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Nonlinear random vibrations of a sandwich beam adaptive to electrorheological materials

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1. Introduction

An adaptive configuration is a structure whose response to excitation can be controlled in real time by modification of its mechanical properties through appropriately placed smart materials that are part of the structure [1-3]. Sensors are used to monitor the input and to measure the dynamic response of the structure. When the desired response quantities exceed the designated bounds, they are altered and reduced by modifying the structure's mechanical properties such as stiffness and damping through the controller with electric field inputs [4, 5]. Adaptive structures have demonstrated the potential to outperform conventional structures in various applications such as reduction of sound radiation from vibrating structures, position control for robotic applications, vibration control of large scale structures to seismic and wind excitations, etc. [2, 3].

The electrorheological (ER) material based adaptive structures have been theoretically and experimentally studied in [6 - 12]. Experiments were done to demonstrate the ability to change the dynamic characteristics of the beam – like structure by applying an electric field to the ER material that is sandwiched between two parallel thin beam layers. In most of these cases, the Ross Kerwin Ungar theory developed for viscoelastically damped sandwich beams was used to determine the natural frequencies and loss factors [6 - 12]. These theoretical and experimental studies were limited to linear response.

Nonlinear dynamic response and control capabilities of ER material adaptive sandwich beams exposed to random inputs are investigated in this study. The configuration of the sandwich beam considered is shown in Fig. 1 where the ER material is sandwiched between two parallel aluminum beams. The beams are simply supported at the ends and the sides are sealed to contain the ER material. The top beam is exposed to a stationary random pressure. The electric field is assumed to be uniform and applied across the two face beams. The response of the ER material is assumed to be in pre-yield region where a three-parameter solid viscoelastic model is used to characterize the material behavior [13 - 15]. The ER material constitutive equations are incorporated into the governing differential equations for nonlinear vibrations of a sandwich beam. The time histories of random pressure acting on the top beam are simulated as stationary and Gaussian random processes. A Galerkin - like procedure and numerical integration in time domain are utilized for the solution of the coupled nonlinear system of partial differential equations. Numerical results include modal frequencies, displacement response time histories, spectral densities and root mean square response. A comparison between theoretical and experimental predictions is presented.

2. Mathematical model of nonlinear sandwich beam

Consider a simply supported thin sandwich beam shown in Fig. 1. The two face beams are made of isotropic elastic material with thickness of h_1 each. The core layer is ER material of thickness h_2. A uniform electric field is applied across the two face beams that bound the ER material. In developing the governing equations of motion, the following assumptions are made: ER material behavior is in pre-yield region for all levels of electric field, the elastic constants E_f and G_f of the face beams are large in comparison to the storage modulus G_c of the ER core, there is no slipping the face beams and the ER core layer, the continuity of transverse displacement across the thickness of the sandwich beam is preserved. On the basis of these assumptions, the face parallel stresses in the core and their effects on the deformation of the sandwich configuration can be neglected. Thus, the ER material core caries no longitudinal normal or shear stress, but offers resistance only to transverse normal and shear stresses.

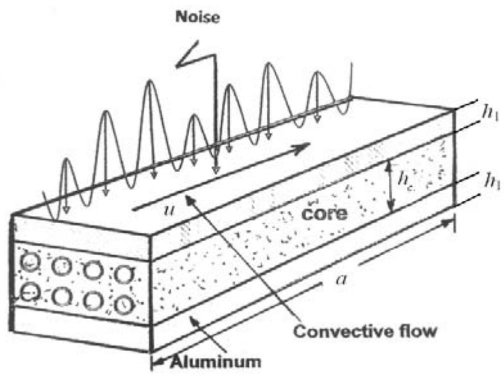

Fig. 1 Geometry of the sandwich beam

The ER material behavior in pre-yield region is represented by a three-parameter solid model [13 - 15]. The stress-strain relationship in time domain can be written as

$$\dot{\tau} + p_1\tau = q_1\gamma + q_2\dot{\gamma} \tag{1}$$

where

$$p_1 = \frac{K_1 + K_2}{C_1} \tag{2}$$

$$q_1 = \frac{K_1 K_2}{C_1} \tag{3}$$

$$q_2 = K_2 \tag{4}$$

in which τ is shear stress; γ is shear strain; K_1, K_2 and C_1 are functions of the electric field strength V.

The face layers of the sandwich construction are treated as thin beams with finite deformation effects taken into account. The random load P^r is applied to the top beam. Quantities referring to the top beam, bottom beam, and the ER material core layer are indicated by superscripts t, b, and c, respectively. The longitudinal inertia and the rotary inertia of the face beams are neglected. The details of the derivation of the equations of motion of the nonlinear vibrations of the sandwich plate and of the sandwich beam are presented in [16]. It has been shown that the governing equations of motion for nonlinear vibrations of a sandwich beam reduce to

$$N_x \frac{\partial^2 w}{\partial x^2} + \frac{2}{h} M_x \frac{\partial^2 e}{\partial x^2} - 2D_f \frac{\partial^4 w}{\partial x^4} - 2C_f \dot{w} + \frac{\partial V_x}{\partial x} + P^r = \left(2\rho_f h_1 + \rho_c h_2\right)\frac{\partial^2 w}{\partial t^2} \tag{5}$$

$$N_x \frac{\partial^2 e}{\partial x^2} + \frac{2}{h} M_x \frac{\partial^2 w}{\partial x^2} - 2D_f \frac{\partial^4 e}{\partial x^4} - 2C_f \dot{e} + \frac{4}{h_2}\left(E_c e + C_c \dot{e}\right) + P^r = 2\rho_f h_1 \frac{\partial^2 w}{\partial t^2} \tag{6}$$

$$\dot{V}_x + p_1 V_x = h\left(q_1 + q_2 \frac{\partial}{\partial t}\right)\left(\alpha + \frac{\partial w}{\partial x}\right) \tag{7}$$

where

$$N_x = \frac{h_1 E_f}{a\left(1 - v_f^2\right)} \int_0^a \left[\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial e}{\partial x}\right)^2\right] \tag{8}$$

$$M_x = D\left(\frac{\partial \alpha}{\partial x} + \frac{2}{h}\frac{\partial w}{\partial x}\frac{\partial e}{\partial x}\right) \tag{9}$$

$$V_x = D\left(\frac{\partial^2 \alpha}{\partial x^2} + \frac{2}{h}\left(\frac{\partial e}{\partial x}\frac{\partial^2 w}{\partial x^2} + \frac{\partial w}{\partial x}\frac{\partial^2 e}{\partial x^2}\right)\right) \tag{10}$$

The vertical displacement w, rotations e and α are defined as

$$w = \frac{w^t + w^b}{2}, e = \frac{w^t + w^b}{2}, \alpha = \frac{u^t + u^b}{h} \tag{11}$$

where $D = (1/2) h_1 h^2 E_f / \left(1 - v_f^2\right)$ is effective stiffness of sandwich beam, $D_f = E_f h_1^3 / 12\left(1 - v_f^2\right)$ is stiffness of face beams. The parameters E_f, C_f, E_c, C_c are elastic modulus of the face beams, damping coefficient of the face beams, modulus of elasticity of the core in transverse direction, damping coefficient of the core in transverse direction, respectively. The parameters a, h_1, h_2, ρ_f, ρ_c, v_f are beam length, thickness of the core, material density of face beams, material density of the core, Poison's ratio of the face beams, respectively. The combined thickness $h = h_1+$ $+h_2$.

Using modal expansion, the solutions of w, e, α can be written in the following form

$$w(x,t) = \sum_m A_m(t) sin \frac{m\pi x}{a} \tag{14}$$

$$e(x,t) = \sum_m Z_m(t) sin \frac{m\pi x}{a} \tag{15}$$

$$\alpha(x,t) = \sum_m B_m(t) cos \frac{m\pi x}{a} \tag{16}$$

Substituting these solutions into Eqs.(5)-(10), integrating Eqs. (5)-(7) each weighted by $sin\frac{k\pi x}{a}$, $sin\frac{k\pi x}{a}$, $cos\frac{k\pi x}{a}$ respectively, yields the following sets of coupled nonlinear differential equations for modal amplitudes A_k, Z_k, B_k.

$$\frac{-h_1 E_f \pi^4 k^2}{2a^4\left(1 - v_f^2\right)} A_k \sum_m \sum_n \left(A_m^2 + Z_m^2\right) n^2 + \frac{2}{h}\sum_m \sum_n B_m Z_n \Gamma_{mnk}^B + \frac{2}{h}\sum_m \sum_s \sum_n A_m Z_s Z_n \Gamma_{msnk}^M + D\left(\frac{k\pi}{a}\right)^3 B_k + \sum_m \sum_n A_m Z_n \Gamma_{mnk}^V - 2D_f\left(\frac{k\pi}{a}\right)^4 A_k - 2C_f \dot{A}_k + P_k = M_1 \ddot{A}_k$$

$$-2C_f \dot{A}_k + P_k = M_1 \ddot{A} \tag{17}$$

$$\frac{-h_1 E_f \pi^4 k^2}{2a^4\left(1 - v_f^2\right)} Z_k \sum_m \sum_n \left(A_m^2 + Z_m^2\right) n^2 + \frac{2}{h}\sum_m \sum_n B_m A_n \Gamma_{mnk}^B + \frac{2}{h}\sum_m \sum_s \sum_n A_m Z_s A_n \Gamma_{msnk}^M - 2D_f\left(\frac{k\pi}{a}\right)^4 Z_k - 2C_f \dot{Z}_k - \frac{4}{h_2}\left(E_c Z_{kl} + C_c \dot{Z}_{kl}\right) + P_k = M_2 \ddot{Z}_k \tag{18}$$

$$\widetilde{A}\dot{B}_k = -\widetilde{B}B_k - \frac{k\pi}{a}\left(q_1 A_k + q_2 \dot{A}_k\right) + \sum_m \sum_n \left(\dot{A}_m Z_n + A_m \dot{Z}_n + p_1 A_m Z_n\right)\Gamma_{mnk}^{Vx} \tag{19}$$

where

$$M_1 = 2\rho_f h_1 + \rho_c h_2 \tag{20}$$

$$M_2 = 2\rho_f h_1 \tag{21}$$

$$P_k = \frac{2}{a}\int_0^a P^r(x,t) sin \frac{k\pi x}{a} dx \tag{22}$$

The P_k are the generalized random pressure inputs. The coefficients $\Gamma_{mnk}^B, \Gamma_{msnk}^m, \Gamma_{mnk}^v$, $\widetilde{A}, \widetilde{B}, \Gamma_{mnk}^{vx}$ are given in [16].

For the case of uniform pressure distribution over the beam surface, the random pressure can be simulated as [15 - 17].

$$P^r\left(t_q\right) = Re\left[\sum_{r=0}^{M-1} A_r e^{i\varphi_r} e^{i\omega_r t_q}\right] \qquad (23)$$

where

$$A_r = \left[2S_p(\omega)\Delta\omega\right]^{\frac{1}{2}} \qquad (24)$$

in which $S_p(\omega)$ is the spectral density of random pressure $P^r(t)$ and φ_r are random phase angles.

3. Numerical results

Numerical results presented correspond to the ER material based sandwich beam shown in Fig. 1. The beam is assumed of unit width with a = 250 mm, h_1 = 0.4 mm and h_2 = 2.0 mm. The density of ER material is taken as ρ_c = 1060 kg/m^3. The face beams are assumed to be made of aluminum with the following material properties: ρ_f = =2768 kg/m^3, E_f=6.898x10^{10} N/m^2.

Table 1

Values of K_1, K_2, C_1, E_c, C_e

Parameter	Electric Field Strength (kV/mm)			
	$V = 0$	$V = 1$	$V = 2$	$V = 3$
K_1, N/m^2	282.0	68814.0	486090.0	1445240.0
K_2, N/m^2	587.0	181001.0	338828.0	652529.0
C_1, Ns/m^2	10.0	50.0	250.0	600.0
E_c, N/m^2	500.0	44265.9	106965.0	169788.0
C_c, Ns/m^2	8.0	6.0	4.0	2.0

The viscous damping coefficient c_f of the face beams is expressed in terms of modal damping coefficients as

$$\frac{c_f}{\rho_f h_1} = 2\xi_k\omega_k \qquad (25)$$

where ω_k are the natural frequencies of the face beams and ξ_k are nondimensional damping coefficients obtained from

$$\xi_k = \xi_1\frac{\omega_1}{\omega_k} \qquad (26)$$

In the present study, the damping coefficient corresponding to the fundamental mode is taken to be ξ_1 = 0.01.

The experimental data for values of parameters K_1, K_2, C_1 as functions of electric field strength V of the three-parameter solid model for ER materials in pre-yield region are not available. These parameters, given in Table 1, were calculated using the procedures suggested in [9 - 12]. The values for E_c and C_c were taken from [18].

The random pressure input P^r acting on the top beam of ER fluid sandwich structure is assumed to be a uniformly distributed band limited Gaussian white noise:

$$S_p(\omega) = S_0 = \frac{p_0^2}{\Delta\omega}10^{SPL/10} \qquad (27)$$

where p_0 = 2 10^{-5} N/m^2 is the reference pressure, SPL is the sound pressure level expressed in decibels, and $\Delta\omega = \omega_u - \omega_1$ is the frequency bandwidth. Simulations of random pressure used in the following numerical results are obtained from ω_u = 2π x 1000 rad/sec, ω_1 = 0 and several input sound pressure levels.

The natural modal frequencies for the first six sandwich beam modes are given in Table 2. These results correspond to the linear cases of a frequency domain solution and a time domain solution were the natural frequencies were extracted from the power spectral densities that were obtained by taking a Fast Fourier Transform (FFT) of the time domain displacement response. The agreement between the two solutions is relatively close. As can be observed from these results, the natural frequencies of the ER material sandwich beam increase with increasing electric field V. However, the amount of increase in natural frequencies with increasing electric field is much less for higher beam modes.

Table 2

ER fluid sandwich beam modal natural frequencies, Hz

Mode Number	Electric Field Strength, kV/mm							
	$V = 0$		$V = 1$		$V = 2$		$V = 3$	
	F.D.*	T.D.*	F.D.*	T.D.*	F.D.*	T.D.*	F.D.*	T.D.*
1	10.89	10.88	15.08	15.08	23.36	23.32	32.29	32.20
2	43.48	43.48	48.27	48.25	60.46	60.38	76.31	76.26
3	97.75	97.74	102.86	102.75	116.79	116.74	136.58	136.51
4	173.74	173.73	179.32	179.29	194.47	194.42	216.70	216.69
5	271.43	271.43	277.84	277.79	294.14	294.23	318.14	318.05
6	390.84	390.83	398.47	398.40	415.79	415.77	441.24	441.19

F. D. : Frequency Domain Soliution
T. D.: Time Domain Solution.

Transverse displacement response time histories for sound pressure inputs of 90 and 140 dB are presented in Figs. 2 and 3 for several values of electric field *V*. The root-mean-square (RMS) values are also included in these figures. All the vibration response calculations presented in this paper correspond to the mid-span location of the sandwich beam. It can be seen from these results that a significant reduction in vibration response can be achieved at 90 dB *SPL* inputs (mostly linear response) with increasing strength of the electric field that is being applied to the ER material core. As the electric field increases, the stiffness of the sandwich beam increases. This is clearly evident from these results where more response cycles appear. Thus, the dominant natural frequencies increased with increasing values electric field. For sound pressure levels of 140 dB, the sandwich beam response is highly nonlinear. The decrease in vibration amplitudes and the RMS values with increasing electric field is not as large as those obtained for the mainly linear case of 90 dB input. Thus, the ER material is not very effective when response is nonlinear and dominated by the in-plane stretching of the face beams. The ER material seems to be more effective in vibration control for linear vibrations where response is dominated by bending effects of the sandwich beam.

The displacement spectral densities for 90 dB input are given in Fig. 4 for several values of electric field *V*. These spectral densities were obtained by applying the FFT procedures to the response time histories. The results shown in Fig. 4 clearly indicate a decrease in vibration response and dominant peak shifting to higher frequencies with increasing electric field. The dominant peaks in Fig. 4 correspond to the odd modes of the sandwich beam. For a linear case, the even mode response will not appear at the mid-span location of the beam. Displacement spectral densities for the nonlinear case are presented in Fig. 5. The response characteristics tend to show a wide band process for all electric field levels. The distinct peaks that were evident in a linear case are now suppressed and all the modes are highly coupled. This type of behavior has been observed in many previous theoretical and experimental studies of nonlinear random vibrations of beams, shells, etc. [19 - 22].

Theoretical and experimental studies of ER material based adaptive sandwich beam have been presented in [8 - 10]. The following geometric and material parameters were used: $a = 381$ mm, $h_1 = 0.79$ mm, $h_2 = 0.5$ mm, $\rho_c = 1700$ kg/m^3, $\rho_f = 2700$ kg/m^3, $E_f = 7.0\text{x}10^{10}$ N/m^2.

The beam was actuated by a point force at $x = 115$ mm and the transverse displacements were measured and then calculated at the $x = 231$ mm. These results excerpted from [12] are shown in Fig. 6.

Corresponding to the same geometric and material properties frequency domain solutions of the linear case were obtained in this study. These results are shown in Fig.7. It can be seen from Figs. 6 and 7 that the general trend of vibration response of the ER material sandwich beam is similar to that obtained in [10-12]. Displacement response for electric field input of 3.5 kV/mm is given in Fig. 8 for a direct comparison between the results obtained in this study and those given in [12]. It can be seen that the trend of frequency shift and the decrease in vibration amplitude obtained in this study agrees with the results presented in [12].

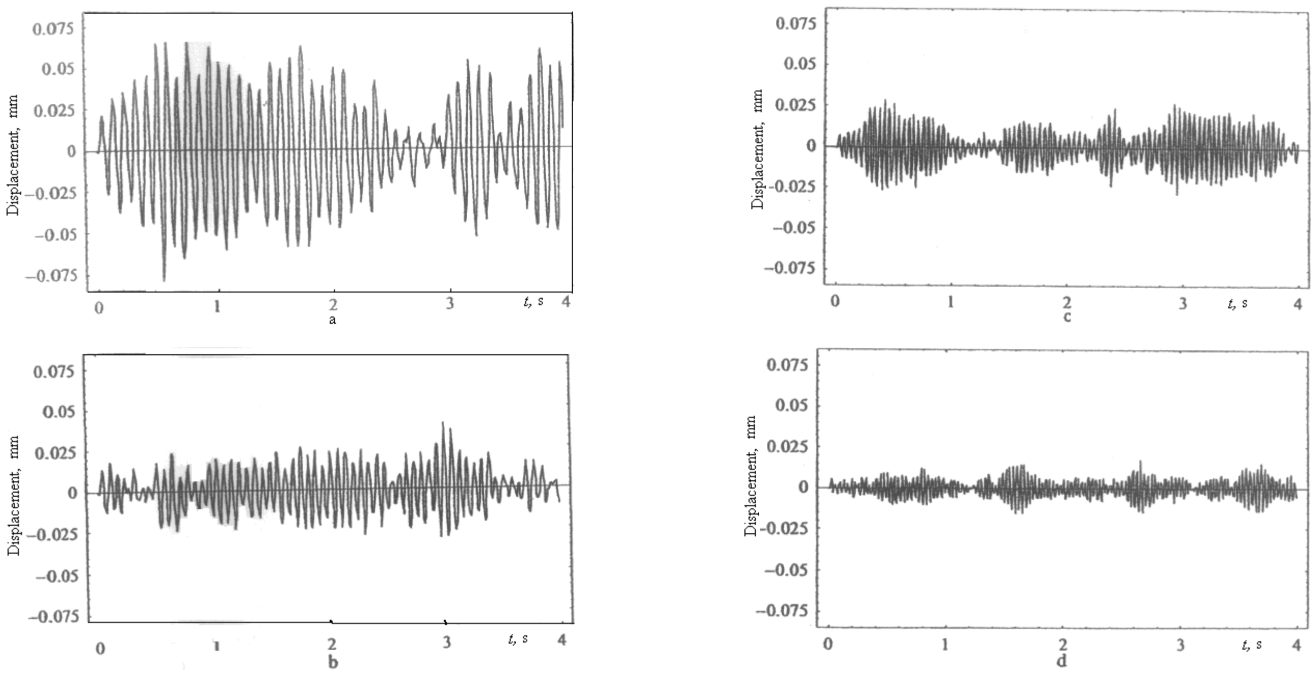

Fig. 2 Displacement response time history for SPL = 90 dB: (a) $V = 0$ kV/mm, RMS = 0.036 mm; (b) $V = 1$ kV/mm, RMS = 0.017; (c) $V = 2$ kV/mm, RMS = 0.09 mm; (d) $V = 3$ kV/mm, RMS=0.006 mm

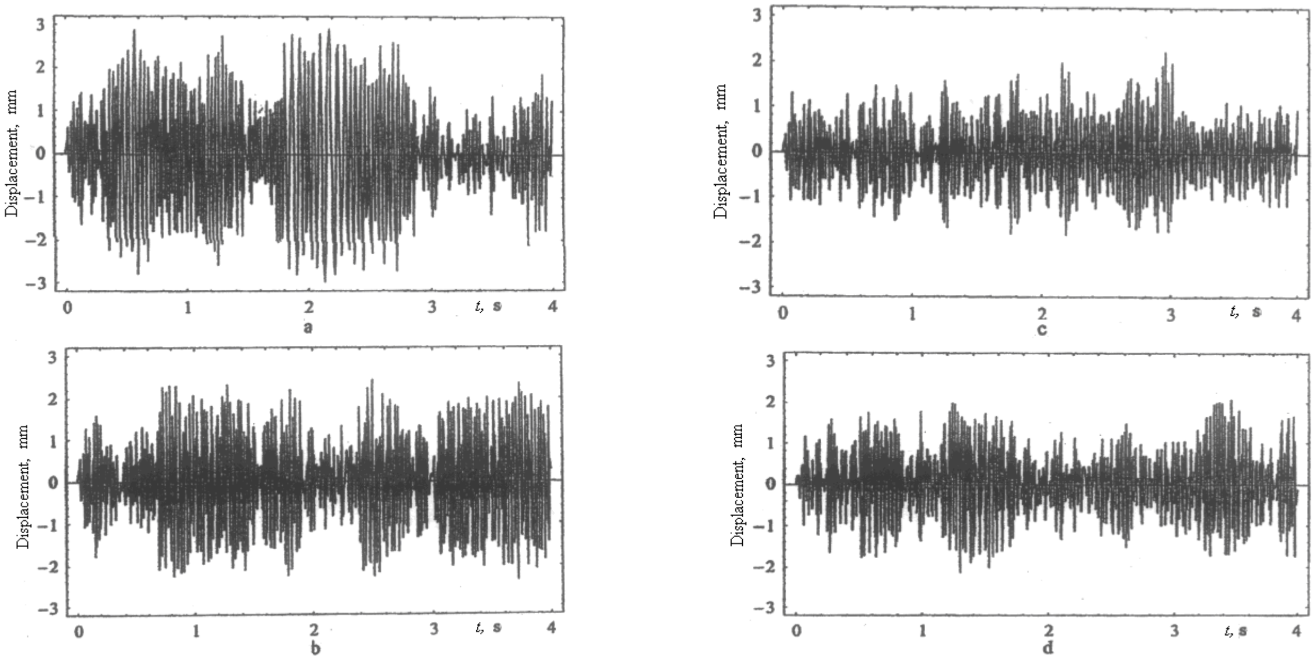

Fig. 3 Displacement response time history for SPL = 140 dB: (a) $V = 0$ kV/mm, RMS = 1.37 mm; (b) $V = 1$ kV/mm, RMS = 1.05; (c) $V = 2$ kV/mm, RMS = 0.97 mm; (d) $V = 3$ kV/mm, RMS = 0.88 mm

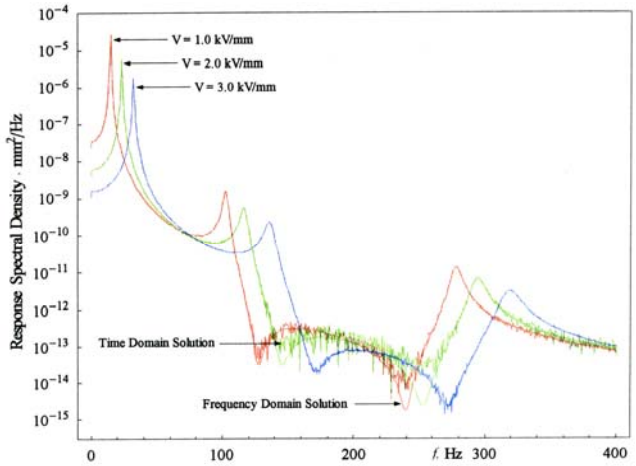

Fig. 4 Displacement spectral density at SPL = 90 dB

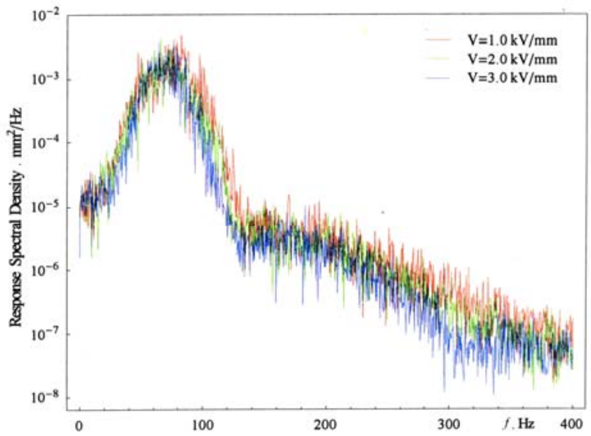

Fig. 5 Displacement spectral density at SPL = 140 dB

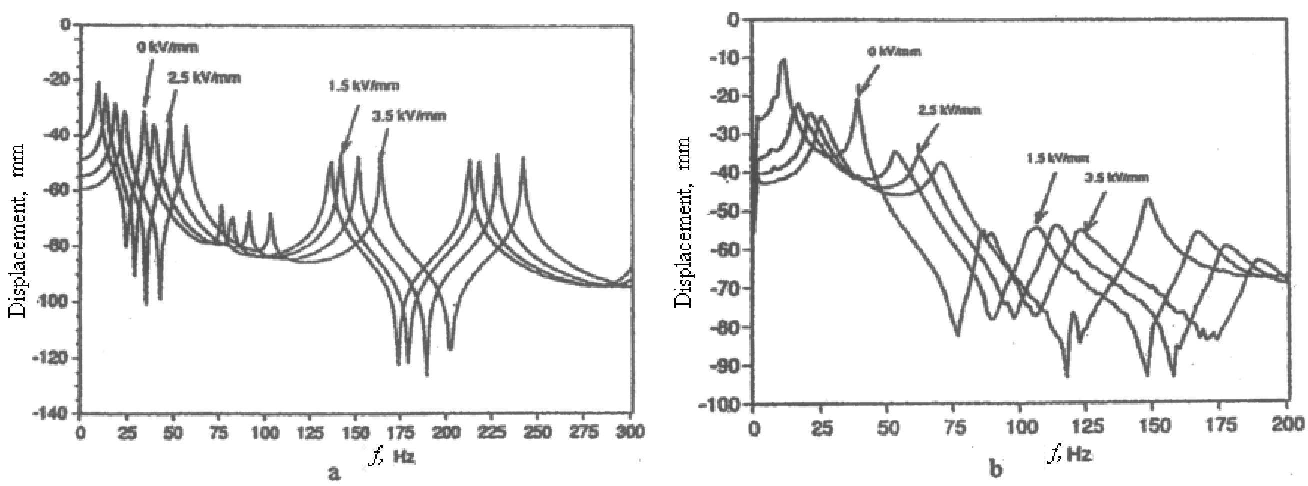

Fig. 6 Theoretical (a) and experimental (b) frequency response of adaptive beam subjected to actuation at one location, for electric fields of 0, 1.5, 2.5, 3.5 kV/mm, by Yalcintas, et al

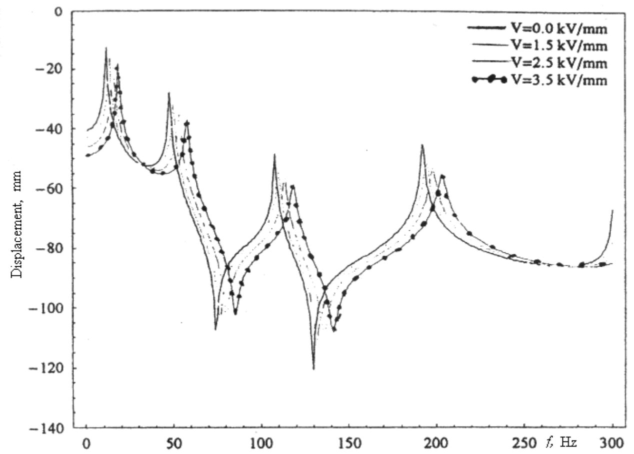

Fig. 7 Frequency response of adaptive beam for electric fields of 0, 1.5, 2.5, 3.5 kV/mm

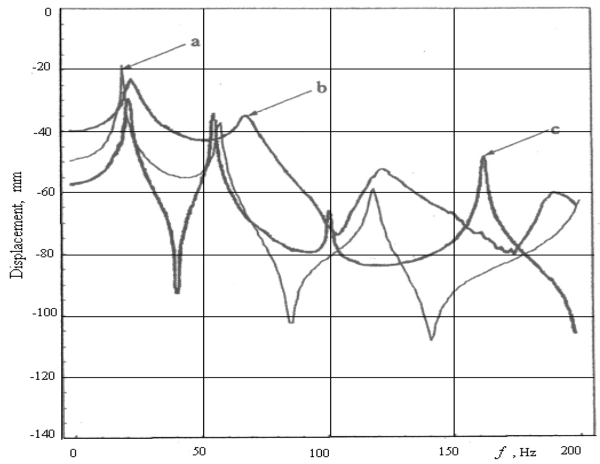

Fig. 8 Comparison of transverse displacements at a point on the ER material adaptive beam for the electric field of 3.5 kV/mm: (a) result from model used in this study; (b) experimental result by Yalcintas, et al.; (c) theoretical result by Yalcintas, et al.

4. Conclusions

Nonlinear dynamic response and vibration control capabilities of ER material based adaptive sandwich beams were investigated. For low and moderate random force inputs where vibration response is mainly linear, displacement amplitudes and RMS values were significantly reduced with the application of an electric field to the beam. Modal frequencies and resonant peaks in the power spectral density shifted to higher frequencies with increasing electric field. This indicates that the sandwich bean became more stiff upon application of the electric field. It has been shown that the ER material based sandwich beam has variable and controllable characteristic and the core layer adaptive to stiffness and damping variations could be effective in suppressing random vibrations. The results presented in this study qualitatively agreed with theoretical and experimental observations obtained by other investigators. For high input levels, the response is nonlinear and the distinct resonant peaks that were evident for linear response are no longer present in the displacement spectral densities. The modes are strongly coupled and the dominant response peaks coalesce into a broad band type behavior. Suppression of vibrations by ER material action is not very effective for nonlinear response cases. Further improvements in the understanding of ER material behavior and analytical modeling is needed for application to vibration control of structures that exhibit large and nonlinear vibrations.

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NAUDOJANT ELEKTROREOLOGINES MEDŽIAGAS PAGAMINTŲ SLUOKSNIUOTŲ STRYPŲ NETIESINIAI ATSITIKTINIAI VIRPESIAI

R e z i u m ė

Pateikti naudojant elektroreologines medžiagas pagamintų sluoksniuotų strypų netiesinių dinaminių procesų ir virpesių kontrolės analitinių tyrimų rezultatai. Prieštakio būvio elektroreologinių medžiagų savybėms aprašyti panaudotas trimatis erdvinis klampus ir tamprus modelis. Judėjimo lygčių sistema sudaryta derinant netiesines tampraus plono strypo lygtis su elektroreologinių medžiagų būvio lygtimis. Atsitiktinės apkrovos, veikiančios strypą laikui bėgant, aprašytos naudojant stacionarių atsitiktinių procesų skaičiavimus. Jungtinių netiesinių dalinių diferencialinių lygčių sprendimui panaudotas Galiorkino metodas ir skaitmeninis intregravimas laike. Keičiant elektrinį lauką, ištirtas elektroreologinės medžiagos šerdies kaupimo modulio pasikeitimo efektas tiesiniu ir netiesiniu atveju. Skaitiniai rezultatai apima pasiskirstymą laike, vidutinius kvadratinius dydžius ir galios spektro tankį. Parodytas elektroreologinių medžiagų dažnio modos keitimasis, virpesių amplitudės nuslopinimas ir sluoksniuotiems strypams gaminti naudojamų elektroreologinių medžiagų aktyvi virpesių kontrolė.

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NONLINEAR RANDOM VIBRATIONS OF A SANDWICH BEAM ADAPTIVE TO ELECTRORHEOLOGICAL MATERIALS

S u m m a r y

An analytical study of nonlinear dynamic response and vibration control capabilities of Electrorheological (ER) materials based adaptive sandwich beam is presented. A three-parameter solid viscoelastic model is used to describe ER material behaviour in pre-yield regime. The governing equations of motion are developed by combining the nonlinear elastic thin beam equations and the ER material constitutive relations. The random loads acting on the beam are developed in time domain utilizing simulation procedures of stationary random processes. A Galerkin-like approach and numerical integration in time domain are used to solve the coupled nonlinear partial differential equations. The effect of changes of core ER material storage modulus on vibration response to applied electric field is investigated for linear and nonlinear cases. Numerical results include displacement response time histories, RMS values and power spectral densities. Shift in modal frequencies, suppression of response amplitudes and active vibration control capabilities of ER material based adaptive sandwich beam are demonstrated.

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НЕЛИНЕЙНЫЕ СЛУЧАЙНЫЕ КОЛЕБАНИЯ СЛОИСТНЫХ СТЕРЖНЕЙ ИЗ ЭЛЕКТРОРЕОЛОГИЧЕСКИХ МАТЕРИАЛОВ

Р е з ю м е

Представлены аналитические исследования нелинейных динамических процессов и контроль колебаний электрореологических материалов, применяемых в слоистных стержнях. Трехмерная вязкоупругая модель применена для описания свойств электрореологических материалов в предтекучем режиме. Составлена система уравнений движения комбинируя нелинейные уравнения упругого тонкого стержня с отношениями состояния электрореологических материалов. Случайные нагрузки, действующие на стержнь во времени, описаны используя расчеты стационарных случайных процессов. Метод Галеркина и численное интегрирование во времени применено для решения объединенных нелинейных полудифференциальных уравнений. Исследован эффект изменения накопителя модуля сердцевины электрореологических материалов для линейного и нелинейного случая. Численные результаты содержат в себе сдвиг во времени, РМС величину и мощность спектральной плотности. Показано изменение в модах частот, подавление амплитуды и активная контроль колебаний электрореологических материалов, применяемых в слоистых стержнях.

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