. The respective spectral fractal dimensions of six fractal images (A, B, C, D, E and F) were then obtained. The conjecture is that the ranked results will agree with ranking obtained by both CLA and spectral index methods. Contrarily, the ranked results disagreed with both CLA and spectral trace results. The new method seems superior to both CLA and spectral trace approaches since a higher accuracy and much less computation time is observed. The maximum percentage relative absolute difference is 13.1%, and the computation time is as short as 3 minutes.

Б. Алаби, Т.А.О. Салау, С.А. Оке

ОЦЕНКА КАЧЕСТВА МАШИННОЙ ОБРАБОТКЕ ПОДВЕРГНУТОЙ ПОВЕРХНОСТИ ПРИ ПОМОЩИ ФРАКТАЛЬНОГО АНАЛИЗА

Р е з ю м е

В статье предложен новый метод оценки качества машинной обработке подвергнутой поверхности при помощи фрактального анализа. Это может быть усовершенствованием метода Олаосебикайнена предназначенного для определения качества машинной обработки поверхности при помощи спектрального анализа. Математическая модель основана на подсчете колец Монте-Карло методом последовательного приближения. Он был развит и проверен моделированием полученных результатов на языке программирования Фортран приготовленной программе. Экспериментально исследовалось пять механически обработанных поверхностей, которые были классифицированы учитывая фрактальные размеры установленные для определенной поверхности по ее спектровым полюсам. Заготовки, обработанные разными операциями, имеют определенное свойство, зависимое от подобранного способа обработки (фрезерования, шлифования и т.д.). По имеющимся шести (A, B, C, D, E и F) фрактальным изображениям были установлены соответствующие спектральные фрактальные размеры. Предполагается, что эти оценивающие показатели будут совпадать с оценивающими показателями, полученными методами CLA и спектрального индекса. Наоборот, оценивающие результаты отличаются от результатов полученных методами CLA и спектрального индекса. Новый метод кажется более качественным по сравнению с методом CLA и спектрального индекса: он более точен, значительно уменьшено время, необходимое для подсчетов результатов измерения. Абсолютная разница между результатами измерения упомянутыми методами составляет 13.1%. Расчеты производятся за 3 минуты.

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