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On momentum transfer in a falling turbulent liquid film

S. Šinkūnas*, A. Kiela**
**Kaunas University of Technology, Donelaičio 20, 44239 Kaunas, Lithuania, E-mail: stasys.sinkunas@ktu.lt*
***Kaunas College, Pramonės 22, 50387 Kaunas, Lithuania, E-mail: algimantas.kiela@kauko.lt*

Nomenclature

d - hydraulic diameter of the film, m; d_n - needle probe outside diameter, m; f - cross-sectional area of film flow, m^2; G - liquid mass flow rate, kg/s; g - acceleration of gravity, m/s^2; Ga_R - Galileo number, gR^3/ν^2; R - tube external radius, m; Re - Reynolds number of liquid film, $4\Gamma/(\rho\nu)$; v^* - dynamic velocity, $(\tau_w/\rho)^{1/2}$, m/s; w - local velocities of stabilized film, m/s; $\overline{w}$ - average velocities of stabilized film, m; y - distance from wetted surface, m; y_n - distance from needle probe centre to the wall, m; Γ - wetting density, $kg/(ms)$; δ - liquid film thickness, m; ε - relative film velocity, w_d/w_{stab}; ε_R - relative cross curvature of the film, δ/R; η - dimensionless distance from the wetted surface, y/δ; η_d - dimensionless film diameter, v^*d/ν; ν - kinematic viscosity, m^2/s; ρ - liquid density, kg/m^3; φ - dimensionless film velocity, w/v^*; τ_w - shear stress at the wall, Pa.
Subscripts: cr - critical; d - distributor; n - related to the needle probe; o - initial; s - film surface; $stab$ - stabilized flow; t - turbulent; w - wetted surface; x - longitudinal coordinate.

1. Introduction

Research of momentum transfer in a liquid film flow is important in relation to various engineering aspects. Heat exchangers in which falling films are employed have such advantages as relatively high heat transfer and low energy consumption. The determination of hydromechanical parameters of liquid falling film, emerging from a slit, is an interest in many applications of chemical and power engineering. The prediction of internal or external film flows plays an important role in the design of heat exchangers.

Turbulent film flow has an immense technological importance, because it frequently occurs in normal operating conditions in a variety of heating and cooling devices. Turbulent falling films have not been investigated as widely as their laminar counterparts. The main reason for this is that transport equations for the turbulent flow are more complex, requiring a turbulence model to solve the fluid flow and heat transfer problem. Many useful heat transfer results have been obtained by establishing an analogy between the processes of heat and momentum transfer [1-5]. In study [6] the gas phase turbulence modification in annular flow due to the gas-liquid phase interaction was experimentally investigated. The annular flow passing through a throat section is under the transient state due to the changing cross sectional area of the channel and resultantly the superficial velocities of both phases are changed compared with a fully developed film flow in a straight pipe. By using a constant temperature hot wire anemometer, the measurements for the gas phase turbulence were carried out. The measurements for the liquid film thickness by the electrode needle method were also performed to measure the base film thickness, mean film thickness, maximum film thickness and wave height of the ripple or the disturbance waves.

A complete two-phase model is presented in [7] for film condensation from turbulent downward flow of vapour-gas mixtures in a vertical tube. The model solves the complete parabolic governing equations in both phases including a model for turbulence in each phase, with no need for additional correlation equations for interfacial heat and momentum transfer. The effects of changes to the inlet Reynolds number, the inlet to wall temperature difference on the film thickness and heat transfer are presented and discussed. Local profiles of axial velocities, temperature and gas mass fracture are also presented.

Studies [8-10] present an investigation into turbulent film condensation on a sphere with variable wall temperature. Under the wide range of vapour velocity, the wall temperature and the local film shear stress were considered. Potential flow theory was used to determine the high tangential velocities of the vapour. Furthermore, the papers discuss the influence of shear stress and temperature amplitudes on the local dimensionless film thickness and heat transfer characteristics. Finally, the results developed in these studies were compared with those generated by previous theoretical results.

A laser induced photochromic dye tracer technique has been applied [11,12] to investigate the hydrodynamic structure of free falling liquid films in side a vertical tube over Reynolds number range of 1408-6549. Flow visualization data indicated significant differences between the measured velocity profiles and Nusselt's predictions in wavy turbulent films. Also, the experimental data have shown that wave induced turbulence produces a flat velocity profile within the large disturbance waves and effects the flow in the substrate film depending on the wave amplitude. Velocity distribution of the falling film was investigated [13]. The cylindrical model appeared to be more appropriate over Cartesian model when the film thickness to tube diameter ratio is large. The study showed that wave characteristics depend on the parameters such as dimensionless wave velocity and Reynolds number.

The purpose of the present study is to obtain knowledge for momentum transfer development in the entrance region of turbulent liquid film flow. That is important for heat exchangers design, efficiency, reliability, compactness and economy of materials.

2. Experimental set-up

In order to measure velocity profiles of turbulent film a special device (Fig. 1) was applied. A needle probe of 0.3 mm in outside diameter and 0.18 mm in inside diameter measured velocity profiles in water film. A coordinating mechanism carried out the movement of the needle probe and fixed its zero position with respect to the wall. The needle probe was connected to a vertical glass tube through the flexible tube. Dynamic pressure in the needle probe was determined by the height of water column in the glass tube. In order to increase the accuracy of readings, a clock-faced indicator was fixed to the coordinating mechanism. The location of velocity measuring point in water film was determined by the following formula

$$y = y_n + 0.1d \tag{1}$$

Measurement accuracy for determination of film flow parameters was range of 1.4 - 8.4%.

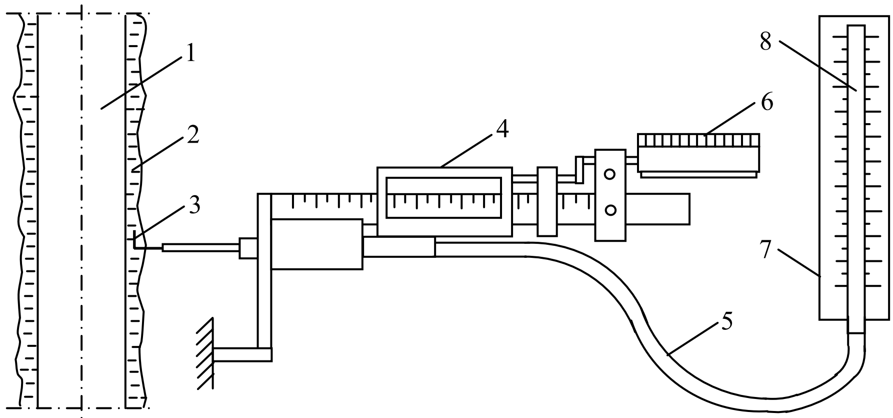

Fig. 1 Device to measure velocity profiles of water film: *1* - wetted surface; *2* - water film; *3* - needle probe; *4* - coordinating mechanism; *5* - flexible tube; *6* - clock-faced indicator; *7* - scale; *8* - glass tube

3. Velocity profiles in the entrance region of a turbulent film

The entrance region of a film flow is assumed when the film average thickness and average velocity becomes stable. The entrance region can be determined by the function $\varphi = f(\eta)$ provided that dynamic velocity v^* is calculated the regularities of stabilized flow and local velocity of the film is real. In that way, defined velocity profiles take up a position above or below profiles of stabilized flow depending on film velocity in the entrance region of the film flow.

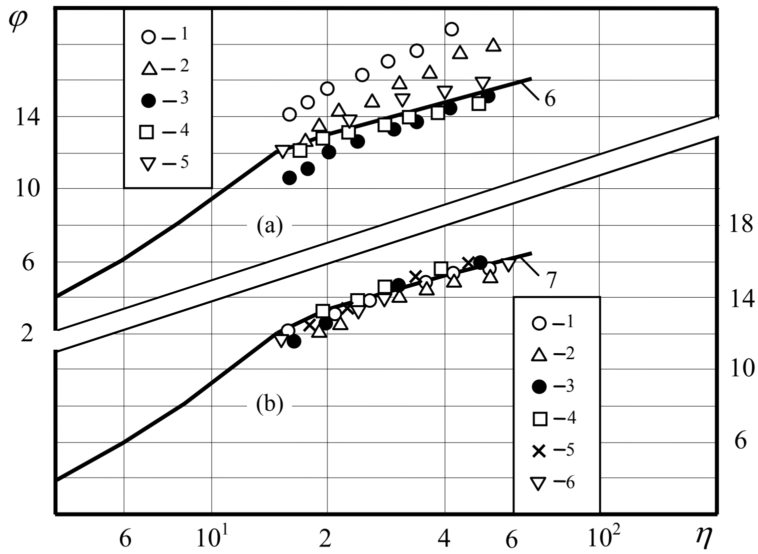

Fig. 2 Velocity profiles of water film in the entrance region of film flow when $Re_{cr} < Re < Re_t$: a) *1* - $Re = 5.7 \cdot 10^3$, $\varepsilon = 1.46$, $x = 0.08$; *2* - $Re = 8.16 \cdot 10^3$, $\varepsilon = 1.2$, $x = 0.08$; *3* - $Re = 5.84 \cdot 10^3$, $\varepsilon = 0.89$, $x = 0.08$; *4* - $Re = 4.87 \cdot 10^3$, $\varepsilon = 1.48$, $x = 0.28$; *5* - $Re = 5.25 \cdot 10^3$, $\varepsilon = 0.91$, $x = 0.28$; 6 – theoretical calculation, stabilized flow, $Re = 5.05 \cdot 10^3$; b) *1* - $Re = 6.2 \cdot 10^3$, $\varepsilon = 1.44$, $x = 0.48$; *2* - $Re = 7.76 \cdot 10^3$, $\varepsilon = 1.21$, $x = 0.48$; *3* - $Re = 6.43 \cdot 10^3$, $\varepsilon = 0.87$, $x = 0.48$; *4* - $Re = 5.43 \cdot 10^3$, $\varepsilon = 1.46$, $x = 1.07$; *5* - $Re = 6.19 \cdot 10^3$, $\varepsilon = 1.28$, $x = 1.07$; *6* - $Re = 5.5 \cdot 10^3$, $\varepsilon = 0.90$, $x = 1.07$; *7* - theoretical calculation, stabilized flow, $Re = 5.05 \cdot 10^3$

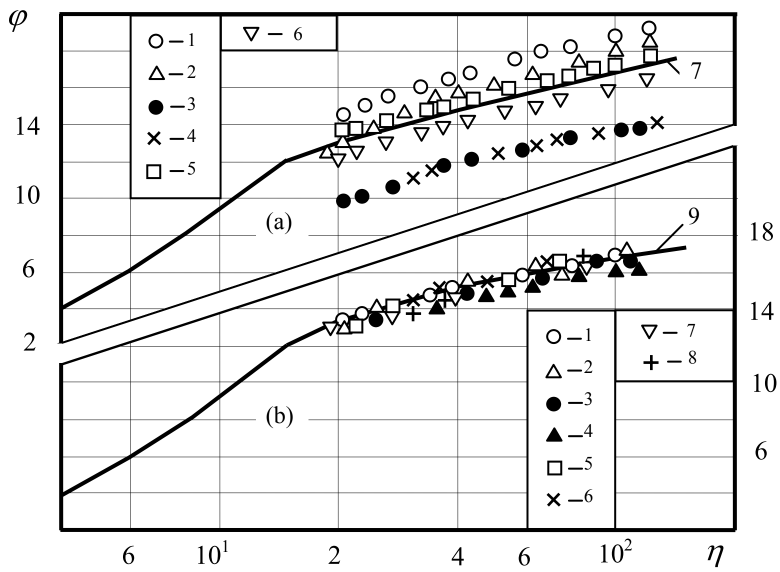

Fig. 3 Velocity profiles of water film in the entrance region of film flow when $Re > Re_t$: a) *1* - $Re = 15.8{\cdot}10^3$, $\varepsilon = 1.14$, $x = 0.08$; *2* - $Re = 13.2{\cdot}10^3$, $\varepsilon = 0.94$, $x = 0.08$; *3* - $Re = 16{\cdot}10^3$, $\varepsilon = 0.69$, $x = 0.08$; *4* - $Re = 41.8{\cdot}10^3$, $\varepsilon = 0.78$, $x = 0.08$; *5* - $Re = 15.8{\cdot}10^3$, $\varepsilon = 1.14$, $x = 0.28$; *6* - $Re = 13.9{\cdot}10^3$, $\varepsilon = 0.72$, $x = 0.28$; *7* - theoretical calculation, stabilized flow, $Re = 35{\cdot}10^3$; b) *1* - $Re = 15.9{\cdot}10^3$, $\varepsilon = 1.14$, $x = 0.48$; *2* - $Re = 16.3{\cdot}10^3$, $\varepsilon = 1.0$, $x = 0.48$; *3* - $Re = 15{\cdot}10^3$, $\varepsilon = 0.69$, $x = 0.48$; *4* - $Re = 37{\cdot}10^3$, $\varepsilon = 0.82$, $x = 0.48$; *5* - $Re = 14.6{\cdot}10^3$, $\varepsilon = 1.16$, $x = 1.07$; *6* - $Re = 13.7{\cdot}10^3$, $\varepsilon = 1.04$, $x = 1.07$; *7* - $Re = 13.8{\cdot}10^3$, $\varepsilon = 0.71$, $x = 1.07$; *8* - $Re = 36.8{\cdot}10^3$, $\varepsilon = 0.82$, $x = 0.71$; *9* - theoretical calculation, stabilized flow, $Re = 35{\cdot}10^3$

It evidently is seen from Figs. 2 and 3, where measured profiles in the entrance region of turbulent film flow are presented. As we can see from Figs. 2 and 3, the initial length film flow depends on parameter $\varepsilon = \overline{w}_d / \overline{w}_{stab}$ and wetting density (*Re*). The influence of relative film velocity ε is larger at the liquid distributor (x = 0.08 m). Along film flow direction the influence of parameter ε diminishes because the film flow stabilization takes place. Analysis of velocity profiles shows that *Re* number has a significant influence on the length of stabilization. The film flow stabilization takes place faster at low *Re* numbers and reverse phenomenon occurs at high *Re* numbers. It could be explained by the fact that at high *Re* numbers the inertia forces act predominantly, therefore the larger region for film flow stabilization is required.

The film mean velocity in the liquid distributor was calculated as follows

$$\overline{w}_d = G / f_d \, \rho \tag{2}$$

The mean velocity for stabilized flow has been calculated by the following equation

$$\overline{w}_{stab} = 1.85 (g\nu)^{1/3} Re^{5/12} \tag{3}$$

4. Average velocity in the entrance region of a turbulent film

Equation for momentum transfer in a turbulent film flow on vertical surface by evaluating its cross curvature can be written as follows

$$\frac{\tau_w}{\rho} = 0.25 g d - \frac{d}{dx} \int_0^{\delta_x} \left(1 + \varepsilon_R \frac{y}{\delta_x}\right) w^2 dy \tag{4}$$

Velocity field in the turbulent film flow can be defined by a simple exponential regularity

$$\frac{w}{w_\delta} = \left(\frac{y}{\delta_x}\right)^n \tag{5}$$

where exponent $n \le 1/7$ as usual.

It is known that when $n \le 1/7$, the regularity of Blazius does conform to the turbulent film flow as well. By taking into account Eq. (5), the integral in Eq. (4) can be written as follows

$$\int_0^{\delta_x} \left(1 + \varepsilon_R \frac{y}{\delta_x}\right) w^2 dy = \overline{w}^2 \delta_x \left(1 + 0.5\varepsilon_R\right) f\left(n, \varepsilon_R\right) \tag{6}$$

where

$$f\left(n, \varepsilon_R\right) = \frac{1 + 0.5\varepsilon_R}{\left(\dfrac{1}{n+1} + \dfrac{\varepsilon_R}{n+2}\right)^2} \left(\frac{1}{2n+1} + \frac{\varepsilon_R}{2n+2}\right) \tag{7}$$

Calculations showed that when exponent $n \le 1/7$, the member $f(n, \varepsilon_R)$ can be equal to 1 within ε_R limits from 0 to 1. Therefore, Eq. (6) with sufficient accuracy can be written as

$$\int_0^{\delta_x} \left(1 + \varepsilon_R \frac{y}{\delta_x}\right) w^2 dy \cong \overline{w}_x^{\,2} \delta_x \left(1 + 0.5\varepsilon_R\right) = \frac{d_x \overline{w}_x^{\,2}}{4} \tag{8}$$

By assuming that $d_x \overline{w}_x = const$, Eq.(8) can obtain the following expression

$$\frac{d}{dx}\int_0^{\delta_x}\left(1+\varepsilon_R\frac{y}{\delta_x}\right)w^2dy = 0.25d_x\bar{w}_x\frac{d\bar{w}_x}{dx} \tag{9}$$

Then equation of moment transfer in the turbulent film flow will appear as follows

$$\frac{\tau_w}{\rho} = 0.25d_x\left(g-\bar{w}_x\frac{d\bar{w}}{dx}\right) \tag{10}$$

Shear stress on the wetted surface can be determined by the equation

$$\tau_w = \rho\frac{\tau_w}{\rho} = \rho v*^2 = \frac{\rho v^2}{d_x^{\ 2}}\eta_d^{\ 2} \tag{11}$$

By taking into account that for turbulent film $\eta_d = 0.2Re^{0.875}$, we obtain the following equation

$$\tau_w = 0.04\frac{\rho v^2}{d_x^{\ 2}}Re^{1.75} \tag{12}$$

Considering that $\bar{w}_x = vRe/d_x$, the second parenthesized term in Eq. (10) will appear as

$$\bar{w}_x\frac{d\bar{w}_x}{dx} = -\frac{v^2Re^2}{d_x^{\ 3}}\frac{d}{dx}\left(d_x\right) \tag{13}$$

Then by taking into account Eqs. (12) and (13), we can obtain the following relationship from Eq. (10)

$$\frac{d}{dx}\left(d_x\right) = \frac{0.16Re^{1.75}-\dfrac{gdx^3}{v^2}}{Re^2} \tag{14}$$

By defining as $\dfrac{gd_0^{\ 3}}{v^2} = z_0^{\ 3}$, $\dfrac{gd_x^{\ 3}}{v^2} = z^3$, $0.16Re^{1.75} = b^3$, we obtain the following expression

$$\int_0^x dx = \left(\frac{v^2}{g}\right)^{1/3}Re^2\int_{x_0}^{z}\frac{dz}{b^3-z^3} \tag{15}$$

By integrating Eq. (15) and making simple rearrangements, we obtain the expression for the calculation of relative film thickness in the entrance region for turbulent film

$$\frac{Ga_x^{\ 1/3}}{Re^{5/6}} = 0.57\left\{2\sqrt{3}\left[arctg\frac{2\varepsilon_d+1}{\sqrt{3}} - arctg\frac{2\varepsilon_{d_0}+1}{\sqrt{3}}\right] - ln\frac{\left(1-\varepsilon_d\right)^2\left(1+\varepsilon_{d_0}+\varepsilon_{d_0}^{\ 2}\right)}{\left(1-\varepsilon_{d_0}\right)^2\left(1+\varepsilon_d+\varepsilon_d^{\ 2}\right)}\right\} \tag{16}$$

where $\varepsilon_d = d_x/d$, $\varepsilon_{d_0} = d_0/d$.

The comparison of theoretical results to experimental data, obtained in [1], is presented in Fig. (4).

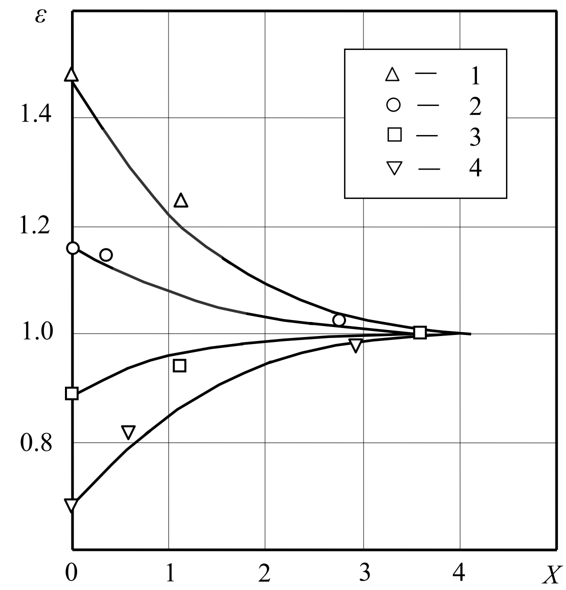

Fig. 4 Variation of relative film velocity in the entrance region of turbulent water film falling down a vertical surface: *1* - Re = 5.5·10³, ε = 1.47; *2* - Re = 15.8·10³, ε = 1.14; *3* – Re = 5.5·10³, ε = 0.9; *4* - Re = 15·10³, ε = 0.7; curves – calculations according Eq. (16); $X = Ga_x^{1/3}/Re^{5/6}$

Theoretically, full stabilization of the film flow takes place at $x = \infty$. Practically, one assumes that stabilization of the film flow comes out to the end, when $\left|1-\varepsilon\right| \cong \left|1-\varepsilon_d\right| \cong 0.01-0.02$, i.e. when the mean velocity (hydraulic diameter) of the film differs from stabilized film velocity by 1 - 2 %. Some results of entrance region length determination for water film according Eq. (16) are presented in Table.

Table

Entrance region length for water film at 20°C when $\left|1-\varepsilon\right| = 0.5$

Re	100	1000	5000	50000
	Length, mm			
$\left\|1-\varepsilon_d\right\| = 0.01$	2.5	54	250	1700
$\left\|1-\varepsilon_d\right\| = 0.02$	2.1	45	206	1400

5. Conclusions

1. Variation of relative film velocity in the entrance region of the turbulent film falling down a vertical surface was estimated analytically. Theoretical results are in a quite good agreement with experimental data.

2. The initial velocity and wetting density has a significant influence on the length of stabilization for turbulent film flow. It is obtained that film flow stabilization takes place at the distance 0.5 m, when $6\cdot10^3 < Re < 7\cdot10^3$ and at 1.0 m, when $Re < 4\cdot10^4$.

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S. Šinkūnas, A. Kiela

IMPULSO PERNEŠIMAS PER GRAVITACINĘ TURBLENTINĘ SKYSČIO PLĖVELĘ

R e z i u m ė

Eksperimentiškai ištirti greičių profiliai pradiniame gravitacinės turbulentinės skysčio plėvelės tekėjimo ruože. Gauti greičių profiliai pradiniame plėvelės tekėjimo stabilizacijos ruože diapazone $4870 < Re < 37000$, esant įvairiems plėvelės ištekėjimo iš skysčio skirstytuvo greičiams. Pateiktas plėvelės greičių profilių matavimo įrangos aprašymas. Greičių profilių tyrimo rezultatai įgalino nustatyti turbulentinės plėvelės tekėjimo stabilizacijos ruožo ilgį, esant įvairiems plėvelės ištekėjimo iš skysčio skirstytuvo greičiams. Teoriškai ištirtas plėvelės ištekėjimo iš skysčio skirstytuvo santykinio greičio kitimas pradiniame turbulentinės plėvelės stabilizacijos ruože. Gauti teoriniai skaičiavimai palyginti neblogai sutampa su eksperimentiniais tyrimo rezultatais.

S. Šinkūnas, A. Kiela

ON MOMENTUM TRANSFER IN A FALLING TURBULENT LIQUID FILM

S u m m a r y

An experimental study of velocity profiles in the entrance region of turbulent liquid film flow is performed. Velocity profiles in the entrance region of film flow were determined at various velocities of the film in liquid distributor for the range of $4870 < Re < 37000$. The description of experimental set-up is presented. Analysis of velocity profiles allowed estimating the length of stabilization for turbulent film flow at various velocities of the film in liquid distributor. Variation of relative film velocity in the entrance region of turbulent film flow is estimated analytically. Theoretical results are in good agreement with experimental data.

С. Шинкунас, А. Кела

ПЕРЕНОС ИМПУЛЬСА В ГРАВИТАЦИОННОЙ ТУРБУЛЕНТНОЙ ПЛЕНКЕ ЖИДКОСТИ

Р е з ю м е

Проведено экспериментальное исследование поля скоростей в турбулентной пленке на участке стабилизации течения. Измерены профили скоростей в пленке на участке стабилизации в диапазоне $4870 < Re < 37000$ при разных скоростях истечения пленки из распределительного устройства. Представлено описание экспериментальной установки для измерения поя скоростей в пленке. Результаты экспериментального исследования поля скоростей позволили установить длину участка стабилизации течения турбулентной пленки при разных скоростей истечения пленки из распределительного устройства. Также проведен теоретический анализ по изменению относительной скорости турбулентной пленки на начальном участке течения. Результаты теоретического расчета вполне удовлетворительно согласуются с экспериментальными данными.

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