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Mathematical modeling of deep drawing force with double reduction of wall thickness

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1. Introduction

Experimental research is used for inspection, correction and verification of numerical results and at stochastic modeling which describes the most realistically machining processes and systems. Technological construction and projecting of the modern machining processes demand to analyze all technical and technological parameters of the process and to apply scientific methods for modeling and defining of optimal conditions of machining processes and systems.

The main goal of modeling and optimization of machining process is to increase productivity, economy, total quality of the product or partial segments (machined surfaces, tool durability, etc.) also to decrease material costs, energy, machining time, and machining costs per one piece of the product.

Using theoretical analytical models it is hard to define precisely parameters of machining processes like: wasting tool, optimal geometric shape, deformation appearance at tool or press die, limitation level of deformation, tribologic (friction) processes, tool loading. In each of the mentioned machining processes a lot of significant factors and theirs interactions were applied. Therefore the application of experiments and analysis of their results is unchangeable in developing the new and improve existing machining processes and systems.

The main goal of modeling is to define mathematical model which is necessary in optimization, simulation, revitalization and controlling of processes and systems.

Regarding that, the main aim of process and system modeling is the construction of mathematical models. The main aim of experimental research is to get exact, approximately correct data which will serve as relationships, necessary for mathematical model. Mathematical model is necessary to start optimization of process. Regarding that, the main aim of modeling and optimization of machining process and systems is cheaper and higher quality production [1].

Mathematical model of deep drawing force with double reduction of wall thickness is presented in this paper.

2. Election of significant factors

For significant factors electing it is used criteria that elected factors are not related and not depends from outside factors. Outside factors are unelected factors which also belong to the process of deep drawing but are not significant for it. Significant factors values have to be applicable for the measuring process. For the process of deep drawing are elected three significant factors deformation φ, diameter ratio d_1/d_2 and friction coefficient μ.

Variation of the factors limits are shown in Table 1 [1 - 4]. The experiment was conducted with the variations factors of two levels. The experiment was repeated three times for each sample.

Table 1

Significant factors of machining

EXPERIMENT LEVEL	Significant factors		
	Deformation φ	Diameter ratio d_1/d_2	Friction coefficient μ
Maximal	1.22	1.055	0.20
Minimal	0.95	1.017	0.10

3. Equipment for researching

In experiment planning it was decided to use three independent changeable input values with two levels which make eight samples. For each sample measuring was repeated three times which makes twenty four measuring times.

This experiment of deep drawing process with double reduction of wall thickness is executed on hydraulic testing machine Amsler 300 kN (Fig. 1). Sensors for measuring friction coefficient (Fig. 2) were installed [5, 6]. The material of raw specimen is CuZn28 which before deep drawing process was prepared by heating, washing and lubricating.

Outer diameter of the raw specimen is d_0=26.4 mm (Fig. 3). Force of this process is measured on the same hydraulic testing machine.

Force measuring equipment in this machining process are presented in Fig. 4 [6]: *1* - hidraulic testing machine, *2* - signal acquisition unit, *3* - monitor for presentation of data and *4* - unit for data processing.

4. Experiment results

After decision which significant factors would be used in this process, the number of variation levels and decision about the number of repeating for each sample, start phase of the experiment process which are presented Figs. 1 - 4.

a b

Fig. 1 Testing equipment: a - hydraulic testing machine, b - specimen grips

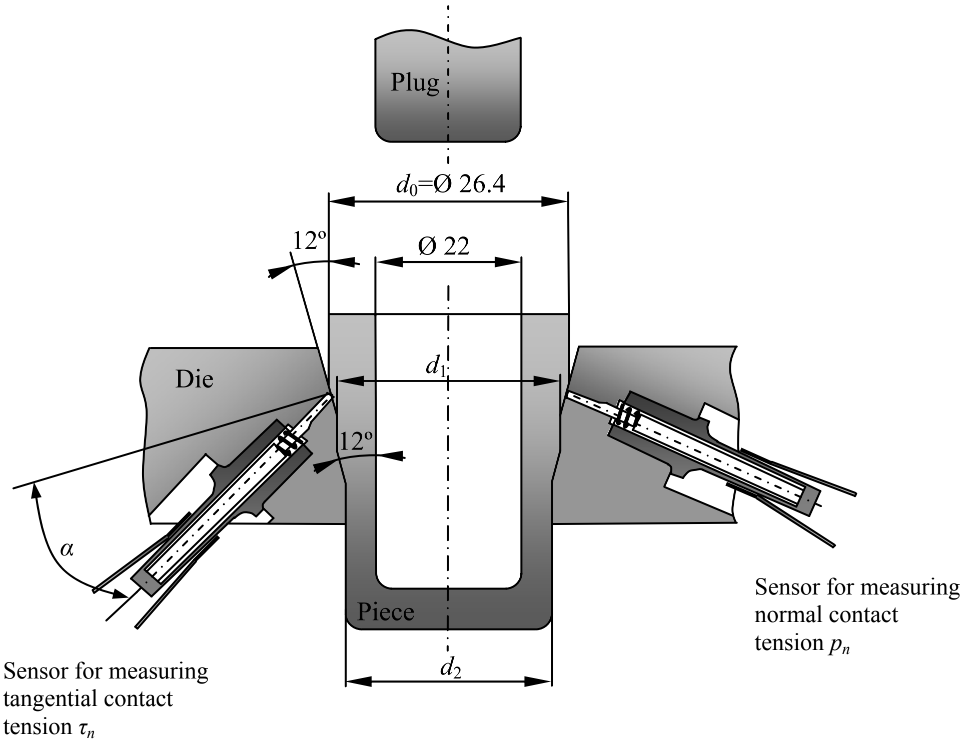

Fig. 2 Process of deep drawing with the system for measuring of contact tensions p_n and τ_n

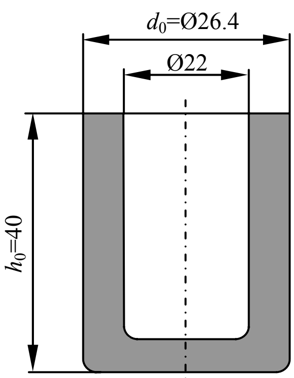

Fig. 3 Raw specimen

Fig. 4 Measuring system of force

Table 2

Experiment results

No of sample	Significant factors data			Results after measuring of deep drawing force			Arithmetic value	Square sum	Degree of freedom	Variation
	φ	d_1/d_2	μ	F_{j1}, kN	F_{j2}, kN	F_{j3}, kN	$\bar{y}_j = \bar{F}$	$\sum_{i=1}^{n}(y_{ji}-\bar{y}_j)^2$	f_j	S_j^2
1.	0.95	1.017	0.10	34	33	34.2	33.7	0.83	2	0.415
2.	1.22	1.017	0.10	46	43	45.2	44.7	4.83	2	2.415
3.	0.95	1.055	0.10	28.1	27.4	27.9	27.8	0.26	2	0.13
4.	1.22	1.055	0.10	42.6	43.1	42.3	42.7	0.33	2	0.165
5.	0.95	1.017	0.20	34.8	34.8	36.1	35.3	1.14	2	0.57
6.	1.22	1.017	0.20	47.1	47.2	46.1	46.8	0.74	2	0.37
7.	0.95	1.055	0.20	29.5	28.1	29.8	29.2	1.66	2	0.83
8.	1.22	1.055	0.20	48	48	47.5	47.8	0.17	2	0.085

Changing the value of all or just some input parameters and repeating the experiment the matrix of output result values is completed. Repeating of experiment in same sample with the same values of significant factors was founded three different results. Final experimental results are given in Table 2. For further procedure it is important to find arithmetical mean value of three repeated experiments. On these bases it is possible to finish inspection of homogeneity of result dispersion.

Inspection of homogeneity of dispersion for sorted level of reliability (P=0.95) has been done by Cohran's criteria [1, 7]

$$K_h = \frac{max\, S_j^2}{\sum_{j=1}^{N} S_j^2} \le K_t(f_j, N) \qquad (1)$$

where K_t is value by Cohran's criteria.

Dispersion (variation) is calculated by the next model (2) (Table 2)

$$S_j^2 = \frac{1}{n_j - 1}\sum_{i=1}^{3}(y_{ji} - \bar{y}_j)^2 \quad j = 1, 2, ..., 8 \qquad (2)$$

$$f_j = n_j - 1 \qquad (3)$$

where f_j is degree of freedom, $n_j = 3$ is the number of repeating for one sample

By Cohran's criteria:

- after calculation process with the results in Table 2

$$K_h = 0.4849$$

- using data from Table 1 founded in reference [1] it is obtained

$$K_t(f_j, N) = K_t(2,8)$$

$$K_t = 0.516$$

Regarding that $K_h = 0.4849 < K_t = 0.516$, process of getting mathematical model to be continued. Cohran's criteria confirms that homogeneity of experimental results dispersion satisfy.

5. Processing of experimental results

After inspection of the homogeneity of experimental results, the next step would be the calculation of regression coefficients. Regression coefficients are calculated using the following formulas (models) [1, 7]

$$b_i = \frac{1}{N}\sum_{j=1}^{N} X_{ij}\bar{y}_j \text{ , } i = 1, 2, ..., k \qquad (4)$$

and

$$b_{im} = \frac{1}{N}\sum_{j=1}^{N} X_{ij} X_{mj} \bar{y}_j \qquad (5)$$

where X_{ij} is the value of X_i in j-th experiment presented in Table 3, y_j is measured value in j-th experiment and N is the number of experiments (samples).

For correct structuring of mathematical model of any process in machining it is very important to choose a correct approximate mathematical model. If the first is not correct it is necessary to repeat all the steps for new approximate model construction.

In this paper the function of deep drawing force correlates with the mathematical model presented in formula (6) where is k=3

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_{12}X_1X_2 + \\ + b_{23}X_2X_3 + b_{13}X_1X_3 + b_{123}X_1X_2X_3 \qquad (6)$$

After calculation of regression coefficients results are presented below

$$b_0 = 38.5,\ b_1 = 7,\ b_2 = -1.625,\ b_3 = 1.275$$
$$b_{12} = 1.375,\ b_{13} = 0.525,\ b_{23} = 0.35,\ b_{123} = 0.4$$

The next step is testing of significances of regression coefficients. Each coefficient should be tested for importance in our mathematical model of deep drawing force (function) (6). In this calculation were use coded values.

Evaluation of significances of regression coefficients is going to be done separately one by one. Regression coefficients which are not significant should be removed from the mathematical model. It is not necessary to do value correction for significant regression coefficients.

Table 3

Code value of process

No of sample	Variables of physical process			Code variable of process			$\overline{y}_j^E$	y_j^R	$\left(\overline{y}_j^E - y_j^R\right)^2$
	φ	d_1/d_2	μ	X_1	X_2	X_3			
1.	0.95	1.017	0.10	**-1**	**-1**	**-1**	33.7	33.7	0
2.	1.22	1.017	0.10	**+1**	**-1**	**-1**	44.7	44.7	0
3.	0.95	1.055	0.10	**-1**	**+1**	**-1**	27.8	27.8	0
4.	1.22	1.055	0.10	**+1**	**+1**	**-1**	42.7	42.7	0
5.	0.95	1.017	0.20	**-1**	**-1**	**+1**	35.3	35.3	0
6.	1.22	1.017	0.20	**+1**	**-1**	**+1**	46.8	46.8	0
7.	0.95	1.055	0.20	**-1**	**+1**	**+1**	29.2	29.2	0
8.	1.22	1.055	0.20	**+1**	**+1**	**+1**	47.8	47.8	0

Two known criteria for the evaluation of significances of regression coefficients can be used: t – Student's criteria or Fisher's criteria and correlation [1, 7] existing between them

$$F(1, f) = t^2(f) \tag{7}$$

For the evaluation of significances of regression coefficients of model b_i, it would be used t's criteria or Student's test.

The formula for significance testing of regressions coefficients b_i with t - criteria is

$$t_{ri} = \frac{|b_i|}{S_{bi}} = \frac{|b_i|\sqrt{N\,n}}{S_y} \ge t_{t(f_y,\alpha)} \tag{8}$$

for $i = 0, 1, 2, ..., k$ or

$$|b_i| \ge \Delta b_i = \pm t_{t(f_y,\alpha)} S_{bi} = t_{t(f_y,\alpha)} \frac{S_y}{\sqrt{N\,n}} \tag{9}$$

for $i = 0, 1, 2, ..., k$

The variation of experimental error can be described by the model

$$S_y^2 = \frac{\sum_{j=1}^{N}\sum_{i=1}^{n}\left(y_{ji} - \overline{y}_j\right)^2}{f_y} \tag{10}$$

or

$$S_{bi}^2 = \frac{S_y^2}{N\,n}, \quad i = 0, 1, 2, ..., k \tag{11}$$

where

$$f_y = \sum_{j=1}^{N}\left(n_j - 1\right) = N\left(n - 1\right) \tag{12}$$

and f_y is total number of degree of freedom, n_j is the number of experiment repeating in j-th line of matrix, when is the same repeating number $n = n_j$. $S_b \cdot t = 0.28183$, in the condition for being significant. The coefficients which satisfy this condition $|b_i| > S_b \cdot t$, are:

$|b_0| = |38.5| > 0.28183$ significant

$|b_1| = |7| > 0.28183$ significant

$|b_2| = |-1.625| > 0.28183$ significant

$|b_3| = |1.275| > 0.28183$ significant

$|b_{12}| = |1.375| > 0.28183$ significant

$|b_{13}| = |0.525| > 0.28183$ significant

$|b_{23}| = |0.35| > 0.28183$ significant

$|b_{123}| = |0.4| > 0.28183$ significant

where

$$t(f_1, \varepsilon) = t(f_E, \alpha) = t(16, 0.05) = 1.75 \tag{13}$$

After significance testing of regression coefficients in the mathematic model, the conclusion is that all regression coefficients are significant. The next step is to return original values for all regression coefficients in model (6). After this model will have the structure presented in formula

$$\begin{aligned} Y = {} & 38.5 + 7X_1 - 1.625X_2 + 1.275X_3 + \\ & + 1.375X_1X_2 + 0.525X_1X_3 + 0.35X_2X_3 + \\ & + 0.4X_1X_2X_3 \end{aligned} \tag{14}$$

Model Eq. (14) presents deep drawing force as the function of significant parameters but in the code value. Used those coded values the values of deep drawing force are calculated. They are presented in Table 3.

6. Structuring of final mathematical model

After calculating and writing down the results of deep drawing force, the next step is to test this mathematical model (14) for adequacy. For this purpose, Fisher's criteria presented below are used

$$F_a < F_t \tag{15}$$

where F_t is value founded in Table 3 in reference [1] where is determinate by level of significance $p(F_a > F_t) = \alpha = 0.05$, or $(1 - \alpha) = 0.95$, it is 95% reliability. Where is

$$F_a = \frac{S_a^2}{S_y^2} \le F_t(f_1, f_2) = F_t(f_a, f_y) \tag{16}$$

for $S_a^2 > S_y^2$

$$F_a = \frac{S_y^2}{S_a^2} \le F_t(f_1, f_2) = F_t(f_a, f_y) \tag{17}$$

for $S_y^2 > S_a^2$.

The value of dispersion of adequacy is determined by the following model

$$S_a^2 = \frac{\sum_{j=1}^{N} n\left(\bar{y}_j^E - y_j^R\right)^2}{f_a} \tag{18}$$

where $f_a = N - k - 1$ is the number of degree of freedom which is related to the dispersion of adequacy.

The corresponding mathematical relation is obtained using the model (16) when $S_y^2 > S_a^2$

$$F_a = \frac{S_a^2}{S_y^2} = 0$$

From this it can be concluded that the mathematical model 100 % describes finished experiment. That is

$$F_a = 0 < F_t = 3$$

$\sum_{j=1}^{N} n\left(\bar{y}_j^E - y_j^R\right)^2$ is a part of model (19). Model (19) presents coefficient of multiple regression. Result of $\sum_{j=1}^{N} n\left(\bar{y}_j^E - y_j^R\right)^2$ equals zero (Table 3). That shows coefficient of multiple regression will equals 1.

When regression model correctly describes the process $R \to 1$. Coefficient of multiple regression is described by the model (19)

$$R = \sqrt{1 - \frac{\sum_{j=1}^{N} \left(\bar{y}_j^E - y_j^R\right)^2}{\sum_{j=1}^{N} \left(\bar{y}_j^E - \bar{y}^E\right)^2}} \tag{19}$$

As the model (14) adequately describes mean value of deep drawing force *F* in relation with significant parameters: φ, d_1/d_2 and μ, then the next proceeding is the conception of final mathematical model. In order to get mathematical model with real coefficients it is necessary to finish decoding of the model using transformation equation [1].

Mathematical model for deep drawing the force where force is related with significant parameters is described in the next model

$$\begin{aligned} F &= 213.4 - 30.46\varphi - 213.58\, d_1/d_2 + 3049.14\mu + \\ &+ 68\varphi\left(d_1/d_2\right) - 3149.88\varphi\mu - 2999.9\mu\left(d_1/d_2\right) + \\ &+ 3115.69\varphi\mu\left(d_1/d_2\right) \end{aligned} \tag{20}$$

The correlation between the force and parameters can be shown graphically Fig. 5, 6 and 7. Diagrams (Fig. 5, 6 and 7) present the correlation of deep drawing force with double reduction of wall thickness with: deformation degree φ, diameter ratio of press die d_1/d_2 and friction coefficient μ.

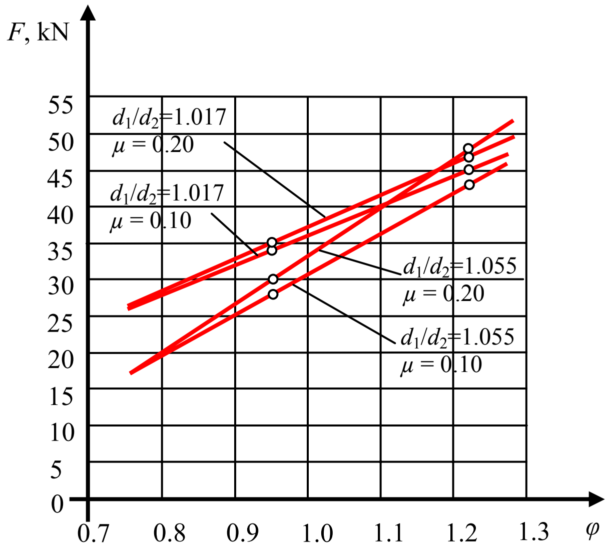

Fig. 5 Theoretical correlation between deep drawings force *F*, kN and deformation degree φ

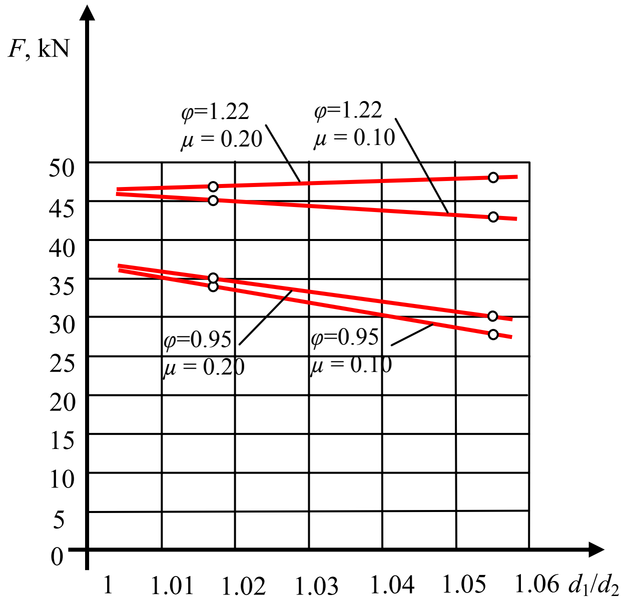

Fig. 6 Theoretical correlation between deep drawing force *F*, kN and diameter ratio d_1/d_2

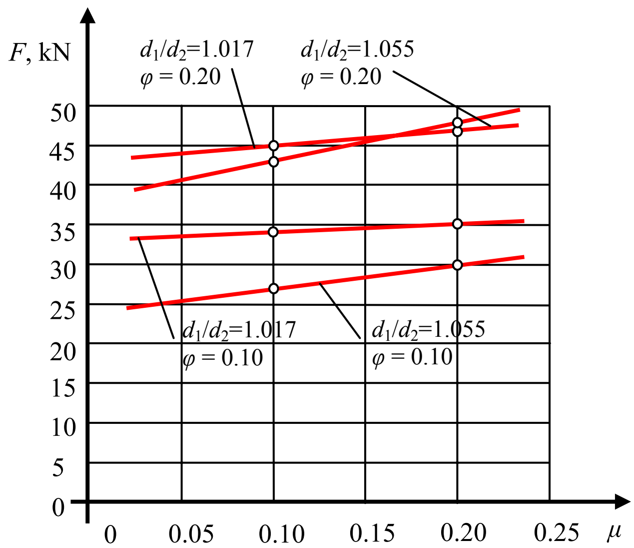

Fig. 7 Theoretical correlation between deep drawings force *F*, kN and friction coefficient μ

7. Conclusions

Experiments of deep drawing process with double reduction in press die of wall thickness were performed.

The following conclusions were made:

- final mathematical model is adequate for describing the process of deep drawing what is confirmed by Fisher's test;
- final mathematical model is absolutely correct for describing the process of deep drawing what is confirmed by multiple regression coefficient. Which is equal 1;
- deep drawing force is in line function of independent input parameters: deformation degree φ, diameter ratio of press die d_1/d_2 and friction coefficient μ;
- final mathematical model should be used for optimization of deep drawing process.

The main goal of the optimization of this mathematical model is to get minimal deep drawing force with the reduction of wall thickness with optimal input parameters (factors). The importance of this force minimizing would be multiple: from minimizing energy consumption to decreasing of intensity of wasting on press die guides and other pieces of press die.

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GILIOJO IŠTEMPIMO JĖGOS, PERPUS SUMAŽINANČIOS SIENOS STORĮ, MATEMATINIS MODELIAVIMAS

R e z i u m ė

Straipsnis skirtas plastinio deformavimo jėgos, susidarančios giliojo ištempimo metu ir perpus sumažinančios sienelės storį, matematiniam modeliavimui. Matematinis modeliavimas įgalina teisingai analitiškai aprašyti deformacijos procesą. Turint analitiškai aprašytą deformacijos procesą, galima skaičiuoti giliojo ištempimo jėgą ir optimizuoti visą deformacijos procesą bei ištempimo jėgą. Straipsnyje aprašoma planuojamo eksperimento eiga, pradedant svarbiausių parametrų įvertinimu, sienelės storio tolygumo kitimo, eksperimente naudojamos įrangos, gautų rezultatų palyginimu su žinomais analiziniais modeliais bei galutinio matematinio modelio giliojo ištempimo jėgai nustatyti sudarymu.

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MATHEMATICAL MODELING OF DEEP DRAWING FORCE WITH DOUBLE REDUCTION OF WALL THICKNESS

S u m m a r y

The main goal of this paper is to get a mathematical model for the process of deep drawing force with the reduction of wall thickness. Mathematical model should have a correct analytic description of this deformation process and along with that a possibility to calculative deep drawing force and after that a possibility of optimization of the whole deformation process and force. In this paper experiment planning is presented, from defining significant factors (parameters), levels of variation, equipment for experiment performing, processing of receiving results with known analytic models and on the end structuring final mathematical model for deep drawing force.

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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ СИЛЫ ГЛУБОКОЙ РАСТЯЖКИ, УМЕНЬШАЮЩЕЙ ТОЛЩИНУ СТЕНКИ В ДВА РАЗА

Р е з ю м е

Статья посвящена математическому моделированию силы пластической деформации уменьшающей толщину стенки образца в два раза при глубокой вытяжке. Математическое моделирование использовано с целью аналитического описания процесса деформации образца. При аналитической оценке процесса деформации возможен подсчет силы глубокой вытяжки, оптимизации ее и всего процесса вытяжки. Описан ход планируемого эксперимента, начиная с оценки основных параметров, изменения равномерности ширины стенки, в эксперименте используемого оборудования. Полученные результаты сравнены с результатами, полученными при использовании известных аналитических моделей и в конечном счете создана математическая модель для определения силы глубокой вытяжки.

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