Magnetic fluid based squeeze film between curved rough circular plates

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Nomenclature

a - radius of the circular plate; p - lubricant pressure; B - curvature parameter of the upper plate; C - curvature parameter of the lower plate; H - magnitude of the magnetic field; $p = -\dfrac{h_0^3 p}{\mu \dot{h}_0 a^2}$ - dimensionless pressure; W - load carrying capacity; $\overline{W} = -\dfrac{h_0^3 W}{\mu \dot{h}_0 a^4}$ - dimensionless load carrying capacity; Δt - response time; $\Delta T = \dfrac{\Delta t W h_0^2}{\pi \mu a^4}$ - non-dimensional response time; α - mean of the stochastic film thickness; σ - standard deviation of the stochastic film thickness; σ^2 - variance; ε - measure of symmetry of the stochastic random variable; $\overline{\sigma} = \sigma/h_0$; $\overline{\alpha} = \alpha/h_0$; $\overline{\varepsilon} = \varepsilon/h_0^3$; $R = r/a$; φ - inclination angle; μ - absolute viscosity of the lubricant; $\overline{\mu}$ - magnetic susceptibility; μ_0 - permeability of the free space; $\mu* = \dfrac{-\mu_0 \overline{\mu} k h^3}{\mu \dot{h}_0}$ - magnetization parameter.

1. Introduction

F.R. Archibald [1] discussed the behavior of squeeze film between various geometrical configurations of flat surfaces. D.F. Hays [2] presented the squeeze film phenomena between curved plates considering curvature of the sine form and keeping minimum film thickness as constant. P.R.K. Murti [3] analyzed the behavior of squeeze film trapped between curved circular plates describing the film thickness by an expression of an exponential function. He based his analysis on the assumption that the central film thickness instead of minimum film thickness as assumed by D.F. Hays, was kept constant. It was established that the load carrying capacity rose sharply with curvature in the case of concave pads. J.L. Gupta and K.H Vora [4] studied the corresponding problem in the case of annular plates. In this analysis the lower plate was considered to be flat. M.B. Ajwaliya [5] dealt with this problem of squeeze film behavior taking the lower plate also to be curved. Two types of geometries namely, annular and rectangular were investigated by H. Wu [6, 7] concerning the squeeze film performance when one of the surface was porous faced. Various bearing configurations such as circular, annular, elliptical, rectangular and conical were analyzed by J. Prakash and S.K. Vij [8]. In this article comparison was made between the squeeze film behavior of various geometries of equivalent surface area. It was established that the circular plates recorded the highest transient load carrying capacity, other parameters remaining the same.

Conventional lubricants were used in all the above studies. The application of a magnetic fluid as lubricant was investigated by P.D.S. Verma [9]. The magnetic fluid consisted of fine surfactant and magnetically passive solvent. Subsequently, the magnetic fluid based squeeze film behavior between porous annular disks was presented by M.V. Bhat and G.M. Deheri [10], where in, they concluded that the application of magnetic fluid lubricant enhanced the performance of the squeeze film. However, here the plates were taken to be flat. But in actual practice the flatness of the plate does not endure owing to elastic, thermal and uneven wear effects. With this end in view M.V. Bhat and G.M. Deheri [11] dealt with the behavior of a magnetic fluid based squeeze film between curved circular plates. The magnetic fluid based squeeze film between curved plates lying along the surfaces determined by secant and hyperbolic function was investigated by R.M. Patel and G.M Deheri [12, 13]. It was found that the application of magnetic fluid lubricant improved the performance of the squeeze film.

It is a well-known fact that after having some run-in and wear the bearing surfaces develop roughness. The roughness often appears random and disordered and does not seem to follow any particular structural pattern. The randomness and the multiple roughness scales both contribute to the complexity of the surface geometrical structure. Invariably, it is this complexity which contributes to most of the problems in studying friction and wear. The random character of the surface roughness was recognized by several investigators who employed a stochastic approach to mathematically model the roughness of the bearing surfaces (S.T. Tzeng and E. Seibel [14], H. Christensen and K.C. Tonder [15 - 17]). K.C. Tonder [18] analyzed theoretically the transition between surface distributed waviness and random roughness. S.T. Tzeng and E. Seibel [14] used a beta probability density function for the random variable characterizing the roughness. This distribution is symmetrical in nature with zero mean and approximates the Gaussian distribution to a good degree of accuracy for certain special cases. H. Christensen and K.C. Tonder [15 - 17] further developed and modified this approach and proposed a comprehensive general analysis both for transverse as well as longitudinal surface roughness based on a general probability density function. H. Christensen and K.C. Tonders method developed the frame work to study the effect of surface roughness on the performance of bearing system in a number of investiga-

tions. (L.L Ting [19], J. Prakash and K. Tiwari [20], B.L. Prajapati [21], S.K. Guha [22], J.L. Gupta and G.M. Deheri [23]). In all these analysis the probability density function for the random variable characterizing the surface roughness was assumed to be symmetric with mean of the random variable equal to zero. However, in general, this may only be true to the first approximation. In practice, due to nonuniform rubbing of the surfaces the distribution of surface roughness may indeed be asymmetrical. Thus, with this idea in view, P.I. Andharia, J.L. Gupta and G.M. Deheri [24] discussed the effect of transverse surface roughness on the performance of a hydrodynamic squeeze film in a spherical bearing using the general stochastic analysis. It was observed that the effect of transverse surface roughness on the performance of the bearing was considerably adverse.

Here we propose to analyze magnetic fluid based squeeze film between two curved transversely rough circular plates, where in, the upper plate lies along the surface determined by hyperbolic function while, the lower plate lies along the surface governed by secant function.

2. Analysis
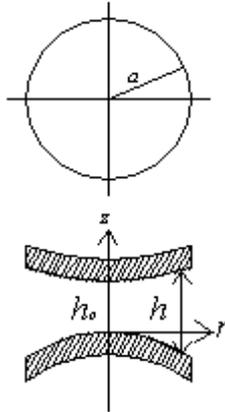
Configuration of the bearing is displayed in Fig. 1.

Fig. 1 Bearing configuration

The bearing surfaces are considered to be transversely rough. The thickness $h(x)$ of the lubricant film is

$$h(x) = \overline{h}(x) + h_s \tag{1}$$

where $\overline{h}(x)$ is the mean film thickness while h_s is the deviation form the mean film thickness characterizing the random roughness of the bearing surfaces. The deviation h_S is assumed to be stochastic in nature and described by the probability density function

$$f(h_s) - c \leq h_s \leq c \tag{2}$$

where c is the maximum deviation from the mean film thickness. The mean α, the standard deviation σ and the parameter ε which is the measure of symmetry associated with random variable h_s are governed by the relations

$$\alpha = E(h_s) \tag{3}$$

$$\sigma^2 = E\left[(h_s - \alpha)^2 \right] \tag{4}$$

and

$$\varepsilon = E\left[(h_s - \alpha)^3 \right] \tag{5}$$

where E denotes the expected value defined by

$$E(R) = \int_{-c}^{c} R f(h_s) dh_s \tag{6}$$

We assume the upper plate lying along the surface determined by

$$Z_u = h_0 \left[\frac{1}{1+Br} \right]; \quad 0 \leq r \leq a \tag{7}$$

approaching with normal velocity $\dot{h}_0 = \dfrac{dh_0}{dt}$, to the lower plate lying along the surface

$$`Z_l = h_0 \left[sec(-Cr^2) - 1 \right]; \quad 0 \leq r \leq a \tag{8}$$

where h_0 is the central distance between the plates, B and C are the curvature parameters of the corresponding plates. The central film thickness $h(r)$ then is defined by

$$h(r) = h_0 \left[\frac{1}{1+Br} - sec(-Cr^2) + 1 \right] \tag{9}$$

Axially symmetric flow of the magnetic fluid between the plates is taken into consideration under an oblique magnetic field

$$\overline{H} = \left(H(r)cos\varphi(r,z), 0, \ H(r)sin\varphi(r,z) \right) \tag{10}$$

whose magnitude H vanishes at $r = a$; for instance; $H^2 = ka(a-r), 0 \leq r \leq a$, where k is a suitably chosen constant so as to have a magnetic field of required strength, which suits the dimensions of both the sides. The direction of the magnetic field plays a significant role since $\overline{H}$ has to satisfy the equation

$$\nabla \overline{H} = 0; \quad \nabla \times \overline{H} = 0 \tag{11}$$

Therefore, $\overline{H}$ arises out of a potential function and the inclination angle φ of the magnetic field $\overline{H}$ with the lower plate is determined by

$$cot\varphi \frac{\partial \varphi}{\partial r} + \frac{\partial \varphi}{\partial z} = \frac{1}{2(a-r)} \tag{12}$$

whose solution is determined from the equations

$$c_1^2 cosec^2\varphi = a - r; \quad z = -2c_1 \sqrt{\left(a - c_1^2 - r \right)} \tag{13}$$

where c_1 is a constant of integration.

The modified Reynolds equation governing the film pressure p can be obtained as [12, 23,25]

$$\frac{1}{r}\frac{d}{dr}\left[rg(h)\frac{d}{dr}\left(p - 0.5\mu_0\overline{\mu}H^2\right)\right] = 12\mu\dot{h}_0 \qquad (14)$$

where

$$g(h) = h^3 + 3\sigma^2 h + 3h^2\alpha + 3h\alpha^2 + 3\sigma^2\alpha + \alpha^3 + \varepsilon \qquad (15)$$

Introducing the nondimensional quantities

$$\overline{h} = h/h_0; \ R = r/a; \ \mu_* = \frac{-\mu_0\overline{\mu}kh^3}{\mu\dot{h}_0}; \ P = -\frac{h_0^3 p}{\mu a^2 \dot{h}_0};$$

$$\boldsymbol{\sigma} = \sigma/h_0; \ \boldsymbol{\varepsilon} = \varepsilon/h_0^3; \boldsymbol{B} = B;, \ \boldsymbol{C} = Ca^2 \qquad (16)$$

and solving the concerned Reynolds equation with the associated boundary conditions

$$P(1)=0; \ \frac{dP}{dR} = -\frac{\mu_*}{2} \quad \text{at } R=0 \qquad (17)$$

we get the nondimensional pressure distribution as

$$P = \frac{\mu_*}{2}(1-R) + 6\int_R^1 \frac{R}{G(h)}dR \qquad (18)$$

where

$$G(h) = \overline{h}^3 + 3\overline{h}^2\alpha + 3\sigma^2\overline{h} + 3\overline{h}\alpha^2 + \varepsilon + 3\sigma^2\alpha + \alpha^3$$

The dimensionless load carrying capacity is given by

$$\overline{W} = -\frac{Wh_0^3}{2\pi\mu a^4\dot{h}_0} = \frac{\mu_*}{12} + 3\int_0^1 \frac{R^3}{G(h)}dR \qquad (19)$$

where the load carrying capacity W is obtained from the relation

$$W = 2\pi\int_0^a rp(r)dr \qquad (20)$$

The response time in dimensionless form becomes

$$\Delta T = \frac{\Delta tWh_0^2}{\pi\mu a^4} = \overline{W}\int_{\overline{h}_1}^{\overline{h}_2}\frac{1}{G(h)}d\overline{h} \qquad (21)$$

where

$$\overline{h}_1 = \frac{h_1}{h_0}; \ \overline{h}_2 = \frac{h_2}{h_0}. \qquad (22)$$

3. Results and discussion

Expression for dimensionless pressure p, load

carrying capacity $\overline{W}$ and response time ΔT are presented in Eqs. (18), (19) and (21) respectively. It is clearly seen that these performance characteristics depend on several parameters such as μ_*, σ, α, ε, B and C. These parameters, respectively, describe the effect of magnetic fluid lubricant, roughness parameters and curvature parameters.

The Eq. (19) suggests that the load carrying capacity increases by $.083 \ \mu_*$. Setting the roughness parameters σ, α and ε to be zero one obtains the performance of a magnetic fluid based squeeze film trapped between curved circular plates lying along the surfaces determined by hyperbolic function and secant function. Further, taking the magnetization parameter as zero this investigation reduces to the study of the squeeze film behavior between curved circular plates.

Figs. 2-6 present the variation of load carrying capacity $\overline{W}$ with respect to the magnetization parameter μ_* for various values of roughness parameters σ, ε and α and the curvature parameters B and C respectively. These figures indicate that the load carrying capacity increases significantly with respect to the magnetization parameter. Further, among the roughness parameters the combined effect of the magnetization parameter and skewness is more pronounced.
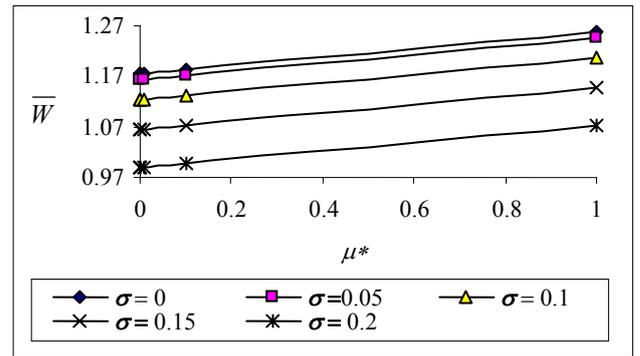

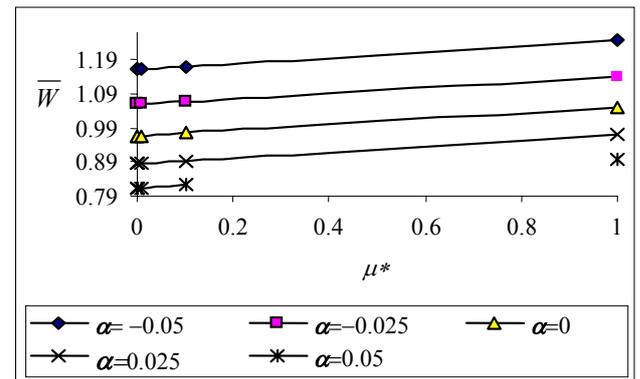Fig. 2 Load carrying capacity with respect to μ_* and σ

Fig. 3 Load carrying capacity with respect to μ_* and α

Figs. 7-9 describe the effect of the standard deviation associated with roughness on the distribution of load carrying capacity. It can be easily observed from these figures that the effect of the standard deviation is considerably adverse, in the sense that the load carrying capacity decreases considerably. This negative effect of σ is littlebit less with respect to the upper plate curvature parameter.

In Figs. 10-12 one can have the effect of variance on the variation of load carrying capacity. These figures tell that α (+ve) decreases the load carrying capacity while α (-ve) increases the load carrying capacity. Further, it is suggested that the combined effect of the upper plate curvature parameter and the negative variance is significantly positive.
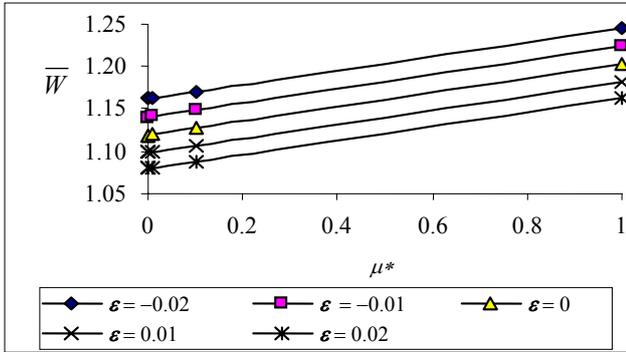

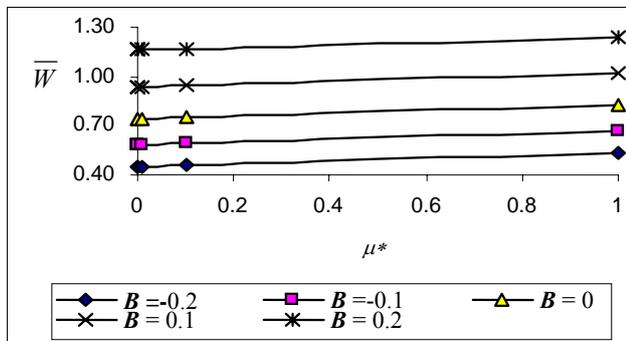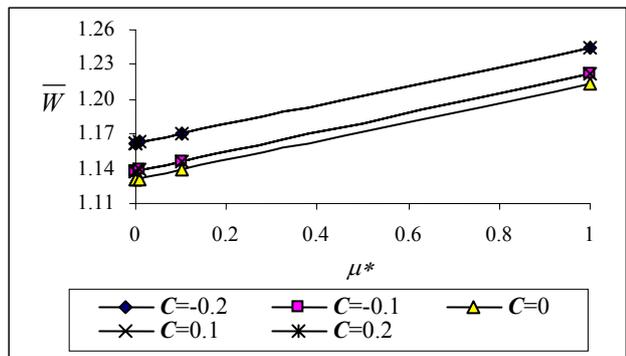Fig. 4 Load carrying capacity with respect to $\mu*$ and ε

Fig. 5 Load carrying capacity with respect to $\mu*$ and B

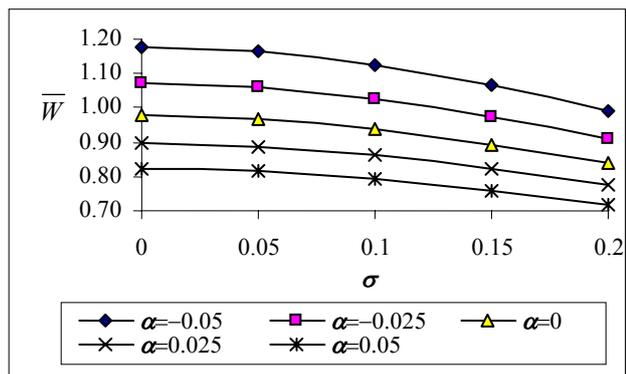Fig. 6 Load carrying capacity with respect to $\mu*$ and C

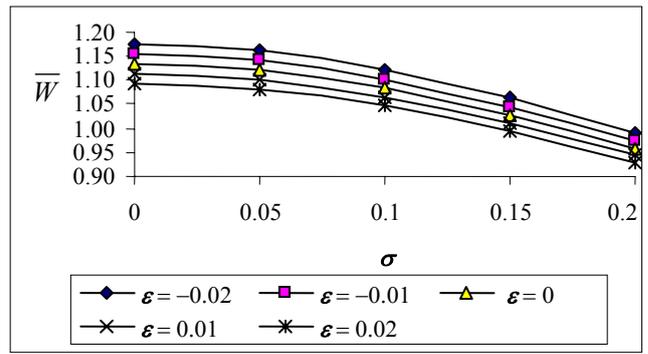Fig. 7 Load carrying capacity with respect to σ and α

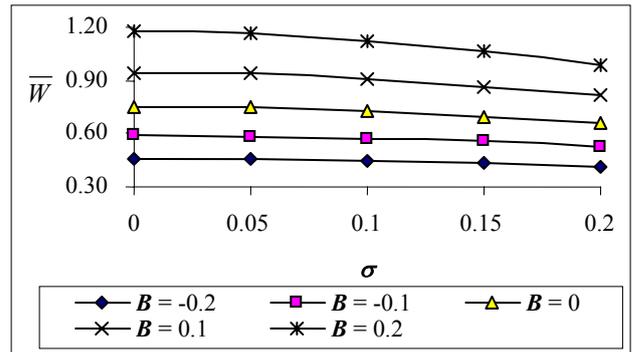Fig. 8 Load carrying capacity with respect to σ and ε

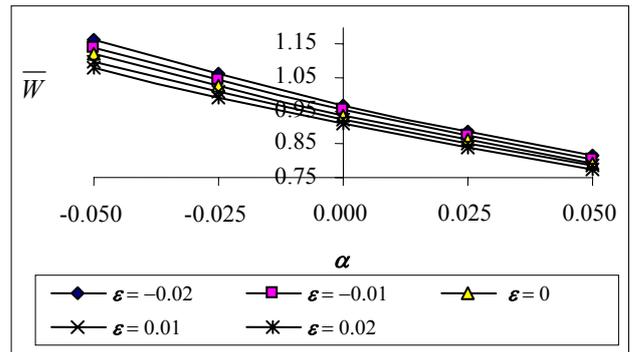Fig. 9 Load carrying capacity with respect to σ and B

Fig. 10 Load carrying capacity with respect to α and ε
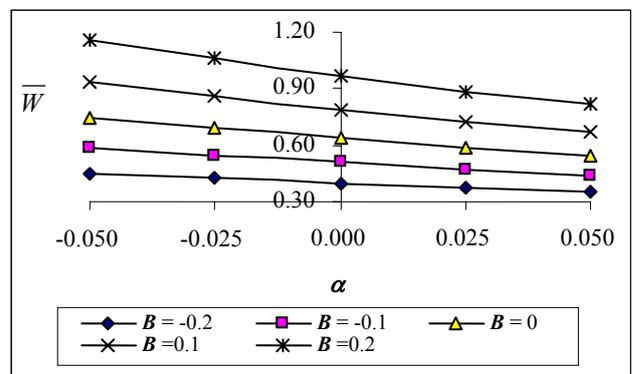

Fig. 11 Load carrying capacity with respect to α and B

Figs. 13-14 show the effect of skewness on the distribution of load carrying capacity. As in the case of variance here also, ε (+ve) decreases the load carrying capacity while the load increases with respect to ε (-ve). In addition, there is the symmetric distribution of the load

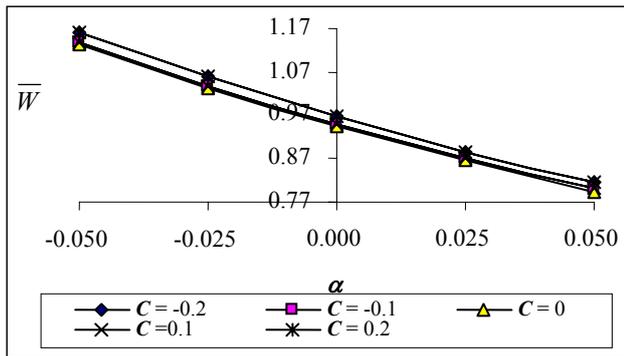carrying capacity with respect to the lower plate curvature parameter.

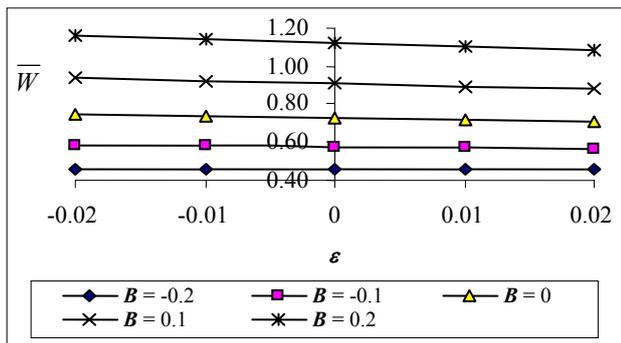Fig. 12 Load carrying capacity with respect to α and C

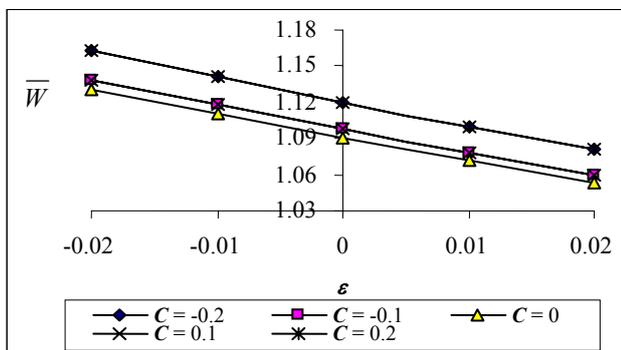Fig. 13 Load carrying capacity with respect to ε and B

Fig. 14 Load carrying capacity with respect to ε and C

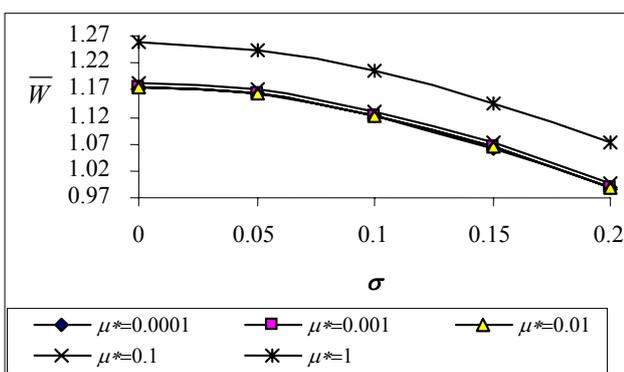The effect of $\mu*$ is almost negligible up to the value 0.01 as shown in the Fig. 15.

Fig. 15 Load carrying capacity with respect to σ and $\mu*$

Interestingly, it is noted that the rate of increase in load carrying capacity with respect to the magnetization parameter is more with respect to lower plate's curvature parameter as compared to the upper plate's curvature parameter. Lastly, the response time $\varDelta T$ follows almost the trends of load carrying capacity.

4. Conclusion

This article reveals that by properly choosing the curvature parameters of both the plates and the magnetization parameter the performance of the bearing system can be enhanced considerably in the case of negatively skewed roughness, especially, when the negative variance is involved. Therefore, this study makes it mandatory that the roughness must be accounted for while designing the bearing system.

Acknowledgement

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MAGNETOREOLOGINĖ SKYSČIO PLĖVELĖ TARP IŠGAUBTŲ APVALIŲ NELYGIŲ PLOKŠTELIŲ

R e z i u m ė

Straipsnyje analizuojama magnetoreologinio skysčio plėvelė, esanti tarp dviejų apvalių išgaubtų, nelygiu paviršiumi plokštelių, kai viršutinės išlenktos plokštės paviršius, apibūdinamas hiperboline funkcija, suartėja su nejudama išlenkta apatine plokšte, kurios paviršius apibūdinamas sekantine funkcija.

Čia kaip tepamoji medžiaga išoriniame magnetiniame lauke, pasvirusiame radialiosios ašies atžvilgiu, naudojamas magnetoreologinis skystis. Guolio paviršiaus radialinis šiurkštis modeliuojamas atsitiktiniu stochastiniu kintamuoju be nulinės jo pastoviosios ir kintamosios dedamųjų reikšmių. Procesą aprašanti Reinoldso lygtis suvidurkinta atsitiktinio šiurkščio parametro atžvilgiu. Siekiant nustatyti slėgio pasiskirstymą, nedimensinė diferencialinė lygtis išspręsta esant tam tikroms kraštinėms sąlygoms, gauti duomenys panaudoti sudarant laikomosios galios lygtį ir nustatant sistemos pereinamąjį laikotarpį. Gauti rezultatai pateikiami grafiškai. Jie parodė, kad tokios guolių sistemos charakteristikos labai pagerėjo, palyginti su guolių sistemomis, tepamomis įprastinėmis medžiagomis. Pastebėta, kad slėgis, sistemos laikomoji galia ir pereinamasis laikotarpis didėja didėjant įmagnetinimo parametrui. Šis tyrimas atskleidė guolio specifinius defektus, atsirandančius dėl skersinių paviršiaus nelygumų, parodė, kad, tinkamai parenkant abiejų plokštelių kreivumo parametrus, galima efektyviai pagerinti guolio eksploatacines savybes esant neigiamai šiurkščio kintamos dedamosios pasiskirstymo asimetrijai.

G. M. Deheri, N. D. Abhangi

MAGNETIC FLUID BASED SQUEEZE FILM BETWEEN CURVED ROUGH CIRCULAR PLATES

S u m m a r y

It has been sought to analyze a magnetic fluid based squeeze film behavior between two curved rough circular plates when the curved upper plate lying along the surface determined by hyperbolic function approaches the stationary curved lower plate along the surface governed by secant function.

The lubricant used is a magnetic fluid in the presence of an external magnetic field oblique to the radial axis. The transverse roughness of the bearing surfaces is modeled by a stochastic random variable with nonzero mean, variance and skewness. The associated Reynolds equation is averaged with respect to the random roughness parameter. The concerned nondimensional differential equation is then solved with appropriate boundary conditions in dimensionless form to get the pressure distribution, which in turn, is used to get the expression for the load carrying capacity paving the way for the calculation of response time. The results are presented graphically. The results suggest that the bearing system registers a considerably improved performance as compared to that of the bearing system working with a conventional lubricant. It is observed that the pressure, the load carrying capacity and the response time increase with increasing magnetization parameter.

This investigation reveals that although the bearing suffers owing to transverse surface roughness in general,

There are ample scopes for obtaining better performance in the case of negatively skewed roughness by properly choosing the curvature parameters of both the plates.

Г.М. Дегери, Н.Д. Абганги

ПЛЕНКА МАГНИТОРЕОЛОГИЧЕСКОЙ
ЖИДКОСТИ НАХОДЯЩАЯСЯ МЕЖДУ ДВУМЯ
ВЫПУХЛЫМИ КРУГЛЫМИ НЕРОВНАМИ
ПЛАСТИНКАМИ

Р е з ю м е

В работе анализируется пленка реологиче-ской жидкости, находящаяся между двумя круглыми выпуклой формы пластинками с шероховатыми по-верхностями. Поверхность верхней выпуклой пластин-ке, которая приближается к нижней выпуклой пла-стинке, характеризуется гиперболической функцией, а поверхность нижней пластинки характеризуется се-кантной функцией. Смазывающим материалом при этом используется магнитореологическая жидкость, находящаяся во внешнем магнитном поле, наклонен-ном по отношению радиальной оси. Шероховатость рабочей поверхности подшипника в радиальном на-правлении моделируется случайной стохастической переменной без нулевого значения ее постоянной и переменной составляющих. Уравнение Рейнольдса, описывающее указанный процесс, усреднено по отно-шению к случайному параметру шероховатости. С це-лью установления распределения давления, безразмер-ное дифференциальное уравнение решено при опреде-ленных конечных условиях, результаты использованы для определения несущей способности и времени пе-реходного процесса системы. Полученные результаты представлены в графическом виде. Результаты иссле-дований показали, что характеристики такого подшип-ника значительно улучшилась по сравнению с систе-мами подшипников, использующих обыкновенные смазочные материалы. Установлено, что давление, несущая способность и продолжительность переходно-го процесса увеличивается при увеличении параметра намагничивания. Исследования позволили обнаружить специфические дефекты подшипника, возникающие из-за поперечных неровностей. Установлено, что при правильном подборе параметров кривизны обеих пла-стин можно эффективно улучшить эксплуатационные свойства подшипника при негативной асимметрии распределении переменной составляющей шерохова-тости.