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Circular scale eccentricity analysis

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1. Introduction

Geodetic instruments for plane angle measurements such as theodolites, tacheometers, etc. in our days are very widely used not only in surveying or geodesy but also in the roads, buildings construction, machine building same as in other branches of industry.

The circular scales are one of the main parts in these instruments serving as reference measure for angle readings. So, their accuracy is very important and is assured by the technology of its manufacture and the means and methods of their accuracy calibration. Both manufacture and the accuracy calibration is influenced by eccentricity problem, its presence during the manufacture and during an accuracy measurement. Many other instruments and processes of roundness measurement are related with the eccentricity problem. Such instruments are roundness measuring equipment and machines, including coordinate measuring machines (CMM), shifts and crankshaft measuring instruments, rotary tables used with metal cutting and measuring machines, special spindle measuring devices, etc. [1, 2]. There are two methods for eccentricity influence elimination from the final result of measurement. The first one includes the use of centring device (stage) for an initial centring of the work piece on the stage eliminating an eccentricity of it according to the axis of rotation of the measuring spindle. The second method is mathematical means eliminating the first harmonics of Fourier series (sine wave) with relevant phase angle and amplitude from the bias curve of the final result of measurement.

Both methods have their own advantages and disadvantages. Mechanical centring devices permit to accomplish the centring precision approximately within 1.0 μm. It is quite enough accuracy for general purposes measurement. Higher accuracy measurements need more precision in centring, sometimes, not exceeding 0.2-0.5 μm. Such accuracy is not easy to achieve. It is necessary to use the stages with very fine mechanical micro displacements and to apply automation means based on the displacement feedback control and piezomechanic actuators [3].

Second method initially seems as the most appropriate as it presents an easy way for its accomplishment by supplying the measuring equipment with relevant software for spectral analysis of the result of measurement and eliminating the sine wave from it. It means, eliminating the error due to eccentricity appearing during the measuring process. Nevertheless, it also is subject of disadvantages; for example, by measurement point-to-point method (as on the CMM) as there can be a shortage of data for correct spectral analysis due to interrupted function of the part's surface between the separate points of measurements. In addition, there can be several sine wave constituents having different phase angles and amplitudes in the graph of final result that can be left in it as the residual harmonic. Such occasions are analysed in this paper.

In Vilnius Gediminas Technical University, Institute of Geodesy the test bench for testing and calibration of geodetic and other angle measuring instruments has been constructed [4]. The test bench incorporates several principles of precise angle measurements (photoelectric angle encoder, multiangular prism with autocollimator, etc.) one of which being the visual circular scale with the photoelectric microscope(s). To achieve the appropriate precision of angle measurements the circular scale was calibrated using the method proposed by the authors of this paper [5, 6]. Therefore the data of the systematic errors of the scale strokes at the pitch of 5° were determined (Table 1, Fig. 1).

Having the data of the scale strokes biases (systematic errors) it is possible to determine the mechanical systematic errors of the elements of the test rig such as the eccentricity of bearings or scale itself [7, 8]. Such analysis

Table 1

Biases (systematic errors) of the scale strokes

Stroke, °	Bias, ″	Stroke, °	Bias, ″	Stroke, °	Bias, ″
0	0.00	120	2.27	240	3.99
5	-0.23	125	2.47	245	3.76
10	-0.12	130	2.87	250	3.81
15	0.14	135	2.40	255	3.56
20	-0.55	140	2.67	260	2.10
25	-0.01	145	2.75	265	2.88
30	0.22	150	2.99	270	3.12
35	1.35	155	3.26	275	3.10
40	2.14	160	13.98	280	4.15
45	1.31	165	3.60	285	2.77
50	0.72	170	2.75	290	2.44
55	1.15	175	2.18	295	0.91
60	-7.42	180	2.30	300	1.68
65	-8.13	185	2.56	305	0.61
70	-26.71	190	2.44	310	0.91
75	-27.41	195	2.77	315	1.04
80	-27.28	200	3.34	320	1.73
85	-28.12	205	3.84	325	0.27
90	-29.97	210	3.92	330	1.20
95	-27.64	215	3.34	335	0.59
100	-12.49	220	3.74	340	-9.40
105	1.48	225	4.04	345	-89.57
110	1.63	230	3.87	350	-2.67
115	1.62	235	3.99	355	0.20

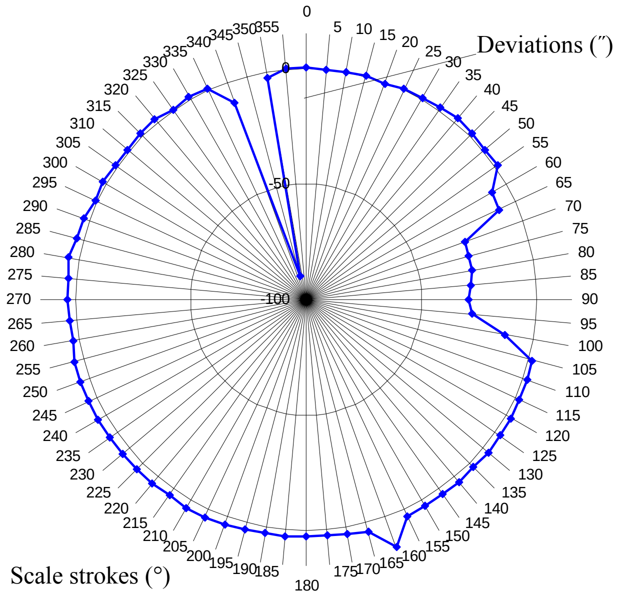

Fig. 1 Determined bias of each scale stroke

of the mechanical parameters of the test rig is further described in the next sections of this paper.

2. Spectral analysis of the systematic errors

One of the main features, influencing the precision of any rotary device is the eccentricity of mounting of rotating or measuring element (eccentricity of bearing, of the measuring scale etc.). Having the scale calibration results (obtained using the suggested calibration method) the eccentricity of the scale (or a disc mounting) can be calculated. For that purpose the Fourier series could be used. However due to some significant biases of the scale strokes (at 60°–100° and 350° strokes, Fig 1) the satisfactory results of calculations could not be reached (those biases effected the harmonics considerably). To fix the problem the calibration graphs were smoothened, compensating the greatest biases numerically only for the harmonics calculations (Fig. 2).

Having the smoothened data and using the Fourier analysis, the harmonics of the measurements could be calculated [9, 10]. The finite Fourier series in our case is

$$\delta\tilde{\phi}\,(\phi) = A_0 + 2\sum_{m=1}^{n-1}(A_m cos 2\pi m f_1 \phi + \\ + B_m sin 2\pi m f_1 \phi) + A_n cos 2\pi m f_1 \phi \qquad (1)$$

The coefficients of the equation for each harmonic can be calculated

$$A_m = \frac{1}{N}\sum_{r=-n}^{n-1}\delta\phi_r cos\frac{2\pi mr}{N} \qquad (2)$$

$$B_m = \frac{1}{N}\sum_{r=-n}^{n-1}\delta\varphi_r\, sin\frac{2\pi mr}{N} \qquad (3)$$

where m is number of harmonic, N is total number of measurements, r is measurement number.

The amplitude of each harmonic

$$R_m = \sqrt{A_m^2 + B_m^2} \qquad (4)$$

Phase shift of each harmonic (regarding to the zero point)

$$\varphi_m = arctg\left(-\frac{B_m}{A_m}\right) \qquad (5)$$

The results of calculations (performed only by means of *Microsoft Excel* software) for each harmonic are shown in Table 2.

Table 2

Harmonics of scale strokes errors calculated using the Furrier series

Source	m	A_m, ″	B_m, ″	R_m, ″	φ_m, °	Input to the mean square value
Average value	0	1.818	0	1.818	0	3.305
1st harmonic	1	-1.593	-0.793	1.779	116.464	12.661
2nd harmonic	2	-0.411	0.276	0.495	56.049	0.980
3rd harmonic	3	-0.043	0.355	0.358	6.948	0.513
4th harmonic	4	-0.374	-0.082	0.383	102.328	0.587
5th harmonic	5	0.042	-0.055	0.069	37.563	0.019
6th harmonic	6	-0.082	-0.145	0.167	150.474	0.111
Full amount						18.177

As can be seen, the most significant amplitude of series can be noticed at a 1st harmonic, which means that the eccentricity has the greatest influence on the scale bias. Graphically the scale systematic errors (corrected) with the 1st harmonic are shown in Fig 2 and 3.

Having the amplitude of the 1st harmonic calculated (Table 1) the eccentricity of one of the elements of the test rig can be determined using equation [11, 12]

$$e_s = R_{scale} sin R_m \qquad (6)$$

and having in mind that the radius of the investigated circular scale is $R_{scale} = 480$ mm, the eccentricity of the scale is $e_s = 4.14$ µm and it lies in line of $116.46° - 296.46°$.

The calculations displayed above determine the

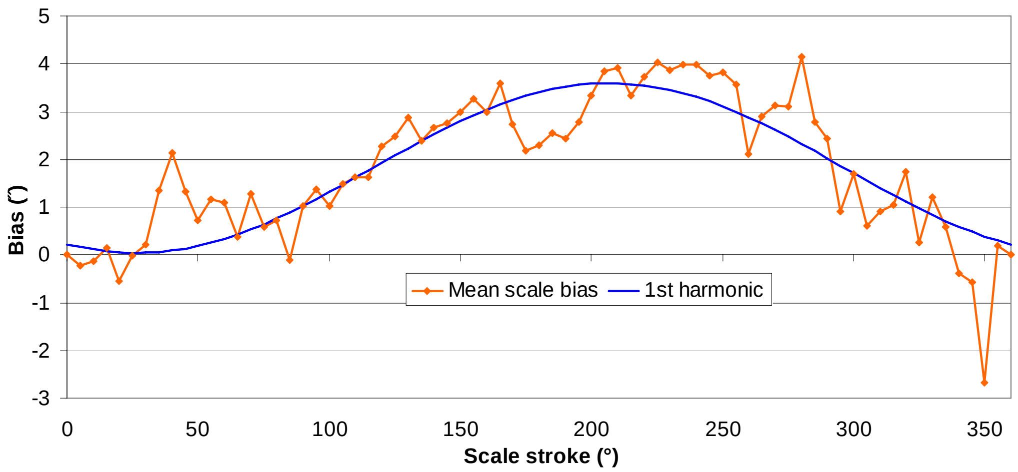

Fig. 2 Smoothened scale stroke bias value with the segregated 1st harmonic

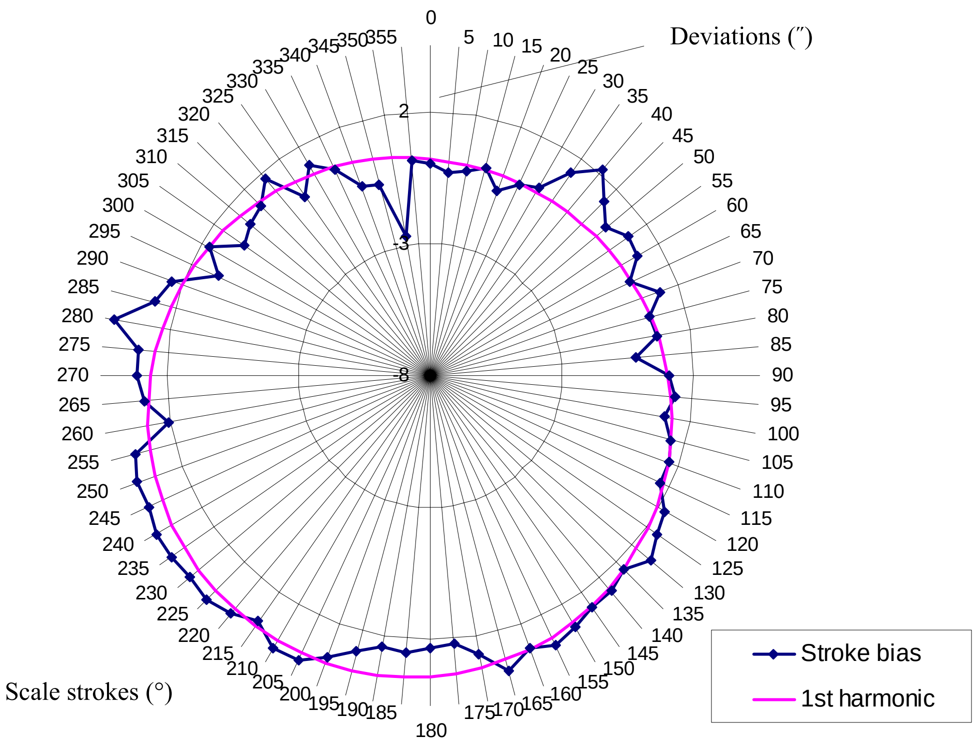

Fig. 3 Determined bias at each scale stroke with the segregated 1st harmonic on a circular graph

harmonics (the 1st of which evaluate the eccentricity of the scale and is the main parameter to determine) of the original calibration data. However the angle measuring device consisting of many mechanical units can produce several 1st harmonics with the different phases due to the different eccentricity of its parts (the eccentricity of mounting, scale position during the stroke cutting) and those harmonics can not be determined using the regular Fourier series [13]. Therefore, after removal of the biases caused by the constant member (0 harmonic) and 1st harmonic the analysis using the Fourier series Eqs. (1)-(5) was performed once again (Table 3).

As can be seen this time the 1st harmonic also exists but it is considerably smaller than the previous one. Also a phase shift can be noted which means that there are components of the eccentricity in the device – probably the disc rotation eccentricity (caused by the bearings or the influence of mechanical disturbance) and the eccentricity of scale (the scale placement itself or eccentricity of the scale strokes caused by the cutting machine) (Fig. 4).

It should be also mentioned that the 2nd–6th harmonics are equal to the ones calculated previously (Table 1) because they were not removed from the calculated scale strokes biases data.

Using the eccentricity calculation equation (6) and having amplitude of the first harmonic, the next first harmonic can be determined as caused by an eccentricity

$$e_s' = 0.157\ \mu\text{m}$$

which lies on the line of $21.68° - 201.68°$. As can be noted the last calculated (after the removal of constant bias and 1st harmonic) eccentricity is considerably smaller than the previously calculated one and produce far smaller bias (Table 2) during the measurements.

Table 3

Harmonics of scale strokes errors calculated using the Fourier series after the removal of the previously calculated constant bias and 1st harmonic

Source	m	A_m, ″	B_m, ″	R_m, ″	φ_m, °	Input to the mean square value
Average value	0	0	0	0	0	0.000
1st harmonic	1	-0.025	0.063	0.067	21.683	0.036
2nd harmonic	2	-0.411	0.276	0.495	56.049	0.980
3rd harmonic	3	-0.043	0.355	0.358	6.948	0.513
4th harmonic	4	-0.374	-0.082	0.383	102.328	0.587
5th harmonic	5	0.042	-0.055	0.069	37.563	0.019
6th harmonic	6	-0.082	-0.145	0.167	150.474	0.111
Full amount						2.174

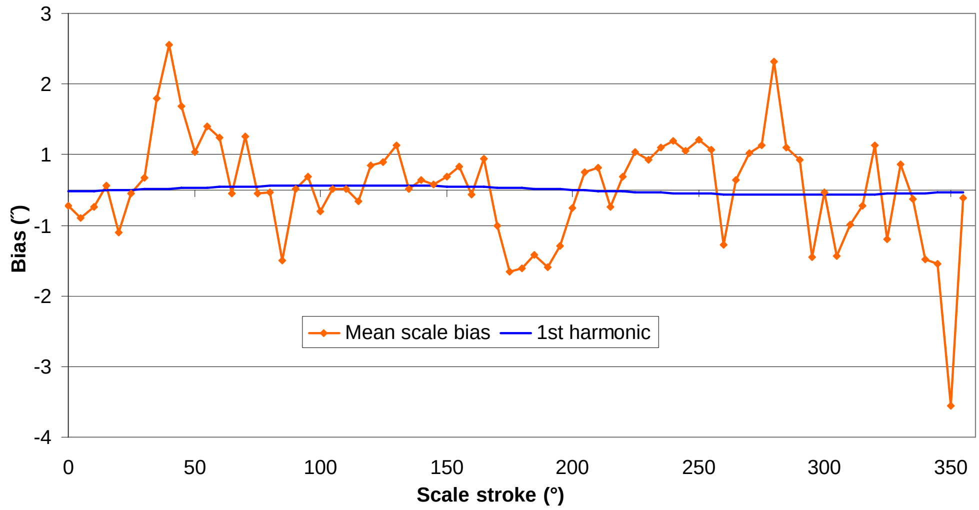

Fig. 4 Smoothened bias of each scale stroke after the removal of previously determined 1st harmonic (Fig 2) with the segregated 1st harmonic

3. Eccentricity of the elements

Having determined dual 1st harmonics and according to the parameters of these harmonics the data of two possible eccentricities, it might be stated that these errors are caused by the eccentricities of two elements of the rotary device. With a high degree of possibility the

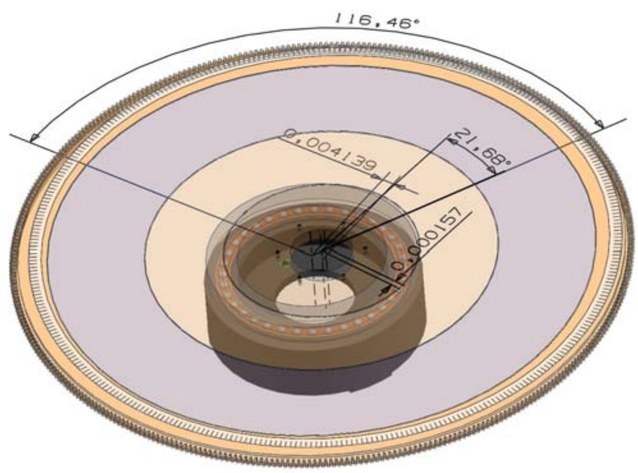

Fig. 5 Values and positions of the eccentricities of the rotary device (isometric)

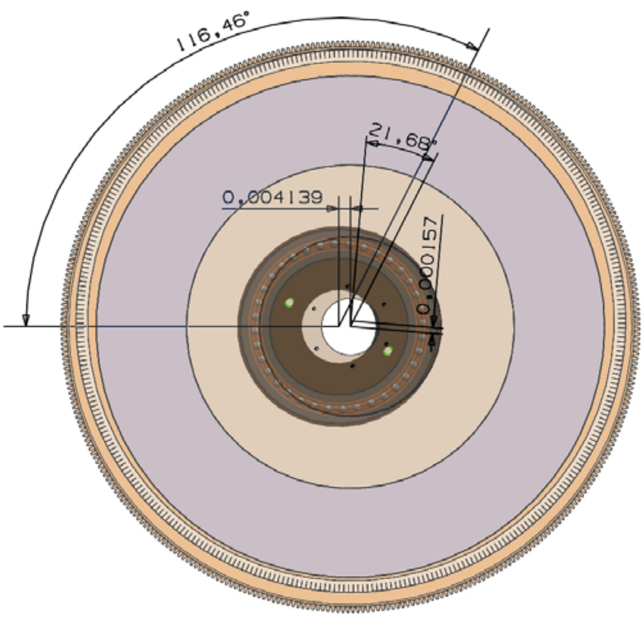

Fig. 6 Values and positions of the eccentricities of the rotary device (planimetric)

greatest eccentricity ($e_s = 4.139$ µm which lies on 116.46° -296.46) has been caused by the ball bearings of the device and the smaller one ($e_s' = 0.157$ µm, lying on $21.68° - 201.68°$) by the eccentricity of the scale itself or by the eccentricity of the scale strokes cutting machine (while manufacturing of the scale). The eccentricities with their positions (regarding the circular scale) are shown in Figs. 5 and 6.

The origin of eccentricities mentioned above (the larger caused by bearings and the smaller by the eccentricity of the scale) is just hypothetical, since for the unambiguous determination of the eccentricities origin further tests should be performed. At the moment only the presence of the eccentricities (and the harmonics) can be stated. The origin of errors is to be determined during future tests.

Using the described method of multiple 1st harmonic determination (dual in our case, but practically it might be triple or even more) eccentricities of the several parts of the rotary devices can be analytically determined and evaluated. This method can be implemented in the analysis of performances of practically any kind of the rotating devices, such as rotary tables, spindles, encoders etc.

4. Conclusions

1. Having the data of the systematic angular position errors of the rotary devices and using the Fourier analysis for the harmonics calculation it is quite possible to determine the eccentricity of the mechanical elements and its influence on the angular position determination (or the positioning).

2. Using the multiple 1st harmonic determination method it is possible to evaluate the eccentricities (and their positions) of the separate mechanical elements of the rotary devices.

3. For precise determination of the origin of the eccentricities some further researches must be carried on.

4. Using the described method the eccentricities and their position were determined on the rotary table constructed in VGTU. These eccentricities are: primary eccentricity $e_s = 4.139$ µm which lies on 116.46–296.46, and secondary $e_s' = 0.157$ µm, lying on 21.68–201.68).

5. In case of the investigated rotary table further researches of the origin of the eccentricities should be carried on, but with a high degree of possibility it might be stated that primary eccentricity ($e_s = 4.139$ µm) is probably caused by ball-bearings and the secondary ($e_s' = 0.157$ µm) by the circular scale (or the scale cutting) itself.

References

1. **Bakšys, B., Kilikevičius S.** Experimental investigation of cylindrical parts robotic vibratory insertion.-Mechanika.-Kaunas: Technologija, 2008, No.2(70), p.32-37.
2. **Spruogis, B., Turla, V., Jakštas, A.** Investigation of radial misalignment influence on dynamics of precise rotor system. -Mechanika. -Kaunas: Technologija, 2007, No.1(63), p.45-49.
3. **Probst, R.** Requirements and recent developments in high precision angle metrology. -In Proceedings of the 186th PTB-Seminar, 5th Nov. 2003, p.124-131.
4. **Bručas, D., Giniotis, V., Petroškevičius, P.** The construction of the test bench for calibration of geodetic instruments.-Geodezija ir kartografija, 2006, XXXII, No.3, p.66-70.
5. **Bručas, D., Giniotis, V.** Preliminary accuracy anglysis of the angular test bench. -Matavimai, 2007, No.2(40), p.27-30.
6. **Giniotis, V.** Measurements of Position and Movement. -Vilnius: Technika, 2005.-215p. (in Lithuanian).
7. **Sydenham, P.H., Thorn, R.** Handbook of Measurement science. Vol. 3: Elements of Change. -Chichester. New York. Brisbane. Toronto. Singapore: John Wiley & Sons, 1992.-600p.
8. **Eliseev, S.V.** Geodetic Instruments and Devices. -Moscow: Nedra, 1973.-390p. (in Russian).
9. **Jenkins, G., Watts, D.G.** Spectral Analysis and its Applications. Vol. 1. -Moscow: Mir, 1972.-310p. (in Russian).
10. **Jenkins, G., Watts, D.G.** Spectral Analysis and its Applications. Vol. 2. -Moscow: Mir, 1972.-286p. (in Russian).
11. **Krivenkov, V.V.** Automated Control and Testing of the Angular and Linear Encoders. -Leningrad: Mashinostroenie, 1984.-247p. (in Russian).
12. **Eidinov, V.J.** Angle Measurements in Machine Building. -Moscow. Gosudarstvennoe Izdatelstvo Standartov, 1963.-413p. (in Russian).
13. **Burdun, G.D.** Linear and Angular Measurements. -Moscow: Gosudarstvennoe Izdatelstvo Standartov, 1977.-413p. (in Russian).

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APSKRITIMINIŲ SKALIŲ EKSCENTRICITETO ANALIZĖ

R e z i u m ė

Straipsnyje pateiktas apskritiminių skalių padalų sistemingųjų paklaidų spektrinės analizės taikymo pavyzdys. Nagrinėtos geodezinių kampų matavimo prietaisų kalibravimo stendo, sukurto Vilniaus Gedimino technikos universitete Geodezijos institute, apskritiminės skalės sistemingosios paklaidos. Taikant spektrinės rezultatų analizės metodą išskirtos rezultatų harmonikos bei pastovioji dedamoji. Remiantis gautais rezultatais (1-osios harmonikos charakteristikomis) nustatyta, kad apskritiminės skalės ekscentricitetas sąlygojamas vieno iš sukamojo disko elementų (tikėtina jog guolio) yra $e_s = 4.14$ µm, bei išdėstytas tiesėje 116.46–296.46. Atėmus gautąją (pirminę) 1-ąją harmoniką iš sistemingųjų paklaidų reikšmių, atlikta antrinė spektrinė rezultatų analizė bei gauta antrinė 1-osios harmonikos reikšmė, remiantis kuria linijoje 21.68–201.68 rastas antrinis ekscentricitetas $e_s' = 0.157$ µm, atsiradęs dėl kito sukamojo disko elemento (tikėtina jog pačios skalės) gamybos netikslumų. Remiantis aprašyta metodika (dvigubu 1-osios harmonikos išskyrimu) įmanoma teoriškai (iš kalibravimo rezultatų) nustatyti kelių besisukančių prietaisų, tokių kaip sukimo staliukai, špindeliai, įvairūs rotoriai, sudedamųjų elementų ekscentricitetus.

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CIRCULAR SCALE ECCENTRICITY ANALYSIS

S u m m a r y

An example of the circular scale strokes systematic errors spectral analysis is described in this paper. The circular scale strokes systematic errors of the geodetic angle measuring instruments calibration device developed in Institute of Geodesy of Vilnius Gediminas Technical University are analysed here. Implementing the spectral analysis method the harmonics and the constant member of the calibration results were segregated. According to the obtained results (features of the 1st harmonic) it was determined that the eccentricity of the scale caused by one of the elements of the rotary disc (probably the bearing) is $e_s = 4.14$ μm and it lies in line of 116.46–296.46. After the removal of the obtained (primary) 1st harmonic from the systematic errors a secondary spectral analysis of the data was performed, and the secondary 1st harmonic was determined, according to which the secondary eccentricity of $e_s' = 0.157$ μm, lying on 21.68–201.68 and caused by another element of the rotary disc (probably the scale itself) was calculated. According to the method described (double segregation of the 1st harmonic) it is possible to determine theoretically (analysing the calibration results) the eccentricities of the elements of different rotating devices such as rotary tables, spindles, various rotors.

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АНАЛИЗ ЭКЦЕНТРИЦИТЕТА КРУГОВЫХ ШКАЛ

Р е з ю м е

В данной статье описывается пример спектрального анализа систематических погрешностей делений круговых шкал. Рассмотрены систематические погрешности шкалы машины для калибрирования геодезических приборов, созданной в Институте геодезии Вильнюсского технического университета им. Гедиминаса. Применяя метод спектрального анализа, выделены гармоники и постоянная слагаемая погрешностей. Опираясь на полученные результаты (характеристики 1-ой гармоники), определено, что эсцентрицитет круговой шкалы, обусловленный одним из элементов поворотного диска (скорее всего подшипника) $e_s = 4.14$ μm и находится на линии 116.46-296.46. Исключив полученную (первичную) 1-ую гармонику из систематических погрешностей шкалы и проведя повторный (вторичный) спектральный анализ результатов, получена вторичная 1-ая гармоника, определяющая вторичный эксцентрицитет $e_s' = 0.157$ μm на линии 21.68-201.68, обусловленный неточностью производства другого элемента поворотного диска (скорее всего самой шкалы). Основываясь на описанной методике (двойном выделении первичной гармоники) возможно теоретически (по результатам калибрирования) определить эксцентриситеты элементов поворотных устройств, таких, как поворотные столики, шпиндели и разнообразные роторы.

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