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Forced vibrations of two plates in fluid and limit eigenmodes

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1. Introduction

Dynamics of elastic plates and shells in fluid have been investigated for more than half century. The presentation of the numerical Immersed Finite Element Method and review of some related methods is given in [1] by Zhang, Gay. Numerical solutions of two-dimensional laminar flows over airfoil are presented by Hafez et al. [2]. Incompressible fluid flow is simulated using a Helmholtz velocity decomposition into potential and rotational components. A Boundary Element and Finite Element Methods are coupled in Young's investigation [3]. The hydrodinamic and centrifugal forces affect elastic blade deformation and the surrounding flow field.

Coincident with the development of numerical methods, the theoretical investigations are being continued. In Ergin, Temarel publication [4] partially filled or submerged cylindrical shell is examined: the eigenmodes and associated frequencies are obtained using a boundary integral equation method together with the method of images. Eigenvalue problems and interaction between sloshing and bulging modes are considered by Amabili [5]. Analysis deals with compressible and incompressible fluids using Rayleigh-Ritz method. The Galerkin method for the hydroelastic vibration of a circular container bottom is applied by Cheung, Zhou [6]. Solution for the velocity potential of liquid motion is given by the method of separation of variables. The same method is applied in Xing's investigation [7], where two-dimensional structure-water interaction system is examined. The Sommerfeld radiation condition at the infinity of the rectangle water domain is investigated. Natural vibrations of a beam-water interaction system are considered by Xing et al. [8] with nondisturbance condition at infinity. A theoretical study, based on the Rayleigh-Ritz method and the finite Hankel transform, is presented by Jeong [9]. Dynamics of a part on a incompressible and compressible air-cushion are analyzed by Bakšys, Ramonas [10,11].

In this paper dynamics of two plates, not connected together, is investigated. But these plates interact with the same ideal incompressible fluid, assumed to be in two-dimensional finite or infinite rectangular domain. Vibrations of the plates in vacuo are independant, but because of the fluid an interconnected mechanical system is formed. The case when some of the eigenvalues in vacuo of different plates coincide (the multiple eigenfrequencies) is closely investigated.

2. Vibrations of plates in vacuo and fluid influence

Deflections of the two plates *AB* and *CD* (Fig. 1), supported at opposite edges, can be approximated $u_n(y,t)=\sum_{r=1}^{2n} q_r(t)\sigma_r(y)$, where $q_r(t)$ are the functions of time and $\sigma_r(y)$ are the set of square integrable functions on the intervals $[\, y_1, y_2\,]$ if $1 \le r \le n$ and $[\, y_3, y_4\,]$ if $n+1 \le r \le 2n$. The base functions $\sigma_r(y)$ satisfy the boundary conditions of the plates $\sigma_r = 0$; $d^2\sigma_r / dy^2 = 0$ if the plates are simply supported. It can be assumed $\sigma_r(y) \equiv 0$ if $y_3 \le y \le y_4$ and $1 \le r \le n$; $\sigma_r(y) \equiv 0$ if $y_1 \le y \le y_2$ and $n+1 \le r \le 2n$. The functions σ_r can coincide with the eigenmodes of the plates in vacuo

$$\begin{cases} \sigma_s(y) = sin\dfrac{\pi s(y-y_1)}{y_2-y_1} \text{if } 1 \le s \le n,\ y_1 \le y \le y_2 \\ \sigma_s(y) = sin\dfrac{\pi(s-n)(y-y_3)}{y_4-y_3} \text{ if } n < s \le 2n,\ y_3 \le y \le y_4 \end{cases} \tag{1}$$

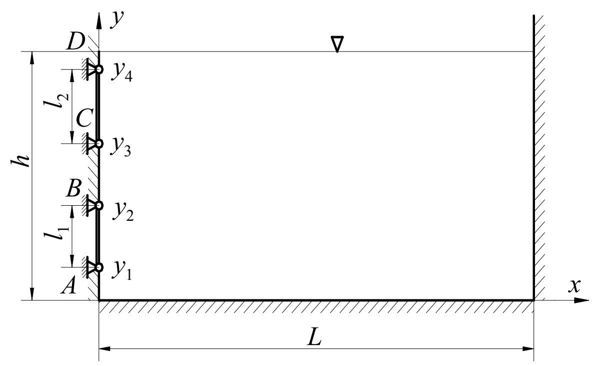

Fig. 1 Two plates *AB*, *CD* and rectangular fluid domain with free surface

But this is not necessary condition. Any complete set of functions when $n \to \infty$ can be used.

Vibrations of the plates can be presented in the matrix equation [12]

$$D\ddot{\overline{q}} + C\overline{q} = 0 \tag{2}$$

where *D* and *C* are the block matrices

$$D = \begin{Vmatrix} D_1 & N \\ N & D_2 \end{Vmatrix},\ C = \begin{Vmatrix} C_1 & N \\ N & C_2 \end{Vmatrix} \tag{3}$$

N is $n \times n$ zero martrix, $\overline{q}$ is $2n$ dimensional column vector. Obviously Eq. (2) can be replaced by two independent matrix equations $D_s\ddot{\overline{q}} + C_s\overline{q} = 0$, *s* = 1, 2, if dynamics of the plates *in vacuo* is under investigation. Entries d_{ji}, c_{ji} of the matrices D_s, C_s are proportional to mass and rigidity of the plates. Dependence on cross-section, density, Young's modulus can be deduced as in [13]. If vibrations are harmonic and $\bar{q} = \bar{g}e^{i\omega t}$, then Eq. (2) is reduced to $(D - \lambda C)\bar{g} = 0$, where $\lambda = \omega^{-2}$ and $\bar{g}$ does not depend on time.

If fluid is ideal and incompressible velocity potential $\varphi(x, y, t)$ satisfies the Laplace equation $\Delta\varphi = 0$ in the fluid domain and the boundary conditions:

1) $\dfrac{\partial\varphi}{\partial t} = 0$ when $y = h$ (free surface),

2) $\dfrac{\partial\varphi}{\partial y} = 0$ when $y = 0$ (rigid bottom of the reservoir),

3) $\dfrac{\partial\varphi}{\partial x} = 0$ when $x = L$ (rigid border),

4) $\dfrac{\partial\varphi}{\partial x} = \dfrac{\partial u}{\partial t}$ when $x = 0$ (the plates and, may be, the rigid border).

By using the separation of variables method when $u = u_s = q_s(t)\sigma_s(y)$ for any $1 \le s \le n$, the velocity potential can be expressed

$$\varphi_s(x, y, t) = \dot{q}_s h \sum_{j=1}^{\infty} a_{js} e^{-\chi_j x} \cos \chi_j y \tag{4}$$

if $L \to \infty$,

$$\varphi_s(x, y, t) = \dot{q}_s h \sum_{j=1}^{\infty} a_{js} \frac{\cosh \chi_j (L - x)}{\sinh \chi_j L} \cos \chi_j y \tag{5}$$

if L is finite and

$$a_{js} = \frac{2s\theta_1}{\pi^2 (j - 0.5)} \frac{\cos \chi_j y_1 - \cos \pi s \cos \chi_j y_2}{(j - 0.5)^2 - s^2\theta_1^2}$$

if $j \ne s\theta_1 + 0.5$,

$$a_{js} = \frac{2 \sin \chi_j y_1}{\pi (j - 0.5)\theta_1} - \frac{\cos \chi_j y_1 - \cos(\chi_j l_1 + \chi_j y_2)}{\pi^2 (j - 0.5)^2}$$

if $j = s\theta_1 + 0.5$, where $\chi_j = \dfrac{2j - 1}{2}\dfrac{\pi}{h}$, $\theta_1 = \dfrac{h}{l_1}$.

Equations (4), (5) are valid also when $s > n$, but y_1, y_2, θ_1 have to be replaced by y_3, y_4, $\theta_2 = h/l_2$ when a_{js} are determinated.

Kinetic energy of the fluid in the reservoir $T_L = -\dfrac{\rho}{2}\oint \varphi \dfrac{\partial\varphi}{\partial n} ds = -\dfrac{\rho}{2}\int_0^h \varphi \dfrac{\partial\varphi}{\partial x} dy,$ where $\varphi = \sum_{s=1}^{2n} \varphi_s(0, y, t)$. If Eqs. (5) or (4) are applied, expression of kinetic energy

$$T_L = \frac{\rho h}{2} \sum_{s=1}^{2n} \sum_{r=1}^{2n} \alpha_{sr} \dot{q}_s \dot{q}_r \tag{6}$$

can be deduced, where

$$\alpha_{sr} = \frac{\pi}{2} \sum_{j=1}^{\infty} (j - 0.5) \frac{a_{js} a_{jr}}{\tanh \chi_j L} \tag{7}$$

ρ is density of the fluid.

When only the first plate is on the border $x = 0$ and the second plate is on the opposite border $x = L$, Eq. (5) is valid only when $s \le n$. When $s > n$ velocity potential

$$\varphi_s(x, y, t) = -\dot{q}_s h \sum_{j=1}^{\infty} a_{js} \frac{\cosh \chi_j x}{\sinh \chi_j L} \cos \chi_j y \tag{8}$$

and kinetic energy

$$T_L = -\frac{\rho}{2} \int_0^h \varphi(0, y, t) \frac{\partial\varphi(0, y, t)}{\partial x} dy + \\ + \frac{\rho}{2} \int_0^h \varphi(L, y, t) \frac{\partial\varphi(L, y, t)}{\partial x} dy \tag{9}$$

because $\partial n = -\partial x$, $\partial s = -dy$ on the border $x = L$. The different expressions of φ_s (Eqs. (5) and (8)) are now valid for $s \le n$ and $s > n$ correspondingly, so the sum $\varphi = \sum_{s=1}^{n} \varphi_s + \sum_{s=n+1}^{2n} \varphi_s$ has to be presented. Two different values have to be inserted into Eq. (9). After such regrouping Eq. (6) can be proved for this case also, but α_{sr} from Eq. (7) remains valid only when $s, r \le n$ or $s, r > n$. If $1 \le s \le n$ and $n + 1 \le r \le 2n$, or $n + 1 \le s \le 2n$ and $1 \le r \le n$ then

$$\alpha_{sr} = -\frac{\pi}{2} \sum_{j=1}^{\infty} (j - 0.5) \frac{a_{js} a_{jr}}{\sinh \chi_j L} \tag{10}$$

The factors α_{sr} in Eq. (6) can be conceived as an interaction between the basic deflections $q_s\sigma_s$ and $q_r\sigma_r$. If both plates are in the same border $x = 0$ of the reservoir (as in Fig. 1 is shown), interaction of the two different basic deflections of the same plate and the two different plates are nearly the same: all factors α_{sr} are presented by the Eq. (7). But when one plate is in the border $x = 0$ and the other plate is in the border $x = L$, then interaction between the basic deflections of the same plate is described by Eq. (7), while interaction between the basic deflections of the different plates in opposite borders are described by Eq. (10). It is principle difference between Eq. (7) and Eq. (10): if $L \to \infty$ then $\tanh \chi_j L \to 1$, but $\sinh \chi_j L \to \infty$, therefore only the interaction between basic deflections of the plates in different borders disappears.

If two plates are considered as a $2n$ dimensional mechanical system and influence of the fluid is defined by Eq. (6), then $2n$ Lagrange's equations, instead of Eq. (2), are [13] $\left(D+\rho dh^2 H\right)\ddot{\bar{q}}+C\bar{q}=0$, and therefore

$$\left[C-\omega^2\left(D+\rho dh^2\right)H\right]\bar{g}=0 \tag{11}$$

where d is width of the plates, perpendicular to the axes x, y (it can be assumed $d=1$ m, Fig. 1). The matrix $H=\left\|\alpha_{sr}\right\|$, $1 \le s$, $r \le 2n$, can be presented as a block matrix

$$H=\begin{Vmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{Vmatrix} \tag{12}$$

where H_{11} and H_{22} are $n \times n$ matrices and present interaction of the basic deflections of the same plate, number 1 or number 2. Entries of these matrices can be calculated from Eq. (7). The matrix H_{12} presents the interaction of the two different plates and have to be solved from Eq. (7) if the plates are in the same reservoir border, and Eq. (10) if the plates are in opposite borders. In any case the matrix H is symmetric as $\alpha_{sr}=\alpha_{rs}$, $1 \le s$, $r \le 2n$.

If the plates are in the opposite borders and $L \to \infty$ then all α_{sr} in matrices H_{12} and H_{21} approach zero, therefore $H_{12} \to N$, $H_{21} \to N$, and the structure of the matrix H is the same as C, D in Eq. (3). Vibrations of every plate in this case is influenced by the fluid, but there is no interaction of both plates. Dynamics of every plate can be investigated by itself. If distance L between the plates is not large or both plates are in the same border (and any L in this case), the interaction matrix H_{12} is not a zero matrix, dynamics of the whole system has to be investigated integrally.

3. Eigenmodes in fluid and forced vibrations

The matrix $D_H=D+\rho dh^2 H=D+\varepsilon m_1 H$, where dimensionless parameter $\varepsilon=\dfrac{\rho dh^2}{m_1}$, m_1 is mass of the first (or it may be the second) plate. Forced vibrations are specified by $D_H\ddot{\bar{q}}+C\bar{q}=\bar{\Phi}e^{i\omega_0 t}$, where vector $\bar{\Phi}$ is amplitude of the harmonic force: $\bar{\Phi}^T\left(y_0\right)=f\left(y_0\right)\left\|\sigma_1\left(y_0\right),\right.$ $\left.\sigma_2\left(y_0\right),\ldots,\sigma_{2n}\left(y_0\right)\right\|$. When the force acts on the first plate, then $y_1 \le y_0 \le y_2$ and $\sigma_r\left(y_0\right)=0$ if $r>n$ – this follows from Eq. (1). Amplitude of the forced vibrations can be solved :

$$\bar{g}_f=\left(C-\omega_0^2 D_H\right)^{-1}\bar{\Phi}\left(y_0\right) \tag{13}$$

if the matrix $\left(C-\omega_0^2 D_H\right)$ is not singular. Solution can be conveniently expressed if the basic functions (1) are applied.

Behaviour of multiple eigenfrequencies of the plates *in* vacuo now will be investigated. If both plates are equal in their height $l_1=l_2=h/2$, but $y_1=0$, $y_2=h/2=y_3$, $y_4=h$ then matrix (12) for $n=2$

$$10H_{11}=\begin{Vmatrix} 1.270 & 0.106 \\ 0.106 & 0.304 \end{Vmatrix},\ 10H_{22}=\begin{Vmatrix} 0.590 & 0.106 \\ 0.106 & 0.260 \end{Vmatrix}$$

$$10H_{12}=\begin{Vmatrix} 0.340 & 0.182 \\ -0.030 & -0.022 \end{Vmatrix} \tag{14}$$

if $L=h$ and both plates are in the same border $x = 0$. If the second plate is in different border $x = L = h$, then H_{11}, H_{22} are the same as in (14), but block matrix of the plates interaction

$$10H_{12}=\begin{Vmatrix} -0.190 & -0.091 \\ -0.013 & -0.007 \end{Vmatrix} \tag{15}$$

When $L=5h$ and both plates are in the same border, all entries of the matrix H are only slightly less than in (14). When the plates are in the different borders and $L=2h$

$$10H_{11}=\begin{Vmatrix} 1.178 & 0.099 \\ 0.099 & 0.304 \end{Vmatrix},\ 10H_{12}=\begin{Vmatrix} -0.038 & -0.018 \\ -0.003 & -0.002 \end{Vmatrix}$$

So, the submatrix H_{11} is approximately the same, while H_{12} diminishes significantly when the distance L between the plates increases.

The case, when two hinged plates are in unlimited half plane fluid domain without free surface [13], can be compared with the similar plates in fluid domain, shown in Fig. 1 of this paper. Although all entries of the matrix H in [14] are 2-3 times higher as the correspondent entries of the H in (14), the relative magnitudes of all these values are approximately the same in both cases. Nevertheless, the case of the plates in different borders has substantially different submatrix H_{12}. It is quite possible that interaction of the plates, when these plates are in different borders, has distintive properties.

When the plates are in different borders of the rectangular domain the height of every plate can be equal to the depth of the reservoir: $y_1=y_3=0$, $y_2=y_4=h$. The two main submatrices are equal:

$$10H_{11}=10H_{22}=\begin{Vmatrix} 2.787 & 0.656 \\ 0.656 & 1.180 \end{Vmatrix}$$

$$10H_{12}=\begin{Vmatrix} -1.002 & -0.392 \\ -0.392 & -0.106 \end{Vmatrix}$$

One can notice more significant relative values of the matrix H_{12}.

When the fluid density diminishes, then influence of the fluid decreases simultaneously with the ε: the eigenfrequencies of Eq. (11) are approaching the eigenfrequencies of the same structure *in vacuo.* Eigenmodes in fluid approache the eigenmodes *in vacuo* if eigenfrequencies of the plates *in vacuo* do not coincide. Calculations where made for two different plates: $l_1=l_2=25$ cm, $h=50$ cm, Young‘s modulus $E_1=E_2=2.1\times10^4$ kN/cm^2, density of the plates $\rho_1=\rho_2=7.8$ kg/dm^3, but thicknesses of the plates are asssumed $\delta_1 = 2.2$ mm, $\delta_2 = 0.55$ mm. Eigenfrequencies of the first plate *in* vacuo are $f_{11} = 82.820$ Hz, $f_{12} = 331.28$ Hz, $f_{13} = 745.38$ Hz, ..., the second – $f_{21} = 20.705$ Hz, $f_{22} = 82.820$ Hz, $f_{23} = 186.34$ Hz, $f_{24} = 331.28$ Hz, ...

Every eigenmode of the plates in fluid can be presented as a sum

$$\sum_{s=1}^{2n} g_{rs}\sigma_s(y) = \sum_{s=1}^{n} g'_{rs}\sigma_s(y) + \sum_{s=1}^{n} g''_{rs}\sigma_{n+s}(y)$$

The first sum determines deflections of the first plate, the second sum – deflections of the second sum. When $\varepsilon \to 0$ and the frequency of the whole mechanical system (the two plates and the fluid) $f_{21}(\varepsilon) \to f_{21}(0) = 20.705$ Hz, all $g_{rs} \to 0$ except one: $g''_{11}\sigma_{n+1}(y) \to \sigma_{n+1}(y)$. A distinctly different limit eigenmodes are for the two multiple eigenfrequencies of the whole system $f_{11}(0) = 82.820$ Hz $= f_{22}(0)$. When $\varepsilon \to 0$ the matched eigenmodes of the eigenfrequencies $f_{11}(\varepsilon) \neq f_{22}(\varepsilon)$ are not approaching the eigenmodes *in vacuo* $\sigma_1(y)$, $\sigma_{n+2}(y)$ correspondingly. These eigenmodes approach the *limit eigenmodes* of the whole mechanical system. If $L = 0.5h$ these normed limit eigenmodes are $u_1 = 0.652\sigma_1 + 0.759\sigma_{n+2}$, $u_{n+2} = -0.280\sigma_1 + +0.960\sigma_{n+2}$; if $L = h$ then $u_1 = 0.563\sigma_1 + 0.826\sigma_{n+2}$, $u_{n+2} = -0.344\sigma_1 + 0.939\sigma_{n+2}$; if $L = 2h$ then $u_1 = = 0.532\sigma_1 + 0.847\sigma_{n+2}$, $u_{n+2} = -0.370\sigma_1 + 0.929\sigma_{n+2}$. All these values are valid if the plates are in the same border. When the plates are in the opposite borders, the limit eigenmoders are different and presented in Table 1. The ordinates of the plates are $y_1 = 0$, $y_2 = h/2 = y_3$, $y_4 = h$.

Table 1

Limit eigenmodes when $l_1 = l_2 = h/2$

L/h	u_1	u_{n+2}
0.25	$0.726\sigma_1 - 0.687\sigma_{n+2}$	$0.230\sigma_1 + 0.973\sigma_{n+2}$
0.50	$0.679\sigma_1 - 0.734\sigma_{n+2}$	$0.261\sigma_1 + 0.965\sigma_{n+2}$
0.75	$0.660\sigma_1 - 0.751\sigma_{n+2}$	$0.274\sigma_1 + 0.962\sigma_{n+2}$
1.00	$0.671\sigma_1 - 0.742\sigma_{n+2}$	$0.266\sigma_1 + 0.964\sigma_{n+2}$
2.00	$0.909\sigma_1 - 0.416\sigma_{n+2}$	$0.114\sigma_1 + 0.994\sigma_{n+2}$
5.00	$1.000\sigma_1 - 0.004\sigma_{n+2}$	$0.001\sigma_1 + 1.000\sigma_{n+2}$

It follows from the Table 1 that when the distance *L* between the plates increases the limit eigenmodes approach the eigenmodes in vacuo. One can observe that the dependence of limit eigenmodes on *L* is quite different when both plates are in the same border of the reservoir. The signs at the σ_1 and the σ_{n+2} indicate a phase difference of the vibration. All matched limit eigenmodes are orthogonal if normed in $L_2(0,h)$. In Table 2 are the matched limit eigenmodes for $y_1 = y_3 = 0$, $y_2 = y_4 = h$, the plates are in the opposite borders. More intense interaction of the plates than in Table 1 can be noted when $L \leq h$. All these calculations are perfomed for *n*=5.

Table 2

Limit eigenmodes when $l_1 = l_2 = h$

L/h	u_1	u_{n+2}
0.25	$0.495\sigma_1 + 0.869\sigma_{n+2}$	$-0.401\sigma_1 + 0.916\sigma_{n+2}$
0.50	$0.598\sigma_1 + 0.802\sigma_{n+2}$	$-0.318\sigma_1 + 0.948\sigma_{n+2}$
0.75	$0.711\sigma_1 + 0.703\sigma_{n+2}$	$-0.240\sigma_1 + 0.971\sigma_{n+2}$
1.00	$0.816\sigma_1 + 0.578\sigma_{n+2}$	$-0.174\sigma_1 + 0.985\sigma_{n+2}$
2.00	$0.988\sigma_1 + 0.156\sigma_{n+2}$	$-0.039\sigma_1 + 0.999\sigma_{n+2}$
5.00	$1.000\sigma_1 + 0.001\sigma_{n+2}$	$-0.000\sigma_1 + 1.000\sigma_{n+2}$

If a base functions σ_j, $j = 1, 2, ...$, are defined and dimension of the vector space 2*n* is fixed then any linearly independent manifold of these functions is equivalent [13]. If two base functions σ_1, σ_{n+2} are replaced by matched limit eigenmodes u_1, u_{n+2}, presented in Tables 1, 2, theoretically no essential change is made. But as a practical matter this can be important: sometimes the process of calculations can be unstable and acceptable only for a low numbers *n* [15].

Solution (13) of the forced vibrations can be approximated only by several terms, the number of which $n_0 << 2n$, if the frequency of harmonic force $f_0 = 2\pi\omega_0$ is in close proximity to the matched eigenfrequences. The exact eingenmodes of the mechanical system (solution of Eq. (11)) can be used for the approximation, but these eigenmodes depend on the fluid density and other parameters of the problem. As an intermediate case, between the complicated exact eigenmodes in fluid and eigenmodes of the plates *in vacuo,* the limit eigenmodes when $\varepsilon \to 0$ can be suggested. If some eigenfrequencies of the plates *in vacuo* coinside, then limit eigenmodes in fluid form the base, different from eigenmodes in vacuo, but still convenient for application.

4. Conclusions

1. Eigenmodes of a two different plates, in contact with singly connected rectangle fluid domain, are found as the eigenvectors of the complex mechanical system : two plates and the fluid as the coupling substance between the plates.

2. If density of the fluid $\rho \to 0$, all eigenfrequencies of the plates in the fluid approach the eigenfrequencies of the plates in vacuo, but not all eigenmodes of the plates in the fluid approach the eigenmodes of the plates in vacuo. The eigenmodes of the multiple eigenfrequencies in vacuo approach the limit eigenmodes.

3. Different base functions can be selected when approximation of the forced harmonic vibration is investigated. The case of the limit eigenmodes as a base functions is discussed and benefits of this choice are pointed out.

4. When some eigenfrequencies of two plates coincide exactly or approximately resonant behaviour of the whole mechanical system, including the fluid, can be expected. This can be a factor explaining significance of the low density fluid to the forced vibrations and peculiar „distribution of the added mass“ on the plates. Not the amount of the added mass should be emphasized, but the change in distribution of this influence.

References

1. **Zhang, L.T., Gay, M.** Imersed finite element method for fluid-structure interactions.-J. of Fluids and Structures, 2007, v.23, p.839-857.
2. **Hafez, M., Shatalov, A., Wahba, E.** Numerical simulations of incompressible aerodynamic flows using viscous/inviscid interaction procedures.-Computer Methods in Applied Mechanics and Engineering, 2006, v.195, p.3110-3127.
3. **Young, Y.L.** Time-dependent hydroelastic analysis of cavitating propulsors.-J. of Fluids and Structures, 2007, v.23, p.269-295.
4. **Ergin, A., Temarel, P.** Free vibration of a partially liquid-filled and submerged, horizontal cylindrical shell.-J. of Sound and Vibration, 2002, v.254(5), p.951-965.
5. **Amabili, M.** Eigenvalue problems for vibrating structures coupled with quiescent fluids with free surface. -J. of Sound and Vibration, 2000, v.231(1), p.79-97.
6. **Cheung, Y.K., Zhou, D.** Hydroelatic vibration of a circular container bottom plate using the Galerkin method.-J. of Fluids and Structures, 2002, v.16(4), p.561-580.
7. **Xing, J.T.** Natural vibration of two-dimensional slender structure-water interaction systems subject to Sommerfeld radiation condition.-J. of Sound and Vibration, 2007, v.308, p.67-79.
8. **Xing, J.T., Price, W.G., Pomfred, M.J., Yan, L.H.** Natural vibration of a beam-water interaction system. -J. of Sound and Vibration, 1997, v.199, p.491-512.
9. **Kyeong-Hoon Jeong** Hydroelastic vibration of two annular plates coupled with a bouded compressible fluid.-J. of Fluids and Structures, 2006, v.22, p.1079-1096.
10. **Bakšys, B., Ramonas, A.** Dynamics of a body vibrating according to the law of harmonics on an air-cushion. -Mechanika. -Kaunas: Technologija, 2006, No.4(60), p.32-39.
11. **Bakšys, B., Ramonas, A.** Dynamics of a part based on an incompressible air-cushion. -Mechanika. -Kaunas: Technologija, 2002, No.1(33), p.27-34.
12. **Iakovlev, S.** Submerged fluid-filled cylindrical shell subjected to a shock wave: Fluid-structure interaction effects. -J. of Fluids and Structures, 2007, v.23, p.117-142.
13. **Rektorys, K.** Variational Methods in Mathematics, Science and Engineering. -D. Reidel Publishing Company, Dordrecht, 1980.-571p.
14. **Kargaudas, V.** Vibrations of deformable plates in fluid and limit eigenmodes. -Proc. of the 4rd International Conference.-Strength, Durability and Stability of Materials and Structures SDSMS'04. -Kaunas. -2007, p.45-52.
15. **Mikhlin, S.G.** Numerical Treatment of Variational Methods.-Moscow, Nauka, 1966 (in Russian).

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DVIEJŲ PLOKŠČIŲ PRIVERSTINIAI VIRPESIAI SKYSTYJE IR RIBINĖS SAVOSIOS FORMOS

R e z i u m ė

Tiriama dviejų tarpusavyje nesusijusių tamprių plokščių dinamika. Šios plokštės ribojasi su stačiakampe sritimi, pripildyta idealaus nespūdaus skysčio, todėl sąveikauti gali tik per skystį. Tiriama plokščių virpesių formų sąveika ir tos sąveikos priklausomybė nuo plokščių tarpusavio padėties. Išskiriamas kartotinių savųjų dažnių vakuume atvejis, analizuojamos ribinės savosios formos, kai skysčio tankis artėja prie nulio. Skaičiuojant mechaninės sistemos priverstinius virpesius, siūloma šias ribines savąsias formas laikyti bazinėmis funkcijomis, nurodomi tokio pasirinkimo pranašumai.

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FORCED VIBRATIONS OF TWO PLATES IN FLUID AND LIMIT EIGENMODES

S u m m a r y

Dynamics of two elastic plates, not connected together, is investigated. These plates are in contact with singly connected rectangular domain of ideal incompressible fluid, so coupling of the plates is possible only through the fluid. Interaction between plate modes vibrations and their dependence on the relative position of the plates is investigated. The case of multiple in vacuo eigenfrequencies is examined and limit eigenmodes of the system , when density of the fluid vanishes, are presented. The limit eigenmodes as base functions for approximation of forced harmonic vibrations are suggested and benefits of this choice are pointed out.

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ВЫНУЖДЕННЫЕ КОЛЕБАНИЯ ДВУХ ПЛАСТИН В ЖИДКОСТИ И ПРЕДЕЛЬНЫЕ СОБСТВЕННЫЕ ФОРМЫ

Р е з ю м е

Исследуется динамика двух пластин, несоединенных между собой непосредственно. Эти пластины граничат с идеальной несжимаемой жидкостью в прямоугольной области, поэтому взаимодействие между пластинами возможно только через жидкость. Исследуется взаимодействие между формами колебаний пластин и зависимость этого взаимодействия от взаимного расположения пластин. Особо исследован случай кратных собственных частот пластин в вакууме, описаны предельные собственные формы, когда плотность жидкости приближается к нулевому значению. Эти предельные собственные формы предлагаются в качестве базисных функций при вычислении приближения вынужденных колебаний. Указаны преимущества такого выбора.

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