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General deformation flow theory

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1. Introduction

While analyzing the relation between stresses and strains, in mechanics of deformed bodies various theories are applied [1-3]. Their character is determined by the body condition. Some theories are applied for brittle bodies, other for elastic bodies, the third group goes to elastic-plastic bodies. Body behaviour under deformation is also considered. One group of bodies under deformation can be characterized by strengthening, on other – by weakening, the third group – by the stability properties. Strength of elastic-brittle body is characterized by the surface of failure [2]. Deformation in plastic body depends on the way of loading and volumetric expansion [4-7]. Deformation of ductile-plastic body is related to time [4, 5]. However, most of the bodies are characterized by elastic-plastic properties, and deformation is complicated in this case.

Theoretical studies show that [2] in case of simple loading the results of deformational theory and yield theory are the same. However, in case of complicated loading the obtained results are different. For investigation, a thin-walled pipe loading by tension and torsion was chosen. In case of linear loading the results of deformational theory and yield theory are the same, and in case of nonlinear loading, the deformational theory shows no difference from the trajectory of loading. Then, yield theory shows dependence on the way of loading [2]. Thus, yield theory fits better than the deformational theory in case of complicated loading.

This study contains the analysis of deformation in context of so-called general deformation flow theory obtained after analyzing classical theories.

2. Survey of deformation theories

Experimental research [2] has shown (Fig. 1) that deformational theory and yield theory do not satisfy the results of certain research as well. Yield theory fits a little better for this case.

When analyzing theoretical solutions and experimental results, imperfection of these theories can be seen, i.e. they do not fully evaluate the event of loading. The most important is that these theories do not evaluate deformation speed and time.

Conditions of emerging elastic strains are characterized by invariants of stress tensor, i.e.

$$f_1\left[I_1(T_\sigma),I_2(T_\sigma),I_3(T_\sigma)\right]=0 \tag{1}$$

As plastic strains do not appear in case of combined tension and compression [2], the condition of elasticity with $I_1(T_\sigma)=0$ can be expressed as follows

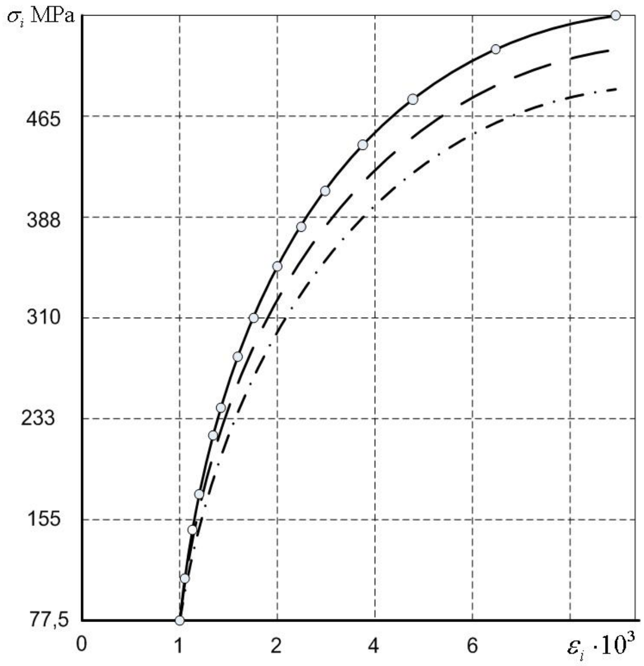

Fig. 1 Dependence between stresses of steel pipe σ and linear strains ε [2]: continuous lines show experimental curve; dash line – yield theory; dash-point line – deformational theory

$$f_2\left[J_2(D_\sigma),J_3(D_\sigma)\right]=0 \tag{2}$$

where $J_2(D_\sigma),J_3(D_\sigma)$ are invariants of stress tensor deviator.

Reuss [1] gives the plasticity condition as

$$f_3\left[J_2(D_\sigma),J_3(D_\sigma)\right]= 4\begin{bmatrix} J_2(D_\sigma)^3-27\left[J_3(D_\sigma)\right]^2- \\ -9\left[J_2(D_\sigma)\right]^2+6\sigma_Y^4J_2(D_\sigma)-\sigma_Y^6 \end{bmatrix}=0 \tag{3}$$

Condition (3) is used more rarely because of its complexity.

If condition (3) are based to determine material plasticity the proposition that the material does not suffer failure because of combined tension or compression is not logical and all three invariants of stress deviator $J_1(D_\sigma),J_2(D_\sigma)$ and $J_3(D_\sigma)$ are significant for failure condition.

Endochronical theory of unelasticity does not evaluate time of loading, and evaluates inner time of deformation [5]. According to the endochronical theory,

$$de_{ij}^{pl}=S_{ij}d\xi;\ d\varepsilon_{ij}^{pl}=d\omega \tag{4}$$

$$\begin{cases} d\xi = f_1\left(\sigma,\varepsilon,\xi\right)d\eta \\ d\omega = f_2\left(\sigma,\varepsilon,\omega\right)d\eta \\ d\eta = \left[\left(1/2\right)de_{ij}de_{ij}\right]^{1/2} \end{cases} \tag{5}$$

where ξ is inner time; ω is unelasticity of expansion; η is parameter of strain; f_1, f_2 are functions reflecting strengthening and weakening.

In endochronical theory of unelasticity, load function F is obtained from equations of volumetric expansion ratio

$$\beta = \partial F / \partial\sigma;\ k\partial F / \partial\sigma = d\varepsilon^{pl} = d\omega$$

where $k = d\xi;\quad k\partial F / \partial S_{ij} = de_{ij}^{pl} = S_{ij}d\xi$ and inner friction ratio $\beta' = \partial F / \partial\sigma = d\omega / \partial\xi$.

From Eqs. (4) and (5) we can obtain that

$$d\varepsilon_{ij}^{pl} = \left[\partial F / \partial\sigma_{ij}\right]d\xi \tag{6}$$

and

$$F = \left(1/2\right)S_{ij}S_{ij} + g\left(\sigma\right) - H_1 = 0 \tag{7}$$

where

$$dg\left(\sigma\right) / d\sigma = \beta' = d\omega / d\xi = f_2\left(\sigma,\varepsilon,\omega\right) / f_1\left(\sigma,\varepsilon,\omega\right) \tag{8}$$

and H_1 is parameter of strengthening.

Differential of parameter H_1 is given as

$$dH_1 = \left(\partial F / \partial\sigma_{ij}\right)d\sigma_{ij} / \partial F / \partial H_1 = S_{ij}dS_{ij} + \beta' d\sigma \tag{9}$$

Because value $1/2\,S_{ij}S_{ij} = J_2\left(D_\sigma\right)$ is the second invariant of stress tensor deviator, and plasticity condition by Misses is also expressed through $J_2\left(D_\sigma\right)$, loading function according to endochronical theory of unelasticity (Eq. 7) supplement the plasticity condition by Misses with members $g\left(\sigma\right)$ and H_1 evaluating deformation during inner time and extend the limits of deformation behind the limit of yielding.

Classical theories of plasticity characterize the surface of elasticity by geometrical line in plane and circle in space, and in endochronical theory of unelasticity, unelastic limit surface is a hypersphere that becomes circle in plane. This happens because the theory of yield plasticity analyzes an ideal elastic-plastic body (no strengthening) and endochronical theory of unelasticity evaluates strengthening.

The theory of ductile plasticity gives the relation between stresses and strains [1] as

$$\left.\begin{aligned} d\varepsilon_{ij} &= D_{ijkm}d\sigma_{km} + d\varepsilon''_{ij} \\ d\varepsilon''_{ij} &= \left(\partial F / \partial\sigma_{ij}\right)d\xi \\ d\xi &= dt / a_0\left(\sigma,\varepsilon\right) \end{aligned}\right\} \tag{10}$$

where D_{ijkm} is tangential moduli of plasticity; t is time; ξ is inner time; $a_0\left(\sigma,\varepsilon\right)$ is plasticity ratio.

Tangential moduli of plasticity are obtained from the following formula

$$D_{ijkm} = 2G\delta_{ik}\delta_{jm} + \left(K - 2G/3\right)\delta_{ij}\delta_{km} \tag{11}$$

where G is shear modulus; K is volumetric modulus.

3. General deformation flow theory

The theory of ductile-plastic can be applied for endochronical theory of unelasticity. For this, inner time or, more certainly, plasticity ratio is to be analyzed. As we already know, speed of deformation in endochronical theory $\|\dot{\varepsilon}\| \to \infty$ and $\|d\dot{\varepsilon}\|/\|d\varepsilon\|$ cannot neither reach infinity nor approach zero. Consequently, ductile ratio will not only depend on stresses σ and strains ε as in theory of plasticity, but on the strain speed ε_{ij} as well.

Then, according to the theory of ductile plasticity can be expressed

$$d\xi = dt/a_1\left(\dot{\varepsilon}\right) \tag{12}$$

This uniform function of ductile ratio $a_1\left(\dot{\varepsilon}\right)$ can be expanded into Taylor's series

$$\begin{aligned} \left[a_1\left(\dot{\varepsilon}\right)\right]^{-r} &= p_0 + p_{ij}\dot{\varepsilon}_{ij} + p_{ijkm}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{km} + \\ &+ p_{ijkmpq}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{km}\dot{\varepsilon}_{pq} + \ldots \end{aligned} \tag{13}$$

where r is power index; p is variable ratios obtained from strengthening function.

Leaving three members in the series, we need to reject members of 1st and 3rd power ($p_{ij} = p_{ijkmpq} = 0$), because for a_1 must not decrease with increasing $\|\varepsilon_{ij}\|$.

Then, applying Eq. (13) we obtain

$$\frac{\|d\dot{\varepsilon}\|}{\|d\varepsilon\|} = \left\|\frac{\partial F}{\partial\sigma_{ij}}\right\|\left(\frac{p_0}{\|\dot{\varepsilon}\|^r} + \frac{p_{ijkm}d\varepsilon_{ij}d\varepsilon_{km}}{d\varepsilon_{pq}d\varepsilon_{pq}}\cdot\|\dot{\varepsilon}\|^{2-r}\right)^{1/r} \tag{14}$$

For the analysis big strain speed $\|\dot{\varepsilon}\| \to \infty$ and fulfilled condition it cannot be equal to infinity or zero, neither $2 - r > 0$ nor $2 - r < 0$ are possible. Only $2 - r = 0$ or $r = 2$ is left.

In this case, Eq. (13) is given as follows

$$\begin{aligned} d\xi &= \frac{dt}{a_1\left(\dot{\varepsilon}\right)} = dt\left(p_0 + p_{ijkm}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{km}\right)^{1/2} = \\ &= \left(p_0dt + p_{ijkm}d\varepsilon_{ij}d\varepsilon_{km}\right)^{1/2} \end{aligned} \tag{15}$$

Because of impulsive deformation the start time dt falls out of Eq. (15) and, consequently, it can be presented as follows

$$d\xi = \left(p_{ijkm}d\varepsilon_{ij}d\varepsilon_{km}\right)^{1/2} \tag{16}$$

Thus, Eq. (6) can be given as

$$d\varepsilon_{ij}^{pl} = \left[\partial F / \partial \sigma_{ij}\right]\left(p_{ijkm} d\varepsilon_{ij} d\varepsilon_{km}\right)^{1/2}$$

Then,

$$\partial F / \partial \sigma = S_{ij} dS_{ij} + d\omega / d\xi + S_{ij} dS_{ij} + \beta' d\sigma = \\ = 2S_{ij} dS_{ij} + d\omega / d\xi + \beta' d\sigma = 0 \quad (17)$$

With $d\omega = d\varepsilon_{ij}^{pl}$ we have

$$d\varepsilon_{ij}^{pl} = -\left(2S_{ij} dS_{ij} d\xi + \beta' d\sigma d\xi\right) \quad (18)$$

Endochronical theory unelasticity [5] describes only a limited surface, but not the surface of plasticity as classical theories of plasticity. Also endochronical theory of unelasticity makes no difference between loading and unloading. Irretrievability of unloading is the main feature of endochronical theory of unelasticity.

Therefore, Eq. (18) calculates the change of plastic strains considered as absolute value and shows the process of strains irrespective of differences between deformations in case of loading and unloading.

Inserting the value of $d\xi$ from Eq. (16) into (18) we obtain

$$d\varepsilon_{ij}^{pl} = \left|\left(2S_{ij} dS_{ij} + \beta' d\sigma\right)\left(p_{ijkm} \times d\varepsilon_{ij} d\varepsilon_{km}\right)^{1/2}\right| \quad (19)$$

This shows additional factors having influence on the deformation process in the suggested deformation flow theory, describing the relation between stress and strain compared to classical theories of plasticity; these factors are inner friction β' and variables p_{ijkm}, describing material strengthening.

With

$$p_{ijkm} = n$$

where n is strengthening ratio analyzing the case when $i = j = k = m = 1$ we obtain $\beta' = \frac{1}{\sqrt{n}}$.

Therefore, the Eq. (19) can be expressed as

$$d_{\varepsilon ij}^{pl} = \sqrt{n} S_{ij} dS_{ij} + d\sigma \quad (20)$$

where $S_{ij} = \sigma_{ij} - \delta_{ij}\, \sigma/3,\ \sigma = \sigma_{kk}/3$.

4. Experimental

Let us compare calculation results obtained under deformational theory, theory of yield and general deformation flow theory with the experimental ones.

Experimental reseach was made with steel pipe in three cases. Diameter sample for testing was 16 mm, wall tickness $t = 2$ mm; modulus of elasticity $E = 2 \cdot 10^5$ MPa. The test was provided for three grade steel: 1) steel 35 – materials characteristics: yield stress $\sigma_{0.2} = 280$ MPa, ultimate strength $\sigma_u = 508$ MPa, limit deformation $A_5 = 17\%$, $Z = 45\%$, strengthening ratio $n = 0.7$; 2) steel 45 – materials characteristics: yield stress $\sigma_{0.2} = 323$ MPa, ultimate strength $\sigma_u = 588$ MPa, limit deformation $A_5 = 14\%$, $Z = 40\%$, strengthening ratio $n = 0.5$; 3) steel 10G2 – materials characteristics: yield stress $\sigma_{0.2} = 265$ MPa, ultimate strength $\sigma_u = 421$ MPa, limit deformation $A_5 = 21\%$, $Z = 50\%$, strengthening ratio $n = 0.6$.

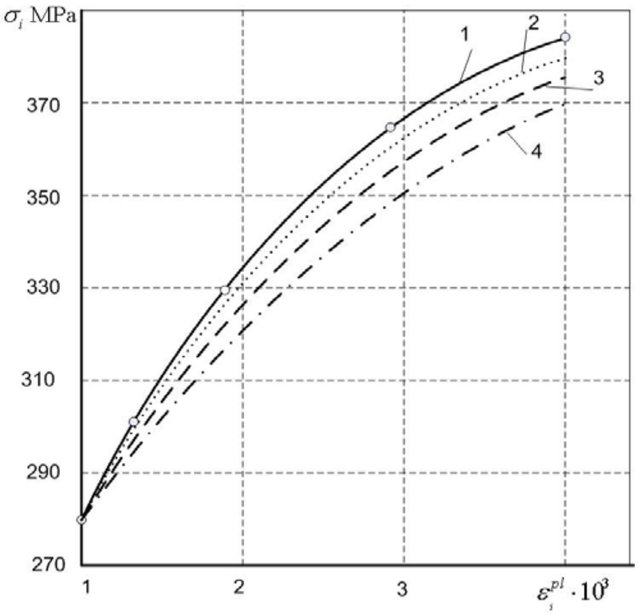

Fig. 2 Curves of stress-strain relation for steel 35: *1* - experimental data; *2* - data under theory of general deformation flow; *3* - data under yield theory; *4* - data under deformational theory

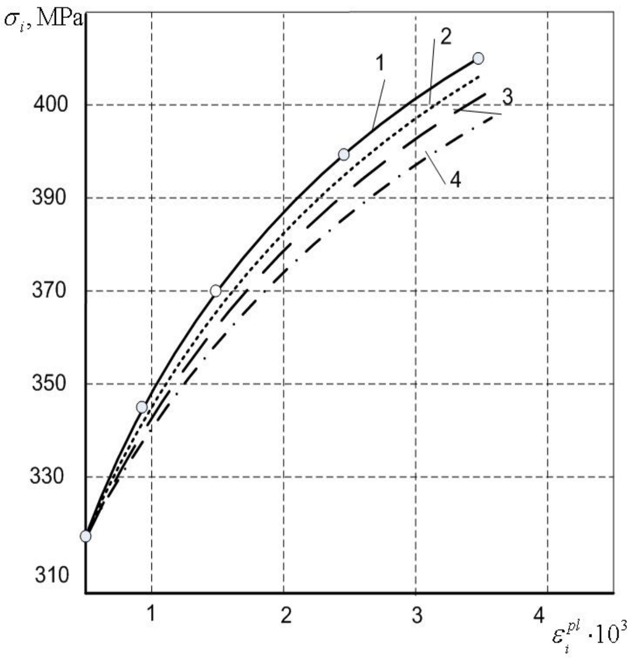

Fig. 3 Curves of stress-strain relation for steel 45: *1* - experimental data; *2* - data under theory of general deformation flow; *3* - data under yield theory; *4* - data under deformational theory

The tests were run on testing machine VEB EU-20 and loaded by bending and torsion.

At first was loaded with torsion and after suffered by bending. Results are reflected in Figs. 2 - 4. The Figs. 2 - 4 represent data obtained using deformational theory $\left(\sigma_i = \frac{\sqrt{3}\sigma_Y}{\sqrt{3\varepsilon^2 + \gamma^2}}\varepsilon_i\right)$; yield theory $\left(q = \frac{\sigma}{\sigma_Y}; t = \frac{\varepsilon}{\varepsilon_Y}; s = \frac{\tau}{\tau_Y} = \frac{\sqrt{3}\tau}{\sigma_Y}; dt = \frac{dq}{1-q^2}\right)$ – integral value theory and the theory of general deformation flow (20) with integral values.

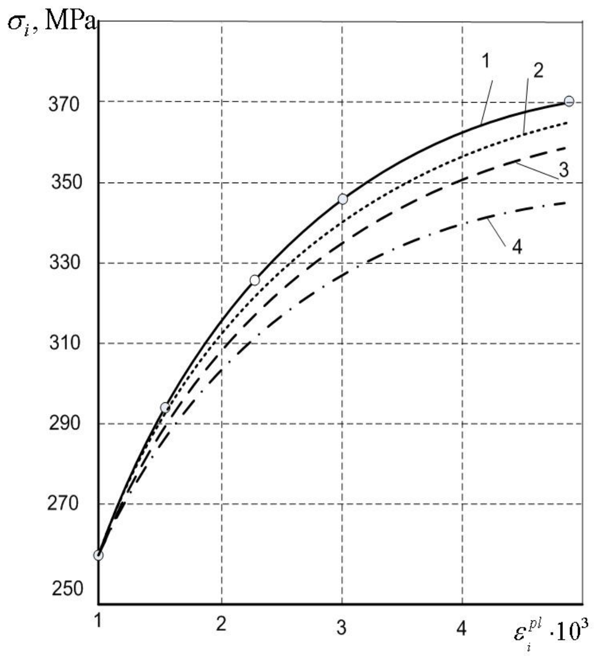

Fig. 4 Curves of stress-strain relation for steel 10G2: *1* - experimental data; *2* - data under theory of general deformation flow; *3* - data under yield theory; *4* - data under deformational theory

As the curves (Figs. 2 - 4) represent, the curve obtained under general deformation flow theory is the closest to the curve of experimental data. Regularities of relation between stresses and strains obtained under deformational theory are the most distant from real deformation process.

5. Conclusions

1. Neither deformational theory nor the theory of yield plasticity do evaluate the event of deformation and do not fully satisfy experimental results.

2. Elasticity conditions are the best described by criteria of Huber-Mises, however, complicated loading is better satisfied by Reuss' criterion evaluating 2nd and 3rd invariants of stress tensor deviator.

3. Describing the flow of deformation above the limit of yielding based on classical plasticity theories that do not evaluate inner material processes is not accurate enough.

4. General deformation yield theory formulated referring to the theory of plasticity and endochronical theory of unelasticity evaluates overdue of plastic deformations during the loading and processes of inner deformations.

5. When evaluating general deformation, flow theory limited surface can be described more accurately and more accurate criteria of critical strength can be obtained.

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BENDROJI DEFORMAVIMO TĖKMĖS TEORIJA

R e z i u m ė

Straipsnyje pateikta tampriai plastiškų kūnų bendroji įtempių ir deformacijų ryšio teorija. Nagrinėjamos deformacinė ir tekėjimo plastiškumo bei endochroninė netamprumo teorijos. Eksperimentiniai ir teoriniai tyrimai rodo, jog paprastam apkrovimui deformacinė ir tekėjimo teorijų rezultatai sutampa. Tačiau sudėtingo apkrovimo metu gaunami rezultatai yra skritingi. Tiesinio apkrovimo atveju rezultatai pagal deformacinę ir tekėjimo teorijas sutampa, o netiesinio apkrovimo atveju deformacinė teorija nerodo skirtumo nuo apkrovimo trajektorijos. Tuomet tekėjimo teorija rodo priklausomybę nuo apkrovimo kelio. Taigi tekėjimo teorija geriau tinka sudėtingo apkrovimo atveju negu deformacinė teorija. Endochroninė netamprumo teorija leidžia išsamiai įvertinti laiko įtaką medžiagos deformavimo procese. Endochroninėje netamprumo teorijoje apibrėžiamas tik ribinis paviršius, o ne plastiškumo paviršius kaip klasikinėse plastiškumo teorijose. Sudėtingo deformavimo atveju siūloma tinkamiausia bendroji deformavimo tekmės teorija, įvertinanti plastinių deformacijų vėlavimą apkrovimo metu ir vidinio deformavimo procesus.

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GENERAL DEFORMATION FLOW THEORY

S u m m a r y

This article represents the theory of general relation between stresses and deformations in elastic-plastic bodies. Deformation theory, the theory of yield plasticity and endochronical theory of unelasticity are analyzed. Experimental and teoretical studies show that in case of simple loading the results of deformational theory and yield theory are the same. Howevwer, in case of complicated loading the obtained results are different. In case of linear loading the results of deformational theory are the same and in case of nonlinear loading the deformational theory show no difference from the trajectory of loading. Then yield theory shows dependence on the way of loading. Thus, yield theory fits better than the deformational theory in case of complicated loading. Endochronical theory of unelasticity allows full evaluation of time influence on the processes of material deformation. Endochronical theory of unelasticity describes only a limited surface, but not the surface of plasticity as classical theory of plasticity. In case of complicated deformation, as the most applicable the general theory of deformation flow is suggested; this theory evaluates overdue of plastic deformations during the loading and processes of internal deformation.

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ОБОБЩЕННАЯ ТЕОРИЯ ПЛАСТИЧЕСКОГО ДЕФОРМИРОВАНИЯ

Р е з ю м е

В настоящей статье предложена обобщенная теория связи между напряжениями и деформациями для упругопластических тел. Анализируются деформационная теория, теория текучести и также эндохронная теория неупругости. Экспериментальные и теоретические исследования показывают, что при простом нагружении результаты деформационной теории и теории текучести совпадают. В случае сложного нагружения результаты становится различными. При линейном нагружении результаты по деформационной теории и теории текучести совпадают, а при нелинейном нагружении деформационная теория не показывает траектории нагружения. В то время теория текучести указывает на путь нагружения. Таким образом теория текучести при сложном нагружении наиболее приемлема. Эндохронная теория неупругости позволяет более полно оценить влияние времени на процесс деформирования. В отличии от классических теорий пластичности, в эндохронной теории неупругости описывается предельная поверхность текучести. В случае сложного деформирования предлагается обобщенная теория текучести, оценивающая запаздывание пластических деформаций и внутренние процессы.

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