Hexapod leg control algorithm in fault conditions

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1. Introduction

A detailed analysis of a robot workspace space of the robot is based on the locus of points in \mathbb{R}^3 that could be reachable by the tool tip. In using the tool tip as a reference point, the effects of both the major axes used to position the wrist and the minor axes used to orient the tool must be included.



Fig. 1 Hexapod robot structure

When viewed as a subset of \mathbb{R}^3 , the shape or geometry of the work envelope varies from robot to robot. However, the work envelope can also be viewed within the framework of joint space \mathbb{R}^n . In joint space, the work envelope is typically characterised by bounds on linear combinations of joint variables. The constraints of this nature generate a convex polyhedron in \mathbb{R}^n named the joint-space work envelope.



Fig. 2 Hexapod robot leg

Let q_{min} and q_{max} be the vector in \mathbb{R}^n the joint limits and let A be a $[m \times n]$ joint coupling matrix. Then the set of all values that the joint variables q can assume is called the joint-space work envelope. It is denoted Q and is of the form [1]

$$Q = \left\{ q \in \mathbf{R}^{n} : q^{\min} \le Aq \le q^{\max} \right\}$$
(1)

Then the set of all values that the joint variables q_i can assume is called the joint-space work envelope. It is

denoted Q and it is of the form [1]:

A = I represents noninteraxial coupling. The jointspace work envelope Q is the locus of points in \mathbb{R}^3 than can be reached by the tool tip. The locus of all points reachable from at least one tool orientation is referred to as the total work envelope, or simply the work envelope and the locus of points reachable from an arbitrary tool orientation is called the dextrous work envelope [1, 2].

In our case, workspace analysis of the leg tip of a hexapod robot (Fig. 1) is made.

Trajectory planning refers to the planning of a sequence of world points (x(t), y(t), z(t)) to be occupied by the end-effector of the robot (in our case, the leg tip) and the planning of a sequence of the values to be taken on by a specific joint variable q_i .

Vector and matrix algebra are utilised to develop a systematic and generalised approach to describe and represent the location of the links of a robot structure with respect to a fixed reference frame [3, 4]. Since the links of a robot structure may rotate and/or translate with respect to the reference coordinate frame, a body-attached coordinate frame will be established along the joint axis for each link. The direct problem of kinematics is reduced to finding a transformation matrix that relates the body-attached coordinate frame to the reference coordinate frame.

The study of the curves or trajectories is essential for understanding robot kinematics, since all points of the robot move on curves.

Let us consider the Rotate-Rotate-Rotate mechanical structure as it is shown in Fig. 2. The structure presented in Fig. 2 is nonredundant structure of a hexapod robot leg because the joints variable number $(\theta_k^1, \theta_k^2, \theta_k^3)$ as well as the operational coordinate number (x, y and θ_z) are equal to 3. The structure can be a part of poliarticulated hyper-redundant robotic arm [5], a part of nonredundant robotic arm or can be a robotic arm itself.

The point $M_k^3(x_k^3, y_k^3)$ belongs to a specified trajectory and their values are known. Index *k* represents actual step in the evolution on the trajectory. So

$$q_{k} = \left[q_{k}^{1}, q_{k}^{2}, q_{k}^{3}\right]^{T} = \left[\theta_{k}^{1}, \theta_{k}^{2}, \theta_{k}^{3}\right]^{T}$$
(2)

For simplicity we consider that the length of those 3 elements of the arm is the same: $l_1 = l_2 = l_3 = l$. In this paper it is proposed to establish values of the angles $\theta_k^1, \theta_k^2, \theta_k^3$ as well as differences $\Delta \theta_k^1, \Delta \theta_k^2, \Delta \theta_k^3$ which are basic for generating commands to the actuators in terms of a good walking (finding optimal motions) and in terms of blocking of some robot leg segments. Practically it is an inverse problem of kinematics. Input and output variables of the proposed algorithm are shown in Fig. 3.



Angles $\theta_k^1, \theta_k^2, \theta_k^3$, displacements $\Delta \theta_k^1, \Delta \theta_k^2$, $\Delta \theta_k^3$ and coordinates of the points M_1 and M_2 (which are necessary in workspace analysis for avoiding some existing obstacles) are determined on the base of the angles $\theta_{k-1}^1, \theta_{k-1}^2, \theta_{k-1}^3$ from previous step, on the base of desired coordinates x_k^3 , y_k^3 of the leg tip and on the base of some information related to physical structure (segments length, maximal and minimal limits of angular displacement and blocking status of some segments). Fault detection and isolation for any systems are presented in [6 - 8]. Specific methods for fault detection and diagnosis are presented in [9 - 13]. The algorithm proposed by the authors allows, if the blocking exists, either a correct positioning by other displacements of the unblocked segments (if it is possible) or a positioning in an acceptable proximity of the desired coordinates by minimising optimal criteria.

2. Algorithm for unblocked joints

Let us consider leg structure of the hexapod robot shown in Fig. 2. We wish the positioning of the leg tip in the point $M_k^3(x_k^3, y_k^3)$ without any specification of the orientation. Coordinates of the point M_k^3 are

$$x_k^3 = l\sin\theta_k^1 + l\sin\left(\theta_k^1 + \theta_k^2\right) + l\sin\left(\theta_k^1 + \theta_k^2 + \theta_k^3\right)$$
(3)

$$y_k^3 = l\cos\theta_k^1 + l\cos(\theta_k^1 + \theta_k^2) + l\cos(\theta_k^1 + \theta_k^2 + \theta_k^3)$$
(4)

Because the inverse problem of kinematics has infinity of solutions, let us consider some supplementary conditions imposed by optimal working, avoiding of blocking, a. s. o. (e.g.: $\theta_k^1 + \theta_k^2 + \theta_k^3 = \theta_k^* = \text{constant},$ $\theta_k^1 = \theta_k^2 = \theta_k^3$, $\theta_k^3 = \theta_k^* - \text{imposed}$, a. s. o.).

In some situations when passing from the point M_{k-1}^3 to the next point M_k^3 because of its advantageous position, movement of all the 3 elements is not necessary, as consumption economy of energy is possible.

If L_1 , L_2 , and L_3 note the elements having the length l, 1 logic movement status and 0 logic rest status, all the possible situations above mentioned are

 $(L_1 \ L_2 \ L_3) \rightarrow (0 \ 0 \ 0), (0 \ 0 \ 1), (0 \ 1 \ 0), (1 \ 0 \ 0),$

 $(0\ 1\ 1),\ (1\ 0\ 1),\ (1\ 1\ 0),\ (1\ 1\ 1).$

The proposed algorithm is presented in the following step sequences: STEP 1: It is read the robot parameter set:

$$l, \, \theta_0^i, \, \theta_{\min}^i, \, \theta_{\max}^i, \, i = 1, 2, 3.$$

STEP 2: k = 1

STEP 3: Are specified x_k^3 , y_k^3 , θ_k^* .

STEP 4: If
$$(x_k^3)^2 + (y_k^3)^2 \le 9l^2$$

then Jump to STEP 6

else Jump to STEP 5

STEP 5: It is displayed "IMPOSSIBLE TO REACH THE POINT"

Jump to STEP 3.

STEP 6: If $(x_k^3 - x_{k-1}^2)^2 + (y_k^3 - y_{k-1}^2) = l^2$

then Jump to A001

else Jump to STEP 7

STEP 7:

If
$$(x_k^3 - x_{k-1}^1)^2 + (y_k^3 - y_{k-1}^1)^2 = (2l\cos(\theta_{k-1}^3/2))^2$$

then Jump to A010

else Jump to STEP 8

STEP 8:
$$\alpha = a \tan \frac{\sin \theta_{k-1}^2 - \sin \theta_{k-1}^3}{1 + \cos \theta_{k-1}^2 + \cos \theta_{k-1}^3}$$

 $l_k^{123} = l \left[\cos \left(\theta_{k-1}^2 - \alpha \right) + \cos \alpha + \cos \left(\theta_{k-1}^3 + \alpha \right) \right]$
If $\left(x_k^3 \right)^2 + \left(y_k^3 \right)^2 = \left(l_k^{123} \right)^2$
then Jump to A100
else Jump to STEP 9

STEP 9: If
$$(x_k^3 - x_{k-1}^1)^2 + (y_k^3 - y_{k-1}^1)^2 \le 4l^2$$

then Jump to A011

else Jump to STEP 10
STEP 10:
$$l_k^{12} = 2l \cos(\theta_{k-1}^2 / 2)$$

If $(l_k^{12} - l)^2 \le (x_k^3) + (y_k^3) \le (l_k^{12} + l)^2$
then Jump to A101
else Jump to STEP 11

STEP 11:
$$l_k^{23} = 2l \cos(\theta_{k-1}^3 / 2)$$

If $(l_k^{23} - l)^2 \le (x_k^3) + (y_k^3) \le (l_k^{23} + l)$

$$(1)^{2}$$

then Jump to A110

else Jump to A111

A001:
$$\theta_k^1 = \theta_{k-1}^1; \ \theta_k^2 = \theta_{k-1}^2;$$

 $\theta_k^3 = a \tan \frac{x_k^3 - x_{k-1}^2}{y_k^3 - y_{k-1}^2} - (\theta_k^1 + \theta_k^2)$

Jump to STEP 12.

A010:
$$\theta_k^1 = \theta_{k-1}^1; \ \theta_k^3 = \theta_{k-1}^3;$$

 $\theta_k^2 = \theta_{k-1}^2 + \left(atan \frac{x_k^3 - x_{k-1}^1}{y_k^3 - y_{k-1}^1} - atan \frac{x_{k-1}^3 - x_{k-1}^1}{y_{k-1}^3 - y_{k-1}^1}\right)$

Jump to STEP 12.

A100:
$$\theta_k^2 = \theta_{k-1}^2; \theta_k^3 = \theta_{k-1}^3;$$

$$\theta_k^1 = \theta_{k-1}^1 + \left(a \tan \frac{x_k^3}{y_k^3} - a \tan \frac{x_{k-1}^3}{y_{k-1}^3}\right)$$

Jump to STEP 12.

A011:
$$\theta_k^1 = \theta_{k-1}^1$$

 $c_k^3 = \frac{(x_k^3 - x_{k-1}^1)^2 + (y_k^3 - y_{k-1}^1)^2 - 2l^2}{2l^2}$
 $\theta_k^3 = a \tan \frac{\sqrt{1 - (c_k^3)^2}}{c_k^3}$
 $s_k^{21} = \frac{x_k^3 - x_{k-1}^1}{2l} - \frac{\sin \theta_k^3 (y_k^3 - y_{k-1}^1)}{2l(1 + \cos \theta_k^3)}$
 $\theta_k^2 = a \tan \frac{s_k^{21}}{\sqrt{1 - (s_k^{21})^2}} - \theta_{k-1}^1$
Jump to STEP 12.
A101: $\theta_k^2 = \theta_{k-1}^2$

$$c_{k}^{3*} = \frac{\left(x_{k}^{3}\right)^{2} + \left(y_{k}^{3}\right)^{2} - \left(l_{k}^{12}\right)^{2} - l^{2}}{2ll_{k}^{12}}$$

$$\theta_{k}^{3*} = a \tan \frac{\sqrt{1 - \left(c_{k}^{3*}\right)^{2}}}{c_{k}^{3*}}; \ \theta_{k}^{3} = \theta_{k}^{3*} - \theta_{k-1}^{2} / 2$$

$$b = 2\left(l_{k}^{12} + l \cos \theta_{k}^{3*}\right);$$

$$c = l \sin \theta_{k}^{3*}$$

$$\theta_k^1 = 2a \tan \frac{b \pm \sqrt{b^2 - 4\left[\left(x_k^3\right)^2 - c^2\right]}}{2\left(x_k^3 + c\right)} - \frac{\theta_{k-1}^2}{2}$$

Jump to STEP 12.

A110:
$$\theta_k^3 = \theta_{k-1}^3$$

 $c_k^{23} = \frac{(x_k^3)^2 + (y_k^3)^2 - (l_k^{23})^2 - l^2}{2ll_k^{23}};$
 $\theta_k^2 = a \tan \frac{\sqrt{1 - (c_k^{23})^2}}{c_k^{23}} - \frac{\theta_{k-1}^3}{2};$
 $b = \frac{(x_k^3)^2 + (y_k^3)^2 - (l_k^{23})^2 + l^2}{2l};$
 $\theta_k^1 = 2a \tan \frac{x_k^3 \pm \sqrt{(x_k^3)^2 + (y_k^3)^2 - b^2}}{b + y_k^3};$

A111:
$$r_{k}^{1} = x_{k}^{3} - l \sin \theta_{k}^{*}; r_{k}^{2} = y_{k}^{3} - l \cos \theta_{k}^{*}$$

 $r_{k}^{3} = \frac{(r_{k}^{1})^{2} + (r_{k}^{2})^{2}}{2l}; c_{k}^{2} = \frac{r_{k}^{3}}{l};$
 $\theta_{k}^{2} = a \tan \frac{\sqrt{1 - (c_{k}^{2})^{2}}}{c_{k}^{2}}$
 $\theta_{k}^{1} = a \tan \left(2 \frac{r_{k}^{1} \pm \sqrt{(r_{k}^{1})^{2} + (r_{k}^{2})^{2} - (r_{k}^{3})^{2}}}{r_{k}^{2} + r_{k}^{3}} \right)$
 $\theta_{k}^{3} = \theta_{k}^{*} - (\theta_{k}^{1} + \theta_{k}^{2})$
Jump to STEP 12.

STEP 12:

If
$$\left(\theta_{\min}^{1}, \theta_{\min}^{2}, \theta_{\min}^{3}\right) \leq \left(\theta_{k}^{1}, \theta_{k}^{2}, \theta_{k}^{3}\right) \leq \left(\theta_{\max}^{1}, \theta_{\max}^{2}, \theta_{\max}^{3}\right)$$

then Jump to STEP 13

else Jump to STEP 16

STEP 13:
$$\Delta \theta_k^i = \theta_k^i - \theta_{k-1}^i$$
; $i = 1, 2, 3$.
 $x_k^1 = l \sin \theta_k^1$; $y_k^1 = l \cos \theta_k^1$
 $x_k^2 = l \sin \theta_k^1 + l \sin(\theta_k^1 + \theta_k^2)$;
 $y_k^2 = l \cos \theta_k^1 + l \cos(\theta_k^1 + \theta_k^2)$

STEP 14: *k* = *k*+1

STEP 16: "IMPOSIBLE DEPLACEMENT FOR *L_i*" ; Jump to STEP 3

3. Algorithm for blocked joints

Let us consider leg structure of the hexapod robot shown in Fig. 2. Because each joint has a sensor, we know exactly the values of each angle $(\theta_k^1, \theta_k^2, \theta_k^3)$ and we can detect if one or more actuators are blocked. The structure proposed by the authors for the synthesis of commands in fault conditions is presented in Fig. 4. We calculate a residual vector r(t) with the relation [6 - 8]:

$$r(t) = \begin{bmatrix} \Delta \theta_k^1 - \Delta \theta_k^{1r} \\ \Delta \theta_k^2 - \Delta \theta_k^{2r} \\ \Delta \theta_k^3 - \Delta \theta_k^{3r} \end{bmatrix} = \begin{bmatrix} r_k^1 \\ r_k^2 \\ r_k^3 \end{bmatrix}$$
(5)

where $\Delta \theta_k^i$ represent the calculated command for the *ith* joint and $\Delta \theta_k^{ir}$ is the real angular movement.

In this situation the algorithm calculates the position of the leg tip under fault condition.

For the synthesis of new commands it must be taken into consideration that the value of $\theta_k^i = \theta^{id}$ is a fixed (uncontrollable) parameter in (3) and (4).

To determine the rest commands we minimise the criterion (6) which represent Euclidean distance

$$J = \left\| \left(x_k^3 - x_k^{3*} \right)^2 + \left(y_k^3 - y_k^{3*} \right)^2 \right\|$$
(6)

where (x_k^3, y_k^3) represent the imposed (desired) position and $(x_k^{3^*}, y_k^{3^*})$ are the best possible position into proximity. To minimise (6) under fault conditions, we have the equations

$$\frac{\partial J}{\partial \theta_k^j} = 0 \tag{7}$$

where $j \neq i$, the *ith* actuator is blocked. If the distance is acceptable, the commands are validated and new angle values will become the inputs (Fig. 3) for the next step, else the algorithm jumps to the step 16.



Fig. 4 Structure for the synthesis of commands for the legs in fault conditions

4. Conclusions

This paper presents an algorithm which allows for the leg structure of a hexapod robot, under the terms of the actuator blocking occurrence during the walking, either a correct positioning (if it is possible) or a positioning in an acceptable proximity of the desired coordinates by minimising optimal criteria (by the adequate commands to the functional elements). A synthesis of the commands to a poli-articulated robotic structure (3 segments) is proposed in this paper. First, a workspace analysis is made, then the algorithm for the actuators in terms of a good walking (finding the optimal motions), is presented.

Because the problem of inverse kinematics has infinity of solutions, some supplementary conditions imposed by optimal work, the avoiding of blocking, a. s. o. are considered. Then, in terms of some leg segments blocking of hexapod robot, an algorithm is proposed. The structure proposed by the authors for command synthesis in fault conditions is based on a residual vector.

If all components of residual vector are null, one can consider that the hexapod robot leg is in good conditions. If one or more components of residual vector are different from zero the possibility to detect the presence of a blocked actuator and isolate it exists.

To determine the commands the criterion which represents the Euclidean distance is minimised.

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ŠEŠIAKOJO ROBOTO KOJŲ VALDYMO ALGORITMAS AVARINĖMIS SĄLYGOMIS

Reziumė

Straipsnyje pristatomas algoritmas šešiakojo roboto kojų struktūrai pozicionuoti (jeigu įmanoma), kai pavara žingsniavimo metu yra blokuojama, arba priimtinu tikslumu pozicionuoti pagal nustatytą koordinatę minimizuojant optimalų kriterijų (atitinkamomis komandomis funkciniams elementams). Pasiūlyta daugiašarnyrio trijų segmentų šešiakojo roboto kojoms duodamų komandų sintezė. Pirmiausia atliktas kojų galų darbo erdvės tyrimas, pasiūlytas pavarų valdymo, esant geroms žingsniavimo sąlygoms (nustatant optimalius judesius) ir esant keleto roboto kojų segmentų blokavimo sąlygoms, algoritmas.

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HEXAPOD LEG CONTROL ALGORITHM IN FAULT CONDITIONS

Summary

In this paper we present an algorithm which allows for the leg structure of a hexapod robot, in terms of the actuator blocking occurrence during walking, either a correct positioning (if it is possible) or a positioning in an acceptable proximity of the desired coordinates by minimising an optimal criteria (by the adequate commands to the functional elements). A synthesis of the commands to a poli-articulated hexapod robotic leg (3 segments) is proposing. First, a workspace analysis of the leg tip is made, then the algorithm for the actuators in the terms of a good walking (finding the optimal motions) and in terms of the blocking of some robotic leg segments is presented.

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АЛГОРИТМ УПРАВЛЕНИЯ НОГАМИ ГЕКСАПОДА В АВАРИЙНЫХ УСЛОВИЯХ

Резюме

В статье представлен алгоритм для позиционирования структуры ноги шестиногого робота при блокировании приводов во время ходьбы и для точного позиционирования (если возможно) или позиционирования с приемлемой точностью по заданной координате, минимизированием оптимального критерия (соответствующими командами для функциональных элементов). Предложен синтез команд ноги многошарнирного шестиногого робота (3 сегмента). В первую очередь проводится анализ рабочей среды стопы ноги, далее представлен алгоритм для приводов при хорошей ходьбе (определение оптимальных движений) и затем при блокировании некоторых сегментов ноги робота.