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The role of friction on size segregation of granular material

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1. Introduction

Granular materials of various forms are ubiquitous in nature and technology. They exhibit a rich variety of phenomena and, depending on circumstances, can have properties similar to either solids, liquids or gases [1], or behave in a completely different manner.

Size segregation is a characteristic feature of the moving granular media [2, 3] and it is important, as well as mixing, in industrial handling of granulated materials. A number of physical mechanisms were suggested to explain the segregation process [4]. Existing theories of continuous mechanics or statistical physics have only limited applications for the description of granular media, and a unified theory encompassing all the granular phenomena is still missing [5, 6]. The possibility to describe and predict the conduct of granulated materials is important for technological applications in industry where bulk materials are routinely handled. On the other hand, granular media is an interesting object for theoretical studies because of its intricate phenomenology despite ostensibly simple purely mechanical nature. Numerical simulations can provide a useful insight into the origins of various aspects of this behaviour.

Research data about the influence of the mechanical properties of granular matter on the segregation process is still scarce. Attempts to create a "granular thermodynamics" framework are commonly limited to simplified models: monodisperse or binary mixtures of particles are researched, binary collisions between the particles are assumed [7]. Experiments involving the particles with different mechanical properties are difficult to implement, but numerical simulations with different parameter sets can be performed easily, even though they are lengthy and require much computing power.

The objective of the present paper is to investigate the mixing and segregation processes inside granular material consisting of spherical particles and the dependency of mixing on the properties of the particle material. Most of the research performed in segregation of granular materials dealt with the situation when the researched material is put to motion by a vibrating bottom wall [2] or mixing takes place inside a rotating cylinder [3]. We modelled a different setup where the material is stirred by a rectangular bar moving forward and backward in the horizontal direction and buried inside the material, the situation frequently encountered in certain components of a real-world equipment, such as forward or backward moving grates widely used in industry [8]. On the other hand, irregular geometry of the simulated system gives rise to special patterns of particle motion revealing an interplay between the different competing mechanisms that could go unnoticed in simpler and more regular geometries. Mixing of granular material is sustained by a periodic motion of a rectangular bar inside a rectangular container. We observe the segregation and stratification of the particles by their sizes and intensity of mixing. Earlier, the influence of mechanical parameters of the particle material upon the intensity of mixing and segregation was researched [9, 10]. The dynamic friction was found to have the most notable influence. Therefore, in this paper we report the results of a more detailed analysis of this influence upon the mixing process in a wider range of the values of dynamic friction coefficient.

2. Setup of the numerical experiment

A number of models have been applied for describing granular media with a varying degree of success [11]. The discrete element method (DEM) proved to be the most accurate, besides, it is simple to implement. In this method, the motions and collisions of each separate particle are tracked using the equations of classical mechanics. The equations of motion are integrated with a constant step considerably shorter than the duration of collision. Spherical particle motion obeys the usual Newton equations:

$$\frac{d\boldsymbol{x}_i}{dt}=\boldsymbol{v}_i,\ \frac{d\boldsymbol{v}_i}{dt}=\frac{\boldsymbol{F}_i}{m_i},\ \frac{d\boldsymbol{w}_i}{dt}=\frac{\boldsymbol{T}_i}{I_i},\ i=\left[1,\ldots,N\right] \tag{1}$$

where $\boldsymbol{x}_i$ is the i-th particle position; $\boldsymbol{v}_i$ is velocity, $\boldsymbol{F}_i=\sum_j \boldsymbol{F}_{ij}+m_i\boldsymbol{g}$ is the total force acting upon the i-th particle; $\boldsymbol{F}_{ij}$ is the force acting upon the i-th particle arising from its collision with the j-th particle; $\boldsymbol{g}$ is the acceleration of gravity; m_i is particle mass; $\boldsymbol{w}_i$ is angular velocity; $\boldsymbol{T}_i=\boldsymbol{d}_{cij}\times\boldsymbol{F}_i$ is torque; $\boldsymbol{d}_{cij}$ is the vector pointing from the center of particle i to the collision point; I_i is moment of inertia which is scalar for spherical particles, N is the number of particles. Orientations of the particles is not updated because it has no sense for spherical shape. The forces acting between the particles during their collision are expressed as follows

$$\boldsymbol{F}_{ij}=\boldsymbol{F}_{n,ij}+\boldsymbol{F}_{t,ij} \tag{2a}$$

$$\boldsymbol{F}_{n,ij}=k_n r_{ij} h_{ij}\boldsymbol{n}_{ij}-m_{ij}\gamma_n\boldsymbol{v}_{n,ij} \tag{2b}$$

$$\boldsymbol{F}_{t,ij}=-\boldsymbol{t}_{ij}min\left(\begin{array}{l}\mu\left|\boldsymbol{F}_{n,ij}\right|,\\ \left|-\gamma_t m_{ij}\boldsymbol{v}_{t,ij}-k_t\boldsymbol{\delta}_{t,ij}\sqrt{r_{ij}h_{ij}}\right|\end{array}\right) \tag{2c}$$

where $\boldsymbol{F}_{n,ij}$, $\boldsymbol{F}_{t,ij}$ are normal and the tangential components, respectively, of the force arising due to collision between the particles i and j, $k_n=\frac{4}{3}\cdot\frac{E}{2(1-\sigma)}$, E is

elasticity modulus of the particle material, σ is Poisson modulus, $r_{ij} = \frac{r_i \cdot r_j}{r_i + r_j}$, $m_{ij} = \frac{m_i \cdot m_j}{m_i + m_j}$ are normalized radius and normalized mass, h_{ij} is the overlap depth of the particle shapes during collisions, $\boldsymbol{n}_{ij}$ and $\boldsymbol{t}_{ij}$ are normal and tangential vectors at the contact point, γ_n and γ_t are normal and tangential (shear) dissipation coefficients, $\boldsymbol{v}_{n,ij}$ and $\boldsymbol{v}_{t,ij}$ are normal and tangential components of relative velocity of the colliding particles, μ is dynamic friction coefficient, $k_t = \frac{8}{3} \cdot \frac{G}{2-\sigma}$, G is shear modulus, $\boldsymbol{\delta}_{t,ij} = \left| \int \boldsymbol{v}_{t,ij} dt \right|$ is contact point slip over the surfaces of the colliding particles, integrated over the total duration of collision. The simulations described below used the time-driven DEM implementation which is described in more details in [11, 12]. The equations of motion (1) were integrated using the 6th-order Gear predictor-corrector scheme [13].

The simulated system consists of a rectangular box whose size is $L_x \times L_y \times L_z = 0.2\,\text{m} \times 0.4\,\text{m} \times 0.2\,\text{m}$ containing $N = 1600$ spherical particles of various sizes with random distribution of radii ranging from $r_{min} = 0.005$ to $r_{max} = 0.015$ m (Fig. 1), and random distribution of particle initial positions (Fig. 2, a). The gravity acceleration vector $\boldsymbol{g}$ is directed downwards along the y axis, with the components $g_y = -10$ m/s^2, $g_x = g_z = 0$. At the lower left corner of the box, a rectangular moving bar is located whose width (in the direction of z axis; perpendicular to the picture plane in Fig. 2) is equal to the width of the box L_z. During the process, the bar moves from its leftmost position (x = 0.05 m) to the right along x axis over the distance of 0.1 m at a constant velocity during the time period T_f, then retracts back to its initial position during the time period T_b, and then the process repeats periodically during the entire simulation time. For all the simulations presented below, the time of the backward motion was equal to the time of the forward motion $T_b = T_f$ and was set to $T_f = 10$ s, and as result, the total period of bar motion is $T = T_f + T_b = 20$ s. The bar motion results in stirring of the particles and their redistribution in the volume of the box.

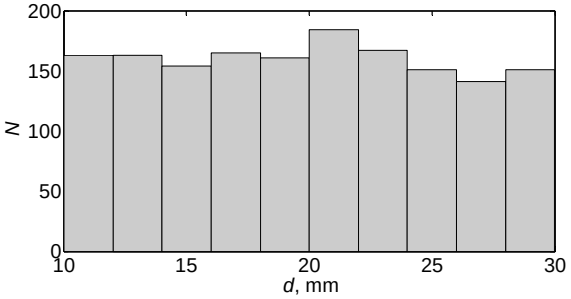


Fig. 1 Distribution of particle diameters d

The simulations were performed for the set of particles that had initially the same initial positions, linear and angular velocities for each run. The mechanical parameters of the particle material are listed in Table. The set of equations (1) was then solved for a certain period of the "simulated" time, yielding the positions and velocities of each particle at the time moments separated by a time step $\Delta t = 5 \cdot 10^{-5}$ s. The duration of the simulation time was set to $n_T \cdot T$, where $n_T = 20$ is the number of the wall motion periods, $t_1 = 0$ s, $t_2 = n_T T$.

Table

Mechanical properties of the particle material

Parameter	Notation	Value
Minimal particle radius	r_{min}	0.005 m
Maximum particle radius	r_{max}	0.015 m
Density	ρ	700 kg/m^3
Elastic modulus	E	10^7 Pa
Poisson modulus	σ	0.2
Normal dissipation coefficient	γ_n	100 s^{-1}
Shear dissipation coefficient	γ_t	100 s^{-1}
Shear modulus	G	3×10 Pa
Period of the stirring bar motion	$T = T_f + T_b$, $T_f + T_b$	20 s
Gravity acceleration module	$\lvert \boldsymbol{g} \rvert$	10 m/s^2

We have found earlier that the processes of mixing and, herewith, size segregation of the particles depend on the parameters of the particles material, of which the coefficient of dynamic friction between the particles has the largest influence [10]. Therefore, the main task of the current work was to investigate the influence of dynamic friction coefficient μ on the processes of particles mixing and size segregation in greater detail. Numerical experiments were carried out with different values of the friction coefficient: $\mu \in \{0, 0.01, 0.05, 0.1\ 0.15, 0.2, 0.5, 0.8, 1\}$. From the resulting data, size segregation parameters were estimated.

3. Simulation results and discussion

Particle mixing occurs in the bulk of granular material. Two types of the mixing processes can be identified:

- "convective" mixing, which is defined by the averaged velocity of the granular material, when the constituent particles travel inside the bulk due to the mean flow;
- "local" mixing, when the particles interchange positions with the neighbours.

Due to the mixing process, particles tend to segregate by size [2, 3, 9, 10]. The reviews of the segregation process, as well as the discussions of the possible segregation mechanisms, can be found in [4] and references therein. Meanwhile, we will focus our attention on the evaluation of segregation phenomena, leaving the problems of mixing process for further research. Only the quan-

titive parameters of segregation will be compared in this work. Size segregation of particles can be observed visually by comparing particle positions at different time instants in Fig. 2. It can be noticed visually that smaller particles tend to sink downwards and accumulate near the bottom as the process goes on.

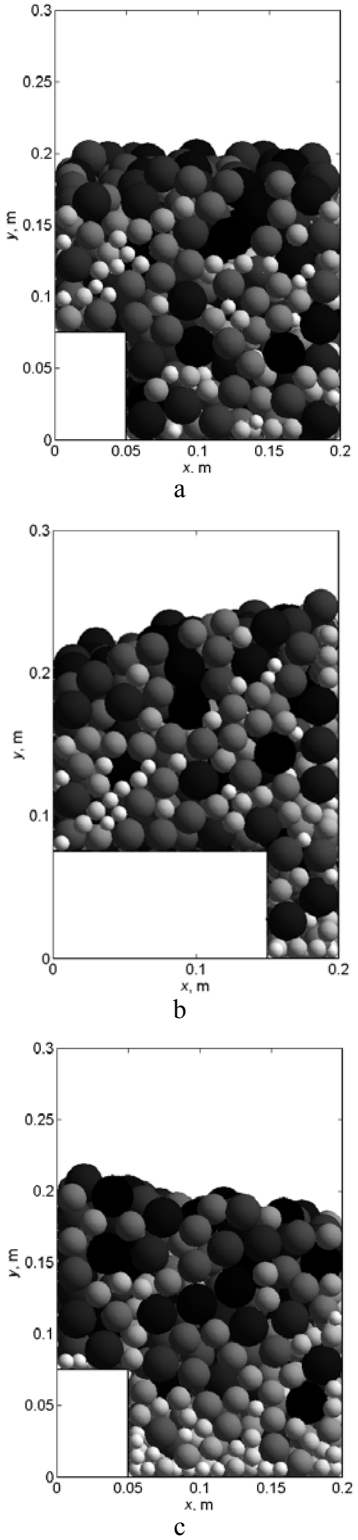


Fig. 2 Evolution of particle positions during the mixing process: initial positions at $t = 0$ s (a), at $t = 10$ s (b) and $t = 400$ s. Particles of different sizes are shown in different shades

Particle segregation by sizes can be estimated quantitatively as the change of their vertical positions (y component of the coordinates, in the direction of gravity vector) during the simulation time depending on particle sizes. The average vertical position of the particles whose radii are in the interval $r_1 \le r_i \le r_2$ is defined as

$$\left\langle y_{r_1,r_2}(t)\right\rangle = \frac{\sum_{i=1}^{N} y_i(t)\delta(r_1 \le r_i \le r_2)}{\sum_{i=1}^{N} \delta(r_1 \le r_i \le r_2)} \tag{3}$$

where a logical function δ was introduced for brevity:

$$\delta(\text{condition}) = \begin{cases} 1, \text{condition is satisfied} \\ 0, \text{otherwise} \end{cases} \tag{4}$$

Let us define the particles whose radii are within the upper 20% of the total interval of the particle radii as “large”, and those with the radii within the lower 20% of the total interval as “small”, i.e., the interval of radii of the small particles is [0.005, 0.007] m, and that of the large particles is [0.013, 0.015] m.

The changes of average vertical position $\langle y_i \rangle$ of “small” and “large” particles are compared to the change of average vertical position of all the particles (including large and small ones). The average vertical position of the “large” particles whose radii r_i are in the interval $r' \le r_i \le r_{\max}$ is defined as

$$\left\langle y_{large}(t)\right\rangle = \left\langle y_{r',r_{max}}(t)\right\rangle \tag{5}$$

and that of the smallest particles $r_{min} \le r_i \le r''$ as

$$\left\langle y_{small}(t)\right\rangle = \left\langle y_{r_{min},r''}(t)\right\rangle \tag{6}$$

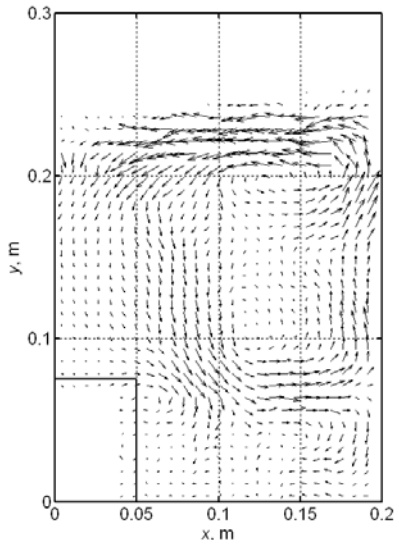
where

$$r'' < r' \ , \ r_{min} = min(r_i), r_{max} = max(r_i) \text{ for } \forall i \tag{7}$$

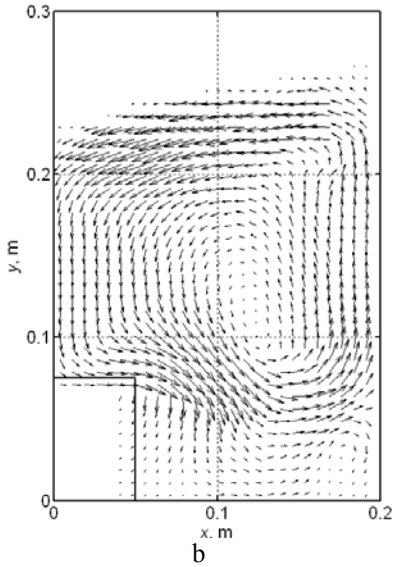
Granular material is affected by periodic motion of the bar, therefore average motion of the granular material becomes cyclic in the plane xy, what is demonstrated by Fig. 3 for the cases of $\mu = 0$, 0.2, and 1.0, where the field of averaged velocity vector $\boldsymbol{U}(x,y)$ is plotted. $\boldsymbol{U}(x,y)$ is the result of averaging the instantaneous velocity field $\boldsymbol{u}(t,x,y)$ over the entire simulation time $[t_1,t_2] = [0,400]$ s as follows

$$\boldsymbol{U}(x,y) = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \boldsymbol{u}(t,x,y)dt \tag{8}$$

where the instantaneous velocity field $\boldsymbol{u}(t,x,y)$ of granular material is estimated as the average velocity within a box of certain size $\Delta = 2r_{max}$ in the vicinity of the point (x,y) for all the positions z:



a



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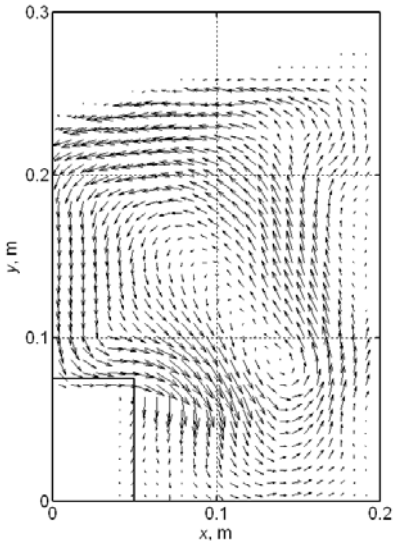

c

Fig. 3 Average velocity fields of the particles for dynamic friction coefficient values $\mu = 0.0$ (a), $\mu = 0.2$ (b) and $\mu = 1.0$ (c)

$$\boldsymbol{u}(t,x,y) = \frac{\sum_{j=1}^{N} \boldsymbol{v}_j(t)\,\delta\left(\left|x_j(t)-x\right| \le \frac{\Delta}{2}\right)\delta\left(\left|y_j(t)-y\right| \le \frac{\Delta}{2}\right)}{\sum_{j=1}^{N} \delta\left(\left|x_j(t)-x\right| \le \frac{\Delta}{2}\right)\delta\left(\left|y_j(t)-y\right| \le \frac{\Delta}{2}\right)} \quad (9)$$

where $\boldsymbol{v}_j(t)$ is the velocity of the j-th particle.

The patterns of particle motion can be illustrated by the flow lines (Fig. 4). At low values of dynamic friction coefficient, the flow structure is complex, comprising a number of small-scale local vortex-like patterns, as seen in Fig. 4, respective cases of the friction coefficient. It can be expected that in this regime, larger structures are unstable because of weak coupling in tangential direction between the neighbouring particles, and the overall motion is fragmented, where the particles exchange places with their neighbours easily. As dynamic friction increases, the larger-scale structures emerge and most of the particles are entrained in a cyclic flow involving basically the entire bulk (Fig. 4; high values of the friction coefficient). Characteristic size of the structures in flow pattern influences the segregation behaviour: small local flows involve predominantly small particles, leading to local mixing, and the segregation is less intense in this case; large-scale flows can also accommodate larger particles [4]. During forward motion of the stirring step, large particles, as well as the small ones, are pushed towards the surface of the packed bed; this upward motion is more correlated in case of strong friction. Once on the top, small particles are more likely to settle down during the ackward motion of the step, and the "void filling" mechanism is the decisive force: small particles are more likely to find a large enough empty space below their current positions than the large ones.

Fig. 5 shows evidently how small and large particles separate vertically during the mixing process. Jagged shape of the curves corresponds to upward and downward motion of the particles following motion of the bar. As the process progresses, the particles tend to approach a certain nearly stationary state when averaged positions of the smallest and the largest particles are influenced only by the stirring bar motion. One might admit that this state would continue infinitely, and some parameters describing segregation and mixing will behave in the same manner. Therefore let the subscript "∞" denote the values of any time dependent parameter $f(t)$ at the time moment $t = t_2$

$$f_\infty = f(t = t_2) \quad (10)$$

when the moving bar is in the initial leftmost position at the end of simulation (Fig. 2, c).

The height of the volume occupied by the particles ("packed bed") changes in many cases of motion of granular material. In our case, the average height $H_B(t)$ of this volume is fluctuating about a nearly constant value synchronously to the bar motion (Fig. 6). It can be noted that final value of the average height $H_{B,\infty}$ depends on friction μ: $H_{B,\infty}$ slightly increases with increasing μ as it is seen in Fig. 7. This result is in accordance with the conclusions of [14] where it was argued, from the considerations of stability of geometrical configurations of a particle system, that the increase of friction decreases the volume fraction of the granular material. Initial height of the packed

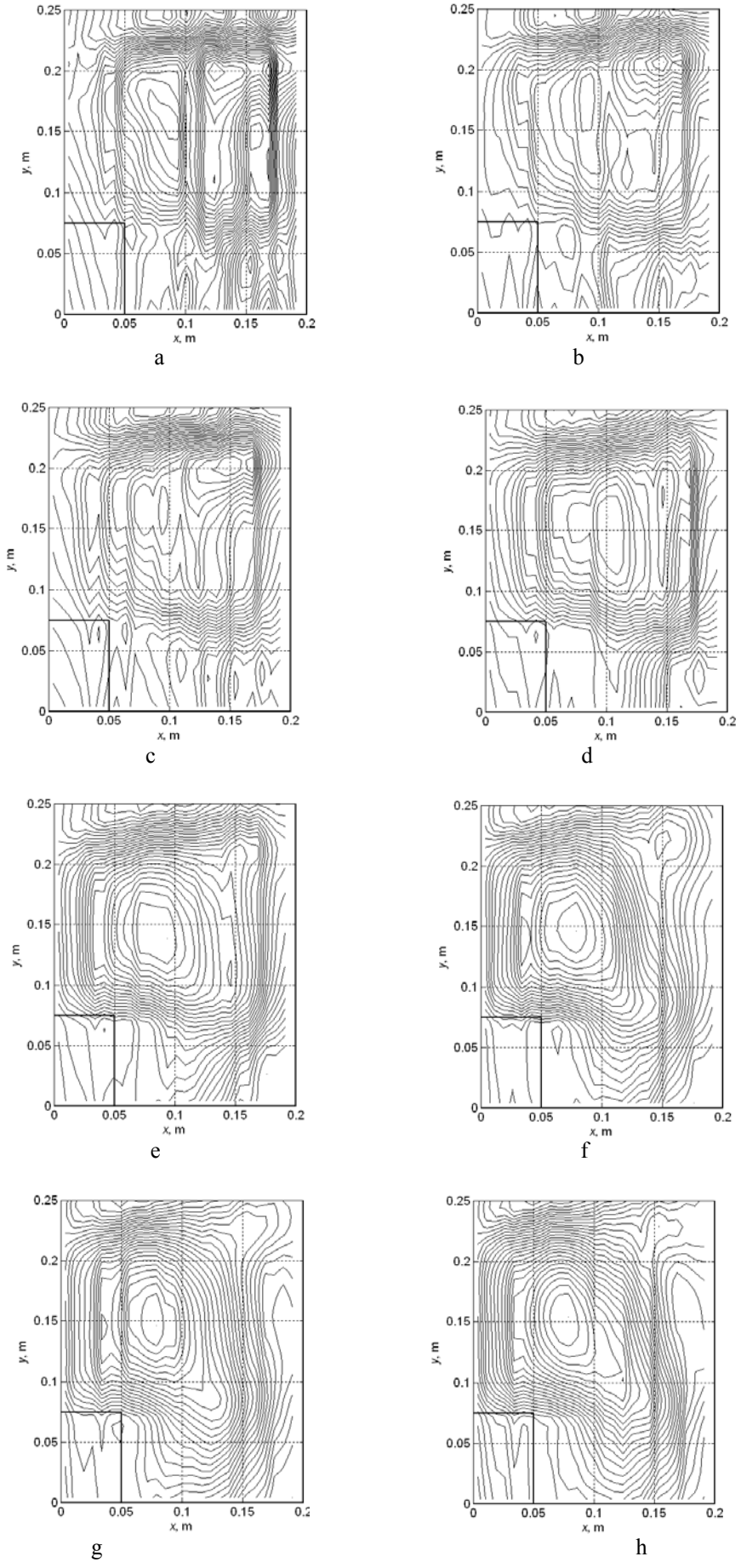

Fig. 4 Flow lines of the particle motion averaged over the simulation time, for different values of the dynamic friction coefficient: $\mu = 0.00$ (a), 0.01 (b), 0.05 (c), 0.10 (d), 0.20 (e), 0.50 (f), 0.80 (g), 1.00 (h)

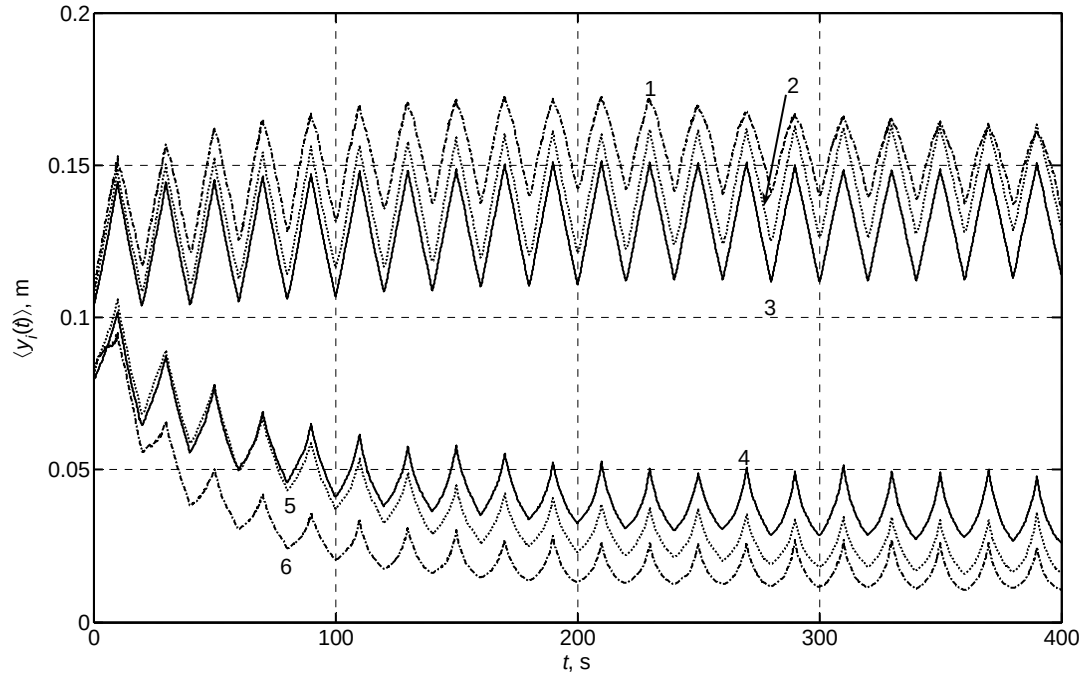

Fig. 5 Time evolution of the averaged vertical positions of the large (curves *1,2,3*) and small (curves *4,5,6*) particles for the values of the dynamic friction coefficient equal to 0.0 (curves *3,4*), 0.2 (curves *2,5*) and 1.0 (curves *1,6*)

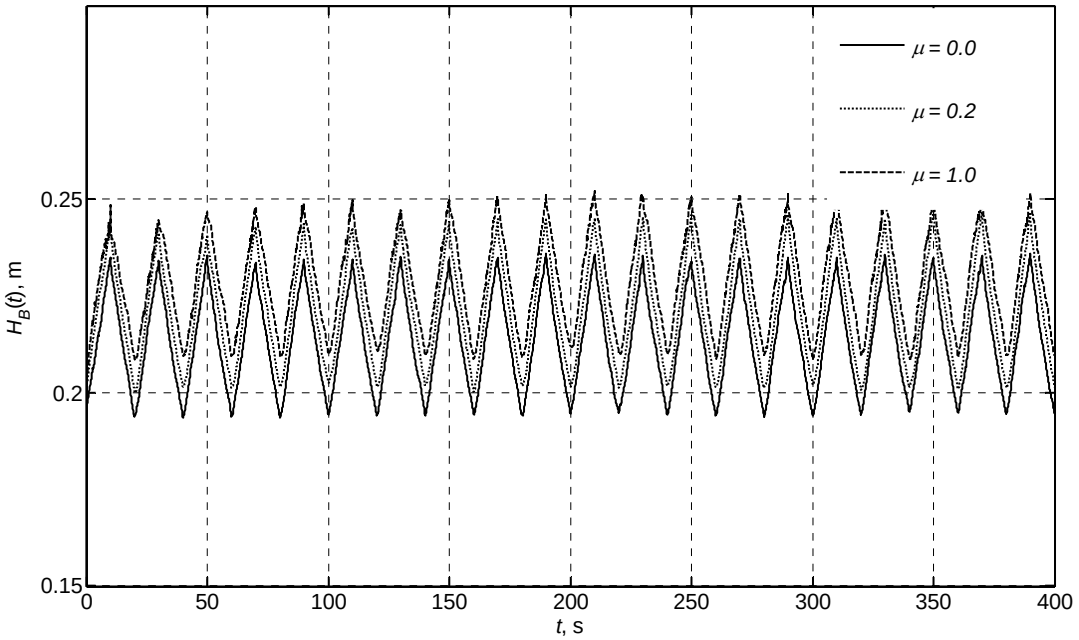

Fig. 6 Time evolution of the average height of the packed bed

bed $H_{B,.initia1}$ is shown on Fig. 7 for comparison. The initial configuration of the particles used as the starting point for all the simulations was prepared at $\mu = 0.2$.

It is reasonable to normalize vertical positions of the particles to $H_B(t)$, in order to discriminate vertical motion of individual particles from the motion of the entire packed bed as a whole:

$$\left\langle Y_{r_1,r_2}(t)\right\rangle = \frac{\left\langle y_{r_1,r_2}(t)\right\rangle}{H_B(t)} \tag{11}$$

The value of $\langle Y(t)\rangle$ is always in the range [0, 1]. In addition, the amplitude of $\langle Y(t)\rangle$ fluctuations becomes smaller in comparison to $\langle y(t)\rangle$ (Fig. 8).

The normalized difference of average vertical positions of the "smallest" and "largest" particles

$$S(t) = \frac{\left\langle y_{\text{large}}(t)\right\rangle - \left\langle y_{\text{small}}(t)\right\rangle}{H_B(t)} = \left\langle Y_{\text{large}}(t)\right\rangle - \left\langle Y_{\text{small}}(t)\right\rangle \tag{12}$$

can be used as the simplest dimensionless parameter to evaluate the intensity of size segregation of particles inside the packed bed and may be used to investigate the segregation process. The value of $S(t)$ is always in the range [-1, 1].

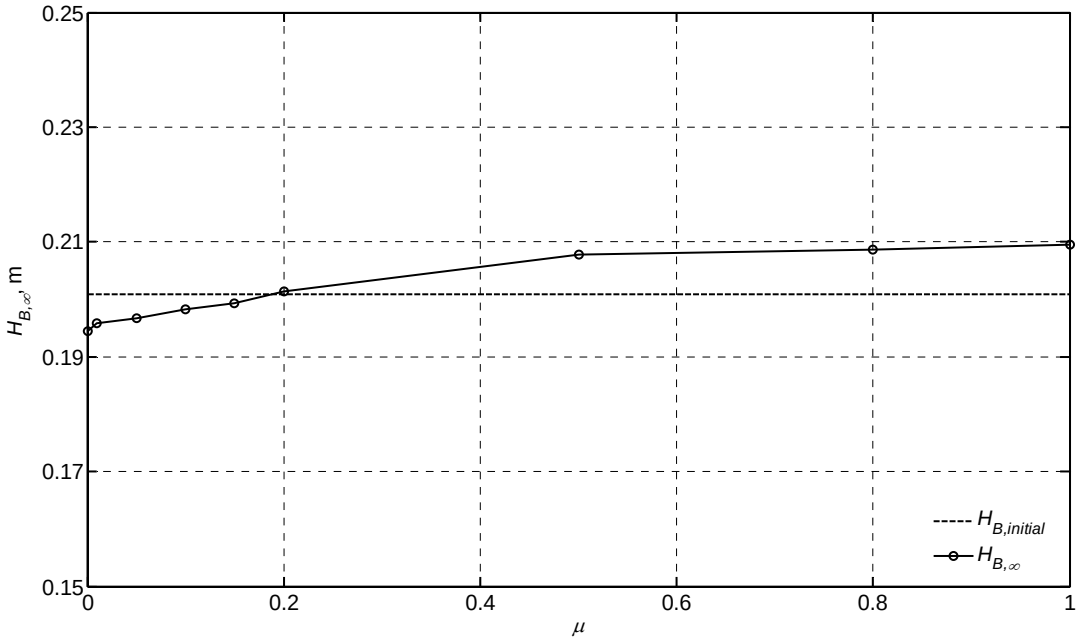

Fig. 7 Dependence of the final height of the packed bed $H_{B,\infty}$ on the dynamic friction coefficient; the initial height of the packed bed $H_{B,.\text{initial}}$ is shown for comparison

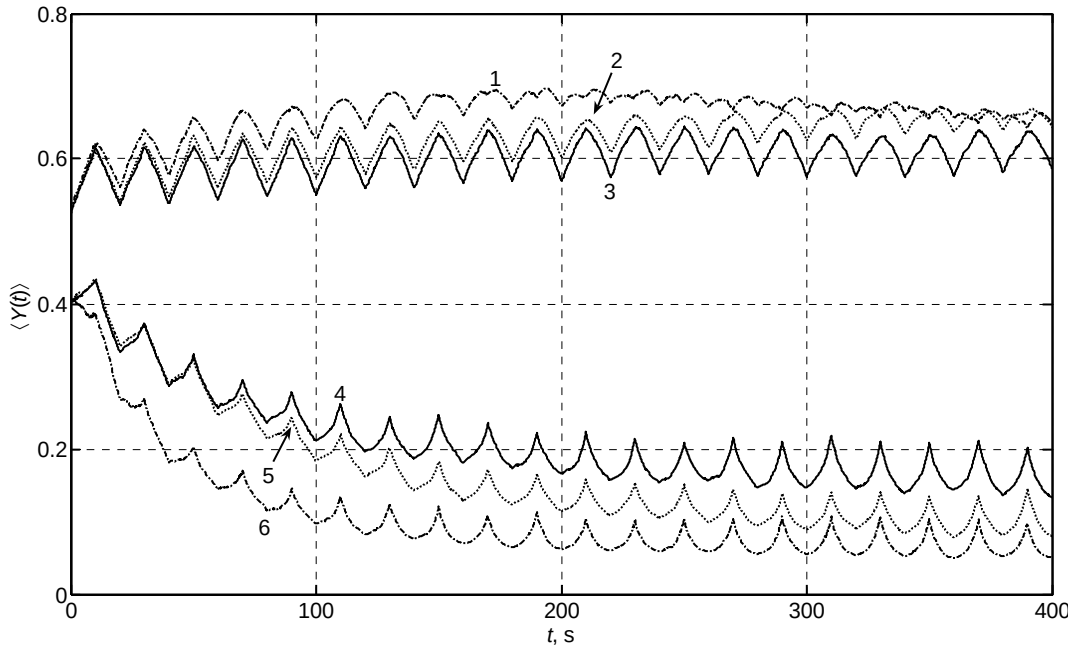

Fig. 8 Time evolution of the normalized average positions of large (curves *1,2,3*) and small (curves *4,5,6*) particles for the values of dynamic friction coefficient $\mu = 0.0$ (curves *3,4*), 0.2 (curves *2,5*) and 1.0 (curves *1,6*)

The evolution of $S(t)$ for some cases of μ is depicted in Fig. 9. The initial value of $S(t)$ is the same for all the simulated problems, but further progress of segregation depends on friction μ and fluctuates according to the bar motion. In order to exclude these fluctuations, Fig. 10 depicts the values of segregation $S(t)$ only at the time moments $t = iT$ $(i = 0,1,2,\ldots)$, when the bar is at the leftmost position, as it can be seen in Figs. 2, a and c.

Evolution of $S(t)$ clearly shows (Fig. 9) that due to local mixing inside the packed bed and convective mixing of the whole packed bed the particles segregate by size until the segregation intensity $S(t)$ reaches the value S_∞ monotonically and remains almost constant thereafter. It is possible to admit a hypothesis that, for the currently investigated system, the increase of $S(t)$ at every time moment t depends on the difference between the current value of $S(t)$ and S_∞ and may be described by the differential equation as follows:

$$\frac{dS(t)}{dt} = A \cdot [S_\infty - S(t)] \tag{13}$$

where the parameter A characterizes the speed of segregation intensity increase. As a first approximation, A is assumed to be independent on $S(t)$.

Solution of Eq. (13) gives theoretical expression for the segregation intensity S_T, where S_T denotes the

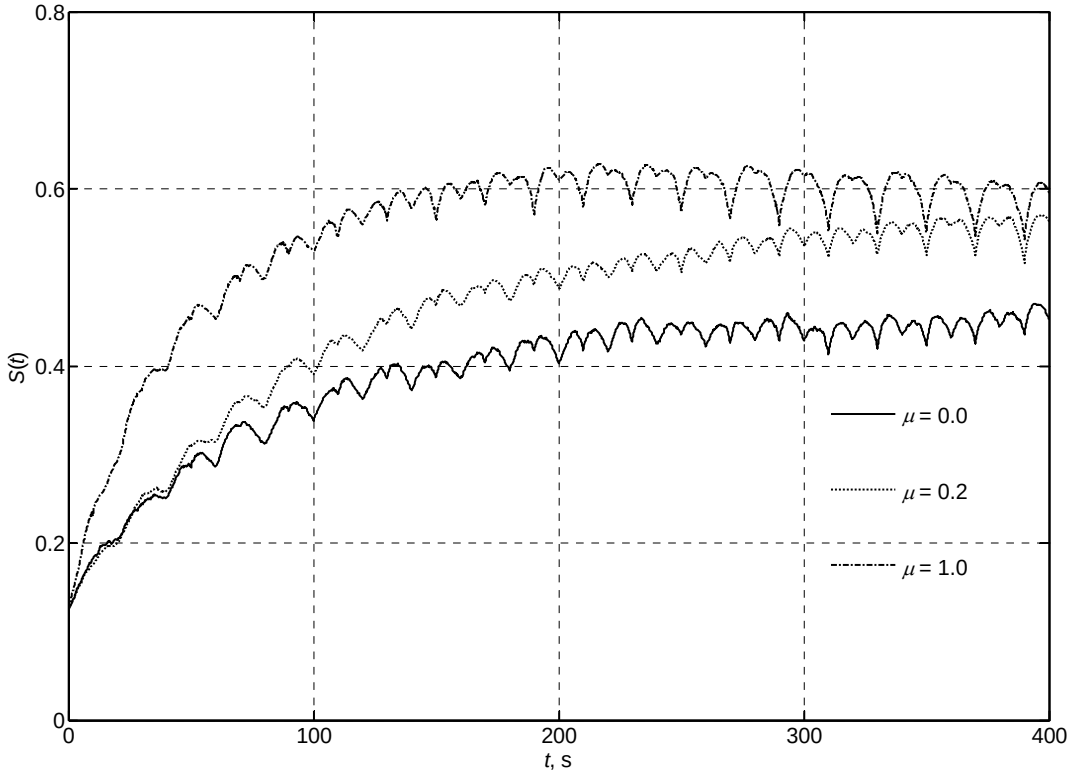

Fig. 9 Time evolution of the segregation parameter $S(t)$ for the values of dynamic friction coefficient 0.0, 0.2 and 1.0

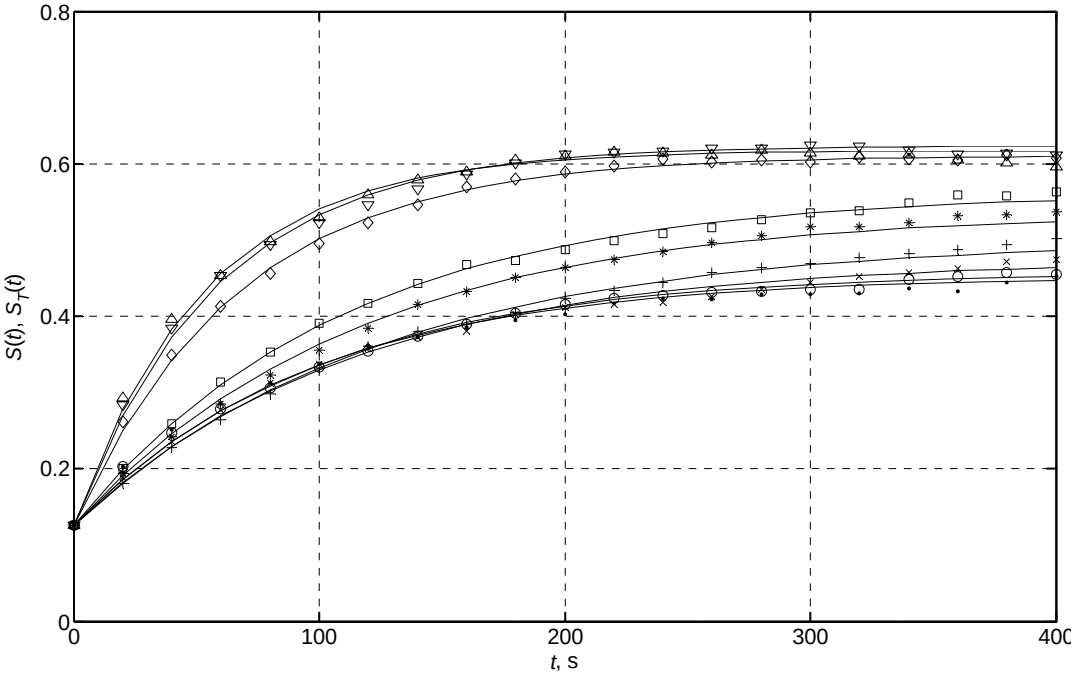

Fig. 10 The values of the segregation parameter $S(t)$ at the time moments $t = iT$ ($i = 0,1,2,\ldots,n_T$) (symbols) and theoretical segregation parameter $S_T(t)$ (lines) for different values of dynamic friction coefficient: • - $\mu = 0.00$, ○ - $\mu = 0.01$, × - $\mu = 0.05$, + - $\mu = 0.10$, * - $\mu = 0.15$, □ - $\mu = 0.20$, ◇ - $\mu = 0.50$, ▽ - $\mu = 0.80$, △ - $\mu = 1.00$

segregation intensity obtained by solving Eq. (13), as opposed to the values calculated from the simulation results

$$S_T(t) = (S_\infty - S_0)\cdot\left[1 - e^{-A(t-t_0)}\right] + S_0, \; t > t_0 \qquad (14)$$

where S_0 is the initial value of $S(t)$ at the time moment $t = t_0$: $S_0 = S(t_0)$. For the simulated problems, $t_0 = t_1 = 0$ s.

In general, parameters A and S_∞ might depend on friction coefficient μ and other material properties of the particles, and geometrical and dynamic parameters of the system such as size and shape of the particles, amplitude and period of the bar motion, bar and box sizes, etc.

The dependence of the coefficients S_∞ and A on friction are displayed in Fig. 11 and Fig. 12. The values of S_∞ for each simulated experiment were estimated from the

simulation data as the last value of $S(t)$ according to Eq. (14). The values of the parameter A were found by fitting $S_T(t)$ to the simulation data displayed in Fig. 9. The resulting curves of $S_T(t)$ are depicted in Fig. 9 in order to compare with the "experimental" points (i.e., the points obtained by the simulation).

S_∞ is increasing almost monotonically with increasing μ (Fig. 11), which shows that for larger friction μ the final value of segregation S_∞ is larger. It might be argued that less frictional particles are more likely to interchange the positions and therefore they are mixing more intensely inside the packed bed which increases the particle diffusivity and, as a result, reduces the final intensity of segregation of the whole packed bed.

Dependence of the parameter A is more complex. Two cases can be distinguished:

- $\mu < 0.1$, when A is decreasing with increasing μ, which means that during initial stages of the process, the segregation is faster for smaller μ;
- $\mu > 0.1$, when A is increasing with increasing μ, which means that during initial stages of the process the segregation is faster for larger μ.

This dependence of A on μ cannot be explained by simple speculations as in the case of the analysis of S_∞ dependence on μ. Since mixing is the driving force of segregation process in the studied system, we must investigate the process of mixing inside the packed bed quantitatively in order to try to explain the dependence of A on μ. Nonmonotonous nature of this dependence indicates a presence of two or more competing mechanisms of size segregation, each of which dominate in different experimental conditions.

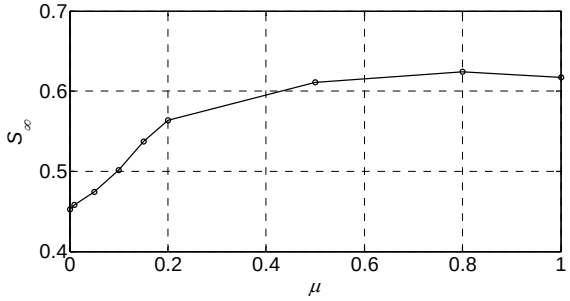

Fig. 11 Dependence of the final value of the segregation parameter S_∞ on the dynamic friction coefficient

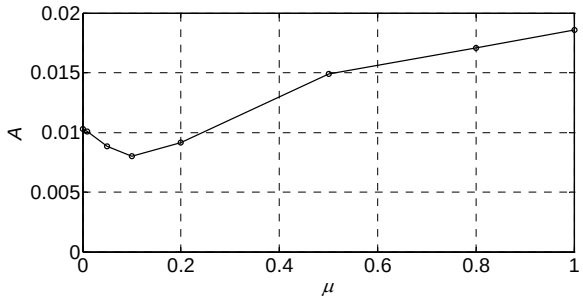

Fig. 12 Dependence of the parameter A on the dynamic friction coefficient

4. Conclusions

It has been shown that dynamic friction coefficient influences the size segregation process in polydisperse granular systems. The equation that characterises the evolution of segregation intensity approaching exponentially the certain steady state value is proposed.

The dependence of the rate of segregation intensity upon dynamic friction coefficient is nonmonotonous: the rate of segregation intensity is higher at lower values of the dynamic friction coefficient and attains its minimum at $\mu \approx 0{,}1$ for the system under consideration. For larger values of the dynamic friction coefficient, the rate segregation intensity starts to increase again with the increasing dynamic friction. This dependence indicates the presence of competing segregation mechanisms, one of which dominates at the given experimental conditions and material properties. The averaged velocity fields and flow lines demonstrate the different patterns of collective flow of the granular matter at different values of dynamic friction. At weak friction, local mixing dominates, while in case of high values of the dynamic friction coefficient, total bulk of the material gets involved in cyclic motion. However, more precise identification of the underlying mechanisms in each case requires further research of "convective" and "local" mixing processes of particles in granular material.

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TRINTIES ĮTAKA GRANULIUOTŲ MEDŽIAGŲ SEGREGACIJAI PAGAL DYDŽIUS

R e z i u m ė

Skaitiniu modeliavimu, naudojant diskrečiųjų elementų metodą, buvo tirtas segregacijos pagal dydžius procesas modelinėje granuliuotos medžiagos sistemoje, sudarytoje iš 1600 įvairaus dydžio apvalių dalelių ir maišomoje stačiakampio laiptelio, periodiškai judančio horizontalia kryptimi. Parodyta, kad dinaminės trinties koeficientas turi įtakos dalelių persiskirstymui pagal dydžius. Pasiūlyta lygtis, aprašanti, kaip laikui bėgant keičiasi ir eksponentiškai artėja prie tam tikros stacionarios vertės segregacijos intensyvumas.

Segregacijos intensyvumo priklausomybė nuo dinaminės trinties koeficiento yra nemonotoniška: segregacijos intensyvumas yra didesnis esant mažoms dinaminės trinties koeficiento μ vertėms ir pasiekia minimumą trinčiai didėjant iki $\mu \approx 0.1$, tačiau, esant didesnėms dinaminės trinties koeficiento vertėms, vėl pradeda didėti. Tokia priklausomybė leidžia manyti, kad, esant atitinkamoms eksperimentinėms sąlygoms ir medžiagos savybėms, vyrauja skirtingi segregacijos mechanizmai. Dalelių vidutinių greičių laukai ir srauto linijos rodo, kad, esant skirtingoms dinaminės trinties koeficiento vertėms, pasikeičia dalelių kolektyvinio judėjimo pobūdis.

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THE ROLE OF FRICTION IN SIZE SEGREGATION OF GRANULAR MATERIAL

S u m m a r y

The process of size segregation in a model system of granular matter, consisting of 1600 spherical polydisperse particles stirred by a rectangular bar moving periodically in the horizontal direction, was modelled by the discrete element method under different values of dynamic friction coefficient of the particle material. It has been shown that the dynamic friction coefficient influences the size segregation process in poly-disperse granular systems. The equation was proposed that characterises the evolution of segregation intensity approaching exponentially a certain steady state value.

The dependence of the rate of segregation intensity upon dynamic friction coefficient μ is nonmonotonous: the rate of segregation intensity is higher at lower values of the dynamic friction coefficient and attains a minimum value as the dynamic friction coefficient increases up to $\mu \approx 0.1$, but starts to increase again for the higher values of the dynamic friction coefficient. This dependence indicates the presence of competing segregation mechanisms, one of which dominates at the given experimental conditions and material properties. The averaged velocity fields and flow lines demonstrate different patterns of collective flow of the granular matter at different values of dynamic friction.

А. Джюгис, Р. Навакас

ВЛИЯНИЕ ТРЕНИЯ НА СЕГРЕГАЦИЮ ПО ВЕЛИЧИНЕ ГРАНУЛИРОВАННОГО МАТЕРИАЛА

Р е з ю м е

Используя метод дискретных элементов, теоретически исследовался процесс сегрегации в модельной системе гранулированного материала, состоящей из 1600 сферических частиц разной величины, приводимых в движение прямоугольной ступенькой, периодически движущейся в горизонтальном направлении. Моделирование проводилось при разных значениях коэффициента динамического трения частиц. Показано, что значение коэффициента динамического трения влияет на процесс перераспределения частиц по величине (сегрегации) в гранулированном материале. Предложено уравнение, описывающее эволюцию степени сегрегации, которая экспоненциально приближается к определенному стационарному значению.

Зависимость степени сегрегации от значения коэффициента динамического трения является немонотонной: степень сегрегации выше при малых значениях коэффициента динамического трения μ и достигает минимальной величины при $\mu \approx 0.1$, но начинает расти опять при повышении значения коэффициента динамического трения. Это указывает на присутствие конкурирующих механизмов сегрегации, которые проявляются при разных условиях процесса и разных свойствах материала частиц. Поля средней скорости перемещения частиц и линии потока показывают, что при разных значениях коэффициента динамического трения возникают разные структуры коллективного движения частиц.

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