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Slider-link driven compressor (III). Optimization

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1. Introduction

This paper deals with the problem of optimization of slider-link driven compressor. The mathemathical model of the compressor was presented in [1, 2]. Primary goal of the study, was to find such set of compressors dimensions that ensures optimum compressor's performance. Initially we intended to use criterion of minimal mechanical losses. Such criterion was used in [3] for the optimization of rotary compressor, and astonishing 14% theoretical COP improvement was reported. However, we failed to get similar results for slider-link driven compressor. As was reported in [2], the losses due to friction in slider link driven compressor do decrease as stroke to diameter ratio decreases. However, this does not mean that compressor's COP will increase since with the increase of pistons diameter losses due to leak through the clearance between the piston and the cylinder will also increase. Attempts to optimize piston stroke to diameter ratio based just on the analysis of friction power does not give satisfactory results.

Optimization of piston stroke to diameter ratio requires more advanced mathematical model, in which losses due to leak between the piston and the cylinder would be taken into account. The design criteria for such optimization should be maximal COP. Further the reworked part of the mathematical model, responsible for calculation of pressure in the cylinder, is presented. With the improved model it was possible to performe optimization of slider-link driven compressor.

2. Gas flow through the clearance

First question for consideration is which equation to choose for the calculation of the blowby rate. The blowby problem refers to the calculation of the amount of the refrigerant which flows from the cylinder head side of reciprocating compressors to the crankcase side through the ring belt. Physical situation for such problem is usually approximated by adiabatic flow through the duct [4, 5]. Thus an approximate relation for the calculation of mass flow rate should be

$$\frac{dm_c}{dt}=\pi D\delta\sqrt{\frac{2k}{k-1}\frac{p_c^2}{RT_c}}\cdot\sqrt{\left(p_c/p_s\right)^{\frac{2}{k}}-\left(p_c/p_s\right)^{\frac{k+1}{k}}} \quad (1)$$

where D is diametre of piston; δ is radial clearance between the piston and the cylinder; T_c, p_c are temperature and pressure of gas in the cylinder; p_s is suction pressure (crankcase side pressure); R is gas constant; $k=c_p/c_v$ is adiabatic exponent.

Such approach is widely axepted, and gives reasonable accuracy for bigger compressors with ring belt as well as for reciprocating internal combustion engines. In the case of small refrigerating compressors, however, the ring belt is not present, and the gas flows through relatively long clearance. Mathematical model for such case was presented in [6]. The model was based on an assumption that the process of the flow is isothermal. In fact that is the case, as the heat exchange between cylinder, piston and gas in the clearance is very intensive. Thus, it can be stated that the temperature of gas in the clearance is equal to the cylinder's temperature.

The flow rate through regular clearance in the case of isothermal process can be calculated using the equation based on Navier-Stokes equations

$$\frac{dm}{dt}=\frac{\delta^3 b}{24\eta l}\frac{p_2^2-p_1^2}{RT} \quad (2)$$

where dm/dt is flow rate through regular clearance; b is clearance width; T is gas temperature; η is gas viscosity.

As can be seen, the flow rate depends highly on the value of clearance δ. Thus irregularities of δ have a big impact on the flow rate.

In real compressor the situation is a bit different. The piston of slider-link driven compressor usually tilts in clockwise or counter-clockwise direction, thus the clearance is irregular. It is really complicated to get an expression based on Navier-Stokes equations for the case of irregular clearance. Even if it were possible, the equation of the flow rate would be too complicated. Therefore the semiempirical equation used for the calculation of blow-by rate through irregular clearance is

$$\frac{dm}{dt}=\frac{\psi\delta^3 b}{24\mu l}\frac{p_2^2-p_1^2}{RT} \quad (3)$$

where coefficient ψ indicates how many times the flow rate through irregular clearance exceeds the one through regular clearance. In [6] the coefficient was determined experimentally. For the range of clearances 3-10 μm and the range of pressures 0.7-1.1 MPa, the determined value of the coefficient is equal to $\psi=1.65$.

3. Calculation of pressure in the cylinder and blow-by losses

Mathematical model for the calculation of pressure in a cylinder and blow-by losses is based on the equations of gas state, thermodynamics of variable mass and gas dynamics. Several presumptions were made.

First of all, the working medium is considered to be homogenous. Thus every external action immediately transfers to every point of the cylinder's space. Temperature, pressure and density of gas in every point of the cylinder are identical for every given moment.

The second presumption is that gas is ideal, thus Clapeyron equation is valid.

The third presumption is that the gas flow through the clearance is isothermal, at constant temperature of the cylinder and the piston. The temperature can be considered constant because of high thermal conductivity and a small size of the cylinder.

Blow-by in expansion and suction processes is not taken into account. Movement of the valves was also not taken into account. Suction valve assumed to open immediately at the beginning of suction process and close immediately at the end of suction process. Discharge valve assumed to open immediately at the beginning of discharge process and close immediately at the end of discharge process.

Energy equation of variable mass is as follows

$$dU = dQ - dL + dE \tag{4}$$

where dU is internal energy change of gas in the cylinder; dQ is heat transferred to gas; dL is work carried out by gas; dE is energy change because of mass change. For compression and discharge processes dE will be negative because of the gas flow through the clearance and valve.

For ideal gas

$$dU = d\left(um\right) = c_v d\left(mT_c\right) \tag{5}$$

where m is mass of gas in the cylinder; u is specific internal energy; c_v is specific heat at constant volume; T_c is temperature of gas in the cylinder.

Processes of suction, compression, discharge and expansion are considered adiabatic. For high speed hermetic compressor with limited cooling such simplification seems reasonable. Thus dQ/dt.

Mechanical work done by the gas

$$dL = p_c dV_c \tag{6}$$

where dV_c is change of the cylinder's volume.

For suction and expansion process the blow-by through the clearance between piston - cylinder is not taken into account. The energy change because of the mass change for suction, compressioin and discharge processes can be expressed respectively

$$dE_s = c_p d\left(m_s T_s\right) \tag{7a}$$

$$dE = -c_p d\left(m_{cl} T_c\right) \tag{7b}$$

$$dE = -c_p d\left(m_d T_c\right) - c_p d\left(m_{cl} T_c\right) \tag{7c}$$

where m_s, m_d and m_{cl} are masses, passed through suction valve, discharge valve and and clearance between the piston and the cylinder respectively (valves considered leak-proof); T_s is suction temperature.

From the equations we can express the following relationship which describes pressure change in the cylinder for (adiabatic) suction, compression and discharge processes respectively

$$\frac{dp_c}{dt} = \frac{R}{V_c c_v}\left[-\frac{c_p}{R} p_c \frac{dV_c}{dt} + c_p \frac{d\left(m_s T_s\right)}{dt}\right] \tag{8a}$$

$$\frac{dp_c}{dt} = \frac{R}{V_c c_v}\left[-\frac{c_p}{R} p_c \frac{dV_c}{dt} - c_p \frac{d\left(m_{cl} T_c\right)}{dt}\right] \tag{8b}$$

$$\frac{dp_c}{dt} = \frac{R}{V_c c_v}\left[-\frac{c_p}{R} p_c \frac{dV_c}{dt} - c_p \frac{d\left(m_d T_c + m_{cl} T_c\right)}{dt}\right] \tag{8c}$$

In the Eqs. (8a-8c) V_c and dV_c/dt are the functions of crankshaft rotation angle:

$$V_c = \frac{S}{2} A_p f_p\left(\varphi\right),\ \frac{dV_c}{dt} = \frac{S}{2} A_p f_p'\left(\varphi\right) \tag{9}$$

where S is piston's stroke; A_p is piston's cross-section area. For slider-link driven compressor functions $f_p\left(\varphi\right)$ and $f_p'\left(\varphi\right)$ can be expressed

$$\begin{aligned} &f_p(\varphi) = 2a_d + 1 - \cos\varphi \\ &f_p'\left(\varphi\right) = \dot{\varphi}\sin\varphi \end{aligned} \tag{10}$$

where φ is crankshafts angle of rotation, $\varphi = 0$ when the piston is at its upper dead center; $\dot{\varphi}$ is crankshafts angular velocity; a_d is relative dead volume. In the model the variable value of $a_d = f\left(D\right)$ was used, since because of non-perpendicularity of the piston end with the increase of its diameter linear dead volume also increases. For example the linear dead volume of compressors, used for validation, increases from 0.13 to 0.14 mm while the piston diameter increases from 22 to 24 mm.

As suction temperature can be considered constant ($dT_s/dt = 0$), the second term of Eq. (8a) becomes

$$c_p \frac{d\left(m_s T_s\right)}{dt} = c_p T_c \frac{dm_s}{dt} \tag{11a}$$

The second term of equation (8b) becomes

$$c_p \frac{d\left(m_{cl} T_c\right)}{dt} = c_p T_c \frac{dm_{cl}}{dt} + c_p m_{cl} \frac{dT_c}{dt} \tag{11b}$$

For discharge process the gas temperature in the cylinder can also be considered constant ($dT_c/dt = 0$). Then the second term of Eq. (8c) becomes

$$c_p \frac{d\left(m_d T_c + m_{cl} T_c\right)}{dt} = c_p T_c \frac{dm_d}{dt} + c_p T_c \frac{dm_{cl}}{dt} \tag{11c}$$

The gas flow through the valves can be calculated using the equation of Saint-Venant and Wantzel which assumes adiabatic gas flow through valves. For suction and discharge valves the equation can be presented in the following forms respectively

$$\frac{dm_s}{dt} = \alpha_s F_s \sqrt{\frac{2k}{k-1}\frac{p_s^2}{RT_s}}\sqrt{r_s^{2/k} - r_s^{(k+1)/k}} \tag{12a}$$

$$\frac{dm_d}{dt} = \alpha_d F_d \sqrt{\frac{2k}{k-1}\frac{p_c^2}{RT_c}}\sqrt{r_d^{2/k} - r_d^{(k+1)/k}} \tag{12b}$$

where F_s, F_d are cross-section areas in the seat of suction and discharge valves; α_s, α_d are flow coefficient for corresponding cross-sections of suction and discharge valves; p_s, p_d, p_c are suction, discharge pressures and pressure in cylinder; definitions of r_s and r_d are

$$r_s = \begin{cases} p_c/p_s \text{ for } p_c/p_s > \left(2/(k+1)\right)^{k/(k-1)} \\ \left(2/(k+1)\right)^{k/(k-1)} \text{ for } p_c/p_s \le \left(2/(k+1)\right)^{k/(k-1)} \end{cases}$$

$$r_d = \begin{cases} p_d/p_c \text{ for } p_d/p_c > \left(2/(k+1)\right)^{k/(k-1)} \\ \left(2/(k+1)\right)^{k/(k-1)} \text{ for } p_d/p_c \le \left(2/(k+1)\right)^{k/(k-1)} \end{cases}$$

Flow coefficients α_s, α_d were calculated according to the following empyrical relationship for flapper type valves [7]

$$\alpha = A \cdot ln\left(s/d + B\right) + C$$

where s is valves lift; d is valves (orifice) diameter; $A = 0.008796$, $B = 0.0679913$, $C = 1.0436686$.

Since the sound velocity at suction conditions $(a_s)_s = \sqrt{kRT_s}$, from Eq. (8a) we have final form of differential equation of pressure change for suction process

$$\frac{dp_c}{dt} = \frac{2\left(a_s\right)_s^2}{SA_p f_p(\varphi)}\frac{dm_s}{dt} - kp_c\dot{\varphi}\frac{sin\,\varphi}{f_p(\varphi)} \tag{13}$$

To obtain pressure in the cylinder $p_c = f(\varphi)$ for suction process, Eq. (13) is solved numerically together with Eq. (12a).

For compression process gas temperature changes adiabatically

$$T_c = T_s\left(\frac{p_c}{p_s}\right)^{\frac{k-1}{k}}; \quad \frac{dT_c}{dt} = \frac{k-1}{k}\frac{T_s}{p_s^{\frac{k-1}{k}} p_c^{\frac{1}{k}}}\frac{dp_c}{dt} \tag{14}$$

The final form of differential equation of pressure change for compression process is obtained from Eq. (8b), taking into account Eqs. (9), (11b) and (14)

$$\frac{dp_c}{dt} = -\frac{kp_c\dot{\varphi}\dfrac{sin\,\varphi}{f_p(\varphi)} + \dfrac{2\left(a_s\right)_s^2}{SA_p f_p(\varphi)}\left(\dfrac{p_c}{p_s}\right)^{\frac{k-1}{k}}}{1 + \dfrac{2m_{cl}}{SA_p f_p(\varphi)}\dfrac{\left(a_s\right)_s^2}{p_s^{\frac{k-1}{k}} p_c^{\frac{1}{k}}}\dfrac{k-1}{k}} \times \frac{dm_{cl}}{dt} \tag{15}$$

To calculate pressure in the cylinder and blow-by losses for compression proces, Eq. (15) is solved numerically together with the equation of blow-by rate through the clearance, which in this case is expressed

$$\frac{dm_{cl}}{dt} = \frac{\psi\delta^3\left(p_c^2 - p_s^2\right)\pi D}{24\mu lRT_w} \tag{16}$$

where $l = L_c - S\left(1 - cos\,\varphi\right)/2$; L_c is length of the cylinder; T_w is temperature of (cylinder) wall.

For discharge process differential equation of pressure change is obtained from Eq. (8c), taking into account Eqs. (9), (11c) and (14)

$$\frac{dp_c}{dt} = -kp_c\dot{\varphi}\frac{sin\,\varphi}{f_p(\varphi)} - \frac{2\left(a_s\right)_s^2\left(p_c/p_s\right)^{\frac{k-1}{k}}}{SA_p f_p(\varphi)}\left(\frac{dm_{cl}}{dt} + \frac{dm_d}{dt}\right) \tag{17}$$

To calculate pressure in the cylinder and blow-by losses for discharge proces, Eq. (17) is solved numerically together with equation of gas flow through the valves (12b) and equation of blow-by rate through the clearance (16).

Presure in the cylinder for suction process was calculated according to the following equation

$$p_c = p_d\left[2a_d/\left(2a_d + 1 - cos\,\varphi\right)\right]^k \tag{18}$$

Angle at which expansion process ends and suction process begins is calculated

$$\varphi_{0s} = Arccos\left[1 - 2a_d\left\{\left(p_d/p_s\right)^{1/k} - 1\right\}\right] \tag{19}$$

At the beginning of suction process pressure in the cylinder is assumed equal to suction pressure $\left(p_c\right)_0 = p_s$. Crankshafts rotation angle φ_{0c} at which suction process ends and compression process begins is obtained by numerically solving Eqs. (12a) and (13). Integration is continued until condition $p_c = p_s$ is satisfied.

Initial conditions for compression process are $\left(p_c\right)_0 = p_s$, $\left(m_{cl}\right)_0 = 0$. Crankshafts rotation angle φ_{0d} at which compression process ends and discharge process begins is obtained by numerically solving Eqs. (15) and (16). Numerical integration is continued until condition $p_c = p_d$ is satisfied.

At the beginning of discharge process pressure in the cylinder is assumed equal to discharge pressure $\left(p_c\right)_0 = p_d$. As an initial value of m_{cl} for discharge process is used the value, obtained at the end of compression process. Numerical integration of Eqs. (17), (12b) and (16) is continued until upper dead point is reached ($\varphi = 2\pi$).

At the end of discharge prosess we obtain final pressure in the cylinder $\left(p_c\right)_{2\pi}$. To improve accuracy of calculations the pressure can be substituted into Eqs. (18), (19), and calculations can be repeated.

At the end of discharge process we also obtain the amount of gas, which passed through the discharge valve and through the clearance between the piston and the cylinder. The values are used for calculation of compressors cooling capacity and blow-by losses.

4 Calculation results

The Fig. 1 shows calculated cooling capacity and Fig. 2 shows relative volumetric blow-by losses subject to pistons stroke to diameter ratio S/D and radial clearance between piston and cylinder δ. In both cases calculations were performed for the compressor with cylinders volume $V = 7.5\,\text{cm}^3$, refrigerant R600a, temperatures and pressures taken as for CECOMAF conditions. Pistons diameter D changes from 25.9 to 21.6 mm, pistons stroke S changes from 14.2 to 20.5 mm.

In spite of significant simplifications in the model, accuracy of the model is high enough – the difference between calculated cooling capacity and measured values is less than 2% (for the compressor with $V = 7.2\,\text{cm}^3$, $S/D = 0.67$ and $V = 8.14\,\text{cm}^3$, $S/D = 0.75$ at CECOMAF conditions). With the increase of piston's diameter blow-by losses also increase. While with the increase of piston's diameter friction power decreases [2], cooling capacity also decreases, and optimization based just on the analysis of friction power is not possible.

From Fig. 2 we also see, that the compressor with higher piston's diameter will be more sensitive to the increase of clearance between the piston and the cylinder. Such the increase develops in a process of prolonged explotation due to wear of the piston and the cylinder. If clearance increases from 6 to 12 μm, due to increased blow-by losses the capacity of compressor will decrease by ~11% for $S/D = 0.55$ and by ~9% for $S/D = 0.95$.

For the optimization of piston's stroke to diameter ratio we used criteria of maximal efficiency. The ratio of calculated cooling capacity to effective power concumption was chosen instead of COP. The parameter is similar to COP, just not taking into account efficiency of the electric motor. The Fig. 3 shows the ratio of calculated cooling capacity to effective power concumption subject to piston's stroke to diameter ratio and radial clearance. Compressors dimensions and conditions are the same as for Fig. 1 and Fig. 2.

As can be seen from Fig. 3, such optimization was succesfull – optimal piston's stroke to diameter ratio was found. For lower values of radial clearance, optimal stroke to diameter ratio is decreasing due to lower blow-by losses. However, it would be hard to recommend such optimal ratio (for implementation by manufacturers). For bigger piston's diameter it may be more difficult to ensure the same clearance, while the clearance between the piston and the cylinder has much higher influence on compressors performance. Bigger piston's diameter means heavier piston, thicker piston's and cylinder's walls, thicker valve plate. Linear dead volume will increase because of nonperpendicularity of piston's end. Finaly, the influence of piston's stroke to diameter ratio is very limited. Maximal gains will not exede 1.5%. In a process of explotation the clearance will increase, shifting optimal stroke to diameter ratio to higher side.

Even if any significant efficiency reserves from the optimization of pistons stroke to diameter ratio were not found, this is not necessarily a bad message for manu-

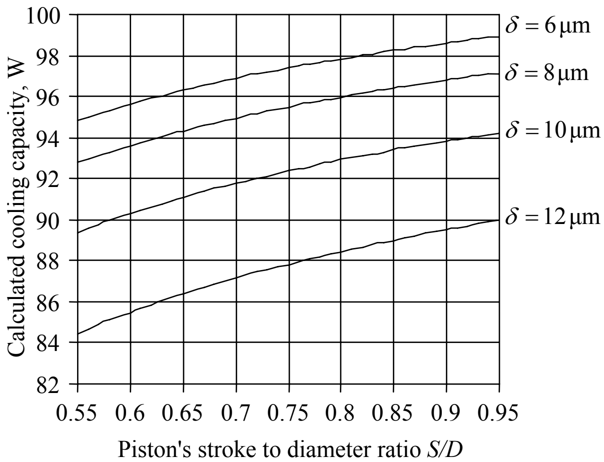

Fig. 1 Calculated cooling capacity subject to piston's stroke to diameter ratio S/D and radial clearance between piston and cylinder δ

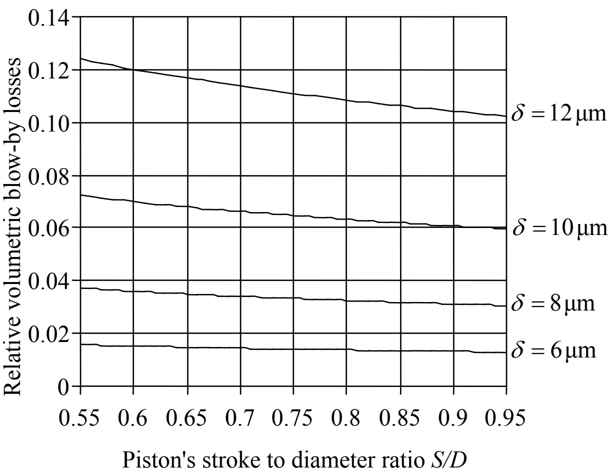

Fig. 2 Relative volumetric blow-by losses through the clearance between piston and cylinder subject to piston's stroke to diameter ratio S/D and radial clearance δ

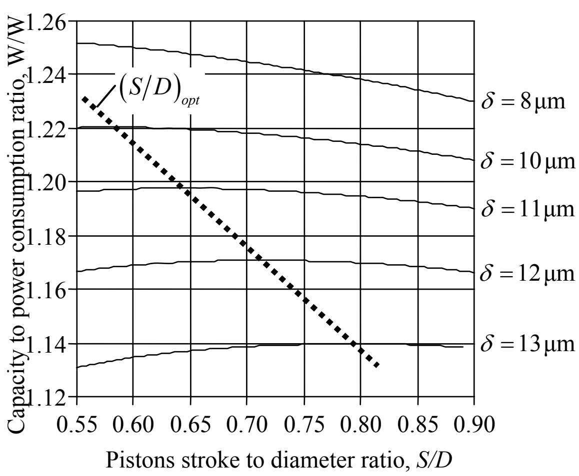

Fig. 3 Ratio of calculated cooling capacity to effective-power concumption subject to piston's stroke to diameter ratio S/D and radial clearance δ; $(S/D)_{opt}$ - optimal stroke to diameter ratio

facturers. They can manipulate pistons stroke and diameter in the manner, which is in best accordance with their design needs and technological requirements. Since the influence of stroke to diameter ratio is minimal, such manipulation most likely will cause no significant penalty on efficiency.

Since no efficiency reserves vere detected, we also do not feel any motivation for further development and improvement of the model. However, the presented model still is universal and accurate enough for wide range of everyday tasks. It can be used for calculating forces, loads, evaluating dynamic behaviour, vibrations and minimization of inertia forces. Even predicted cooling capacity, effective power consumption and efficiency of the compressor are very close to measured values. We hope, that the presented model will be a usefull tool for engineers and developers of slider-link driven compressors.

Conclusions

The remaining manufacturers of “outdated” slider-link driven compressors should concentrate their resources on acquirement and implementation of high efficiency motor.

From the parameters of compressor itself, the highest influence on compressors performance has the clearance between the piston and the cylinder.

Optimization of piston’s stroke to diameter ratio and piston’s eccentricity is possible, but positive effect of such optimizations is limited.

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KULISINIS KOMPRESORIUS (III). OPTIMIZACIJA

R e z i u m ė

Pirmose straipsnio dalyse pristatytas kulisinio kompresoriaus matematinis modelis buvo patobulintas, atsižvelgiant į našumo nuostolius dėl protėkių pro tarpelį tarp stūmoklio ir cilindro. Optimizuotas kompresoriaus stūmoklio eigos ir skersmens santykis, siekiant, kad kompresorius dirbtų kuo efektyviau. Nustatytas optimalus stūmoklio eigos ir skersmens santykis, tačiau šio parametro įtaka yra palyginti nedidelė. Iš nagrinėtų parametrų kompresoriaus efektyvumui daugiausia įtakos turi tarpelis tarp stūmoklio ir cilindro.

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SLIDER-LINK DRIVEN COMPRESSOR (III). OPTIMIZATION

S u m m a r y

The model of slider-link driven compressor, presented in the previous parts of the study, was improved to take into account capacity losses due to blow-by through the clearance between the piston and the cylinder. Pistons stroke to diameter ratio of a compressor was optimized in order to achieve the highest compressor efficiency. Optimal pistons stroke to diameter ratio was determined, but the influence of this parameter found out to be relatively small. From analyzed parameters the highest influence on compressors efficiency has the clearance between the piston and the cylinder.

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КУЛИСНЫЙ КОМПРЕССОР (III). ОПТИМИЗАЦИЯ

Р е з ю м е

Модель компрессора с кулисным приводом, представленная в предыдущих частях исследования, была усовершенствована учетом потерь производительности из-за утечек между поршнем и цилиндром. Произведена оптимизация отношения хода поршня компрессора к диаметру с целью достижения максимальной эффективности компрессора. Найдено оптимальное отношение хода поршня к диаметру, однако влияние этого параметра незначительно. Из исследованных параметров наибольшее влияние на эффективность компрессора имеет зазор между поршнем и цилиндром.

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