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Reliability of a timber structure exposed to fire: estimation using fragility function

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1. Introduction

The principal structural members of many commercial buildings are the so-called heavy timber structures. Their elements are beams, columns, decks, or truss members made from glue-laminated or large-dimension sawn timber. As compared to steel and concrete structures, heavy timber structures are recognized as having good fire resistance. There are many examples of such structures surviving fire exposure without collapse [1]. However, the relatively high fire resistance of heavy timber structures does not automatically mean that they are safe from failures caused by fires.

The fire resistance of timber structures is assessed using the same methods as for other materials. The today's practice of the fire-resistance calculation remains deterministic. However, the fire is, by its very nature, an uncertain phenomenon. This generates a need for a probabilistic modeling of both fire severity and structural response to fire. From the structural point of view, the final result of such a modeling should be the probability of failure (exceedance of a fire limit state). The fire safety of a particular structure can then be verified by comparing the failure probability with some prescribed tolerable value.

The estimation of the failure probability is complicated by the fact that direct statistical data gained from post-mortem investigations of natural fires is sparse and not very accurate [2]. Thermal actions induced on structures by natural fires are usually predicted by applying deterministic computer fire models [3 - 5]. The stochastic (Monte Carlo) simulation is suggested for an assessment of uncertainties related to output of such models [6]. The combined application of deterministic computer fire models and Monte Carlo method is called the probabilistic fire simulation. It can produce information for estimating the failure probability of a specific structure exposed to fire. Hietaniemi [7, 8] provides an example of how such information can be applied to estimating the probability that a glue-laminated beam will fail during fire.

The present paper seeks to refine the procedure of the failure probability estimation by applying results of probabilistic fire simulation. The basic idea is that this estimation should include a fragility function developed for a timber structure under analysis. The failure probability can be expressed and estimated as a mean of fragility function values. It is shown how to estimate the failure probability by computing a sample of fragility function estimates obtained using the results of computer fire simulation. The fragility function can be used in the case where the problem of the failure probability estimation involves measures of both aleatory and epistemic uncertainty.

2. The problem of probabilistic verification of fire safety

The failure of a structure subjected to a fire is verified using three failure criteria, namely, the criteria of load-bearing capacity, insulation and integrity [9, 10]. In case where a load-bearing structure is not intended for performing any insulation function, only the first of these criteria will have to be assessed in the design for fire safety. A verification of the load-bearing criterion is understood as the verification that the fire resistance of the structure (or each part of the structure) is greater than the severity of the fire to which the structure is exposed [1]. The general expression for the verification of the criterion has the form

$$\text{Fire resistance} \geq \text{Fire severity} \tag{1}$$

The above criterion can be verified in the time domain, the strength domain, and the temperature domain. The comparison of the fire severity and fire resistance in the time domain is by far the most common procedure for all types of structures, whereas the temperature domain is not used for timber structures because there is no critical temperature for fire exposed timber [1].

In the deterministic structural analysis for fire safety, the verification of structure in the time domain and strength domain is done by checking the respective inequalities: $t_f \geq t_{req}$ and $m_{d,fi} = r_{d,fi} - e_{d,fi} \geq 0$, where t_f is the calculated time to failure; t_{req} is the required time of fire resistance (required time to failure); and $m_{d,fi}$, $r_{d,fi}$, $e_{d,fi}$ are design values of safety margin, resistance and action effect of the structure under fire situation, respectively. In case of the verification in the strength domain, the deterministic resistance $r_{d,fi}$ and action effect $e_{d,fi}$ should be conservative values ($r_{d,fi}$ should be less than the actual minimum resistance reached during the fire; $e_{d,fi}$ should exceed the "real-world" action effect at the time of the fire). Although the condition $m_{d,fi} \geq 0$ does not contain the time t_{req} specified in the building/design codes, it is stated that the inequalities $t_f \geq t_{req}$ and $m_{d,fi} \geq 0$ give equivalent result as the positive value of the difference $t_{fail} - t_{req}$ corresponds to the positive value of $r_{d,fi}(t) - e_{d,fi}(t)$ at the moment $t = t_{req}$.

The fire severity (destructive potential) of natural (real, not nominal) fires is influenced by many random factors and so can be highly uncertain. Uncertainties can be inherent in the response of structure to fire. For instance, the burning of external wood in a timber structure and its conversion to a layer of char can be uncertain in terms of the char thickness and distribution over the surface of structural element. This will result in uncertainties related to the resistance of timber structure. Therefore it makes sense to measure the fire safety of a structural element by a probability that the element will fail during the time $[0, t_{req}]$ or, in short, by the failure probability $P_f(t_{req})$. This can be expressed as

$$P_f(t_{req}) = P(T_f < t_{req}) = P(M_{fi}(t) \leq 0 \mid \forall t \in [0, t_{req}]) \quad (2)$$

where T_f is random time to failure; $M_{fi}(t)$ is time-dependent and random safety margin at the time t. With the probability $P_f(t_{req})$, the verification of the conditions $t_f \geq t_{req}$ and $m_{d,fi} \geq 0$ can be replaced by checking their probabilistic analogue

$$P_f(t_{req}) \leq P_{f,tol} \quad (3)$$

where $P_{f,tol}$ is tolerable value of failure probability. A verification of the condition (3) will require to calculate an estimate the failure probability $P_f(t_{req})$ for a specific situation of exposure to fire.

In a general way, the failure probability $P_f(t_{req})$ can be expressed through the so-called instantaneous failure probability at time t, namely, $P(M_{fi}(t) \leq 0)$. The failure probability $P_f(t_{req})$ can then be obtained by integrating $P(M_{fi}(t) \leq 0)$ over the interval $[0, t_{req}]$

$$P_f(t_{req}) = t_{req}^{-1} \int_0^{t_{req}} P(M_{fi}(t) \leq 0) \mathrm{d}t \quad (4)$$

The expression (4) is of a general nature and differs from the usual expression of a structural failure probability by the fact that in the latter the function $P(M_{fi}(t) \leq 0)$ is averaged over the design working life of the structure (structural lifetime), t_d, and not the relatively short required time to failure, t_{req} (e.g. [11, 12]). In addition, one can expect that the realizations of the random variables $M_{fi}(t)$ are much easier to predict over the time t_{req} than the time t_d.

To facilitate the estimation of $P_f(t_{req})$ for a timber structure exposed to fire, the expression (4) should be reformulated by taking into account two specific processes:

- the dynamics of natural fire in the room and immediate vicinity of the structure exposed to fire.
- charring of outer layer of timber element and strength and elasticity loss in the residual cross section (central core).

We think the general expression (4) can be made more specific by expressing the failure probability $P_f(t_{req})$ through a fragility function. The general way to do this is to express $P_f(t_{req})$ in the following fundamental form

$$P_f(t_{req}) = \sum_{\text{all } j} P(\text{Failure} \mid \text{Fire severity } j) \times \times P(\text{Fire severity } j) \quad (5)$$

where $P(\text{Failure} \mid \text{Fire severity } j)$ is general expression of fragility function; $P(\text{Fire severity } j)$ is probability that the severity of fire will reach the level j. The fragility function allows to separate the fire modelling problem from the problem of modelling the response of timber structure to fire.

3. General expressions of fragility function

Fully developed or post-flashover fires, which may lead to a failure of exposed structure, are rare and heavy-to-predict phenomena. Local fires of high intensity occurring in large fire compartments with a very large concentration of fire loads (e.g. fires in industrial buildings) are also highly accidental events. Statistical information on thermal actions, which may be induced on a structure by surrounding accidental fire, can not be collected and processed in the same way as information on actions applied during the normal use of the structure.

In many practical problems, a prediction of thermal actions will be dependent on the deterministic computer fire models. Reviews of these models are presented by Rasbach et al [3] and Karlsson & Quintiere [13], among others. The deterministic models rely on the basic assumption that for a given vector of initial conditions, $\boldsymbol{x}$, the outcome $\boldsymbol{y}$ at time t is entirely determined. This can be reflected by the function $\boldsymbol{y}(\boldsymbol{x}, t \mid \boldsymbol{\theta})$, where $\boldsymbol{x}$ is the vector representing the input of a computer fire model and $\boldsymbol{\theta}$ is the vector of parameters of this model.

In general, the vector $\boldsymbol{y}$ can express fire development, its characteristic features, and its consequences. However, it is possible to simulate by means of $\boldsymbol{y}(\boldsymbol{x}, t \mid \boldsymbol{\theta})$ thermal actions applied to a specific structure exposed to a particular fire situation.

With the function $\boldsymbol{y}(\boldsymbol{x}, t \mid \boldsymbol{\theta})$, the safety margin related to the fire limit state function in question can be expressed as $m_{fi}(\boldsymbol{z}, \boldsymbol{y}(\boldsymbol{x}, t \mid \boldsymbol{\theta}))$, where the vector $\boldsymbol{z}$ represents time-independent characteristics of the structure. For a timber structure, components of $\boldsymbol{z}$ will represent original dimensions, mechanical properties of residual cross-section, and time-independent loads applied to the structure during fire. Uncertainty in values of $\boldsymbol{z}$ can be expressed by a joint probability density function $f_{\boldsymbol{Z}}(\boldsymbol{z})$. The time-dependent formation of char and so gradual reduction of resistance of the structure can be expressed through components of $\boldsymbol{y}(\boldsymbol{x}, t \mid \boldsymbol{\theta})$.

In case where both $\boldsymbol{x}$ and $\boldsymbol{z}$ are assumed to be random vectors, the instantaneous failure probability can be expressed as

$$P(M_{fi}(t) \leq 0) = P(m_{fi}(\boldsymbol{Z}, \boldsymbol{y}(\boldsymbol{X}, t \mid \boldsymbol{\theta})) \leq 0) \quad (6)$$

This expression allows to define the most general fragility function of the structure, which is subjected to fire with the initial conditions given by the time-independent vector $\boldsymbol{x}$, namely

$$P(\mathscr{F} \mid \boldsymbol{x}) = = t_{req}^{-1} \int_0^{t_{req}} \left[\int_{\boldsymbol{z}} P(m_{fi}(\boldsymbol{z}, \boldsymbol{y}(\boldsymbol{x}, t \mid \boldsymbol{\theta})) \leq 0)) f_{\boldsymbol{Z}}(\boldsymbol{z}) \mathrm{d}\boldsymbol{z} \right] \mathrm{d}t \quad (7)$$

where $\mathscr{F}$ is a short notation of the random event of failure during the time $[0, t_{req}]$

$$\mathscr{F} = (M_f(t) \leq 0 \; \forall t \in [0, t_{req}]) \quad (8)$$

In the function $P(\mathscr{F} \mid \boldsymbol{x})$, the components of $\boldsymbol{x}$ play the role of demand variables. However, the number of components of $\boldsymbol{x}$ can be large and these variables will not be directly related to the structure, for which the fragility function is developed. In addition, the scenario of fire and so the simulation result $\boldsymbol{y}(\boldsymbol{x}, t \mid \boldsymbol{\theta})$ can be influenced by such random events as breakage of windows or extinguishing of fire by fire brigade. It can be problematic to reflect an occurrence of these events by the vector $\boldsymbol{x}$.

In conventional fragility functions, the demand variables are characteristics of actions for which these functions are developed (e.g. peak ground acceleration of an earthquake, wind speed of a strong wind, weight of snow cover, see Ellingwood et al. [14], Lee & Rosowsky [15]). To date, fragility functions are expressed as ones having no more than two time-independent arguments (demand variables).

The conventional approach to developing fragility functions can be adapted to the case of fire by subdividing the general problem of the failure probability estimation into two tasks:

- a computer simulation of fire, which imitates the exposure of the structure under analysis to fire, or an imitation of fire by full-scale or large-scale experiments (Task 1).
- an estimation of the failure probability $P_f(t_{req})$ by applying results of the previous task (Task 2).

The connecting link between Task 1 and Task 2 can be the signal (time-history) of fire actions, $\boldsymbol{y}(t)$, generated by simulation or recorded in the experiment. The simulated signals $\boldsymbol{y}(t)$ can be generated by means of Monte Carlo method [4, 16].

Reason for the subdivision in the two tasks is that it simplifies the estimation of $P_f(t_{req})$ and opens several theoretical possibilities:

1. Computer fire simulators and theoretical models underlying them can differ in accuracy and the time required for the simulation. The signal $\boldsymbol{y}(t)$ can be obtained by applying competitive models, say, zone models or field models [3]. Two or more competitive models can be used in one problem of estimating $P_f(t_{req})$ by applying the scheme known in QRA as a weighting of alternative models [17].
2. A simulation of the signals $\boldsymbol{y}(t)$ prior to estimating the failure probability $P_f(t_{req})$ may allow to utilize QRA tools used to quantify model uncertainties [18]. These tools can be used, at least in theory, for expressing uncertainties in the signal $\boldsymbol{y}(t)$.
3. The information on a potential fire expressed by the simulated signals $\boldsymbol{y}(t)$ can be augmented by signals $\boldsymbol{y}'(t)$ recorded in a full-scale or large-scale experiment. The signals $\boldsymbol{y}(t)$ and $\boldsymbol{y}'(t)$ can be combined by applying the Bayesian updating scheme.

Components of $\boldsymbol{y}(t)$ may be considered to be time-dependent demand variables. With $\boldsymbol{y}(t)$, the fragility function can be formulated as a probability of the failure event $\mathscr{F}$ conditioned on the given signal $\boldsymbol{y}(t)$

$$P(\mathscr{F} \mid \boldsymbol{y}(t)) =$$
$$= t_{req}^{-1} \int_0^{t_{req}} \left[\int_{\boldsymbol{z}} P(m_{fi}(\boldsymbol{z}, \boldsymbol{y}(t)) \le 0)) f_{\boldsymbol{Z}}(\boldsymbol{z}) \mathrm{d}\, \boldsymbol{z} \right] \mathrm{d}t \tag{9}$$

Then the failure probability $P_f(t_{req})$ can be expressed as a mean calculated over all signals $\boldsymbol{y}(t)$

$$P_f(t_{req}) = E_{\boldsymbol{Y}(t)}(P(\mathscr{F} \mid \boldsymbol{Y}(t))) \tag{10}$$

where $\boldsymbol{Y}(t)$ is time-dependent random vector modeling the aleatory uncertainty related to the signals $\boldsymbol{y}(t)$; $E_{\boldsymbol{Y}(t)}(\cdot)$ is mean value with respect to $\boldsymbol{Y}(t)$.

If it were possible to calculate estimates $P_e(\mathscr{F} \mid \boldsymbol{y}(t))$ of the fragility function $P(\mathscr{F} \mid \boldsymbol{y}(t))$ for individual signals $\boldsymbol{y}(t)$ and the number of these signals, N, were sufficiently large, the failure probability $P_f(t_{req})$ could be estimated by the average

$$P_{fe} = N^{-1} \sum_{j=1}^{N} P_e(\mathscr{F} \mid \boldsymbol{y}(t)) \tag{11}$$

In other words, the mean value $E_{\boldsymbol{Y}(t)}(P(\mathscr{F} \mid \boldsymbol{Y}(t)))$ can be estimated by the average of the fictitious sample $\{P_e(\mathscr{F} \mid \boldsymbol{y}(t)), j = 1, 2, \dots, N\}$. The calculation of the estimates $P_e(\mathscr{F} \mid \boldsymbol{y}(t))$ is dependent on the type of structure exposed to fire. The interaction of timber structures with fire is somewhat simpler than that of steel and concrete structures. We think that this relative simplicity can be of use for obtaining the estimates $P_e(\mathscr{F} \mid \boldsymbol{y}(t))$.

4. Peculiarities of fire fragility of timber structures

A calculation of the estimates $P_e(\mathscr{F} \mid \boldsymbol{y}_j)$ for timber structures can be substantially facilitated by utilizing specific features of fire damage to these structures and making several simplifying assumptions.

1. The interaction of fire with structure occurs as a gradual process of charring over the time $[0, t_{req}]$.
2. Charring leads to a gradual reduction of cross-section and monotonic decrease in time of the section resistance $r_{fi}(t)$ (i.e. $r_{fi}(t)$ is a monotonically decreasing function of t). The monotonic decrease of $r_{fi}(t)$ is caused by monotonic growth of the depth of char front (char depth, in short) $d(t)$.
3. Loads applied to a structure during fire can be assumed to be time-independent random variables. This leads to a time-independence of random action effect e_{fi}. This assumption will not be valid in cases where e_{fi} can be influenced by such processes as evacuation of people and goods during fire, concentration of people trapped by fire in small areas, rapid melting of snow cover due to the thermal effect of fire.
4. The reduction of mechanical properties of residual cross-section during fire is low [9]. This allows to make the assumption that these properties can be modeled as time-independent random variables.
5. The monotonic decrease of section resistance $r_{fi}(t)$ over the time interval $[0, t_{req}]$ and the time-independence of the action effect e_{fi} will result in a monotonic decrease of safety margin m_{fi} given by the difference $r_{fi}(t) - e_{fi}$.
6. The monotonic decrease of safety margin m_{fi} allows to express the probability of failure during the time $[0, t_{req}]$ as the probability of failure at the required time t_{req}.

The facts and assumptions listed above allow to replace the time-dependent safety margin $m_{fi}(\boldsymbol{z}, \boldsymbol{y}(\boldsymbol{x}, t \mid \boldsymbol{\theta}))$

by a time-independent one, namely

$$m_{fi}(\boldsymbol{z}, d(t_{req})) = r(\boldsymbol{z}, d(t_{req})) - e_{fi}(\boldsymbol{z}) \tag{12}$$

where $r(\boldsymbol{z}, d(t_{req}))$ is the resistance of section at the time t_{req}; $e_{fi}(\boldsymbol{z})$ is action effect in the section. The char depth $d(t_{req})$ reached at t_{req} is introduced into the expression of $r_{fi}(\cdot)$ through a simple modification of original cross-sectional dimensions [9]. If, for instance, the original depth and width of a timber beam section exposed to fire on all four sides are the first two components of the vector $\boldsymbol{z}$, the resistance of the section at the time t_{req} should be calculated using the cross-sectional dimensions $z_1 - 2\,d(t_{req})$ and $z_2 - 2\,d(t_{req})$, where $d(t_{req})$ is the char depth at t_{req}.

For a given signal $\boldsymbol{y}(t)$, the char depth $d(t_{req})$ is obtained from the expression

$$d(t_{req} \mid \boldsymbol{\xi}, \boldsymbol{\theta}_\beta) = \int_0^{t_{req}} \beta(\tau, \boldsymbol{y}(t), \boldsymbol{\xi} \mid \boldsymbol{\theta}_\beta)\,\mathrm{d}\tau \tag{13}$$

where $\beta(\cdot)$ is a function relating the rate of charring to the signal $\boldsymbol{y}(t)$; $\boldsymbol{\xi}$ is vector of arguments of $\beta(\cdot)$ expressing physical quantities on which the charring rate depends; $\boldsymbol{\theta}_\beta$ is vector of parameters in the expression of $\beta(\cdot)$. Buchanan [1] provides a short review of literature on the charring rate modeling. The state-of-the-art form of the function $\beta(t, \boldsymbol{y}(t), \boldsymbol{\xi} \mid \boldsymbol{\theta}_\beta)$ is presented by Hietaniemi [7, 8].

Typical components of the vector $\boldsymbol{\xi}$ are density and moisture content of the wood.

The relation (13) means that the damage due to the fire with the given signal $\boldsymbol{y}(t)$ accumulated over $[0, t_{req}]$ can be unambiguously expressed by the single char depth $d(t_{req})$. Consequently, the estimation of the fragility function $P(\mathscr{F} \mid \cdot)$ for the given signal $\boldsymbol{y}(t)$ can be replaced by an estimation of $P(\mathscr{F} \mid \cdot)$ for $d(t_{req})$, namely

$$P(\mathscr{F} \mid \boldsymbol{y}(t)) = P(r(\boldsymbol{Z}, d(t_{req})) - e_{fi}(\boldsymbol{Z}) < 0) \tag{14}$$

To facilitate the calculation of the fragility function estimates $P_e(\mathscr{F} \mid \boldsymbol{y}(t))$ used in the final failure probability estimate (11), one can introduce yet another fragility function

$$P(\mathscr{F} \mid \boldsymbol{y}(t)) = P(\mathscr{F} \mid d(t_{req}), \boldsymbol{z}') \tag{15}$$

where $\boldsymbol{z}'$ is vector including those components of $\boldsymbol{z}$ which express the time-independent loads acting on the structure during the time $[0, t_{req}]$. If these loads are not excluded from the vector $\boldsymbol{z}$, the fragility function $P(\mathscr{F} \mid \cdot)$ can be represented as a single-argument function

$$P(\mathscr{F} \mid \boldsymbol{y}(t)) = P(\mathscr{F} \mid d(t_{req})) \tag{16}$$

The form of the functions (15) and (16) is very close to the one of traditional fragility functions used in the structural reliability assessment [14, 15]. The only difference is that the geometrical quantity $d(t_{req})$ calculated by evaluating the integral (15) is used as a demand variable. A developing of the fragility functions $P(\mathscr{F} \mid d(t_{req}), \boldsymbol{z}')$ or $P(\mathscr{F} \mid d(t_{req}))$ is a standard problem of structural reliability analysis.

An application of the fragility functions $P(\mathscr{F} \mid d(t_{req}), \boldsymbol{z}')$ or $P(\mathscr{F} \mid d(t_{req}))$ can decrease the computational effort necessary to obtain the set of N estimates $P_e(\mathscr{F} \mid \boldsymbol{y}(t))$ used to estimate the failure probability $P_f(t_{req})$. However, these functions can be even more useful in the case where there is a need to quantify uncertainties related to the models used in the analysis, namely, the models $\boldsymbol{y}(\boldsymbol{x}, t \mid \boldsymbol{\theta})$ and $\beta(t, \boldsymbol{y}(t), \boldsymbol{\xi} \mid \boldsymbol{\theta}_\beta)$.

5. Modeling epistemic uncertainties

Expression (11) yields the point estimate P_{fe} of the failure probability $P_f(t_{req})$. This can be treated as a measure of the aleatory uncertainty in the occurrence of the failure event $\mathscr{F}$. However, the key models $\boldsymbol{y}(\boldsymbol{x}, t \mid \boldsymbol{\theta})$ and $\beta(t, \boldsymbol{y}(t), \boldsymbol{\xi} \mid \boldsymbol{\theta}_\beta)$ used to estimate $P_f(t_{req})$ contain large number of parameters (components of $\boldsymbol{\theta}$ and $\boldsymbol{\theta}_\beta$) which can be uncertain in the epistemic sense. In principle, measures of epistemic uncertainty can also be assigned to outputs of these models, namely, the simulated signal $\boldsymbol{y}(t)$ and the charring rate $\beta(t)$.

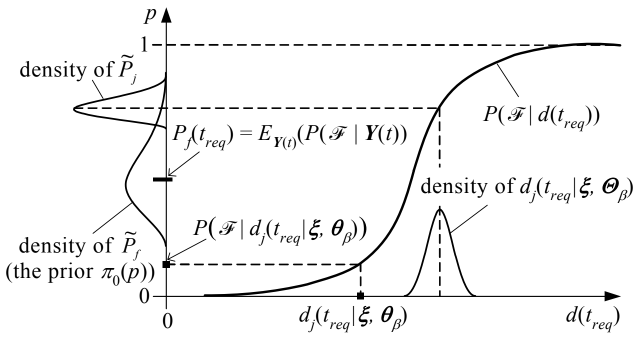

Fig 1 Fragility function values calculated for fixed char depths $d_j(t_{req})$ and uncertain char depths $D_j(t_{req})$

The presence of the epistemic uncertainties generates the need to propagate them and express in the form of epistemic uncertainty related to the failure probability $P_f(t_{req})$. The distribution of the epistemic uncertainty in $P_f(t_{req})$ will quantify the accuracy of estimation of this probability. For the sake of brevity, the present section will consider how to transform the epistemic uncertainty related to components of the parameter vector $\boldsymbol{\theta}_\beta$ of $\beta(\cdot)$ into one related to $P_f(t_{req})$. Along with this uncertainty, the estimation of $P_f(t_{req})$ will have to deal with an aleatory uncertainty in components of the argument vector $\boldsymbol{\xi}$ of $\beta(\cdot)$. The presence of the two different sources of uncertainty will be denoted by the function $\beta(t, \boldsymbol{y}(t), \boldsymbol{\Xi} \mid \boldsymbol{\Theta}_\beta)$, where $\boldsymbol{\Xi}$ and $\boldsymbol{\Theta}_\beta$ are random vectors expressing the aleatory uncertainty in $\boldsymbol{\xi}$ and epistemic uncertainty in $\boldsymbol{\theta}_\beta$, respectively.

Practical estimation of the failure probability $P_f(t_{req})$ can be carried out with simulated realizations $\boldsymbol{y}_j$ of $\boldsymbol{y}(t)$ having the form of the sequences

$$\boldsymbol{y}_j = \{\boldsymbol{y}_j(t_\tau),\ \tau = 1, 2, \ldots, N_\tau\} \tag{17}$$

where the times t_τ are obtained by dividing the time period $[0, t_{req}]$ into a relatively large number N_τ of small internals Δt. The sequences $\boldsymbol{y}_j$ can be generated by a Monte Carlo simulation of fire scenarios as described by Hostikka & Kesti-Rahkonen [6] and Hietaniemi [7, 8].

With the sequence $\boldsymbol{y}_j$, the j th simulated value of the char depth $d(t_{req})$ is calculated by

$$d_j(t_{req} \mid \boldsymbol{\xi}, \boldsymbol{\theta}_\beta) \approx \sum_{\tau=1}^{N_\tau} (\beta(t_\tau, \boldsymbol{y}_j(t_\tau), \boldsymbol{\xi} \mid \boldsymbol{\theta}_\beta)\ \Delta t) \tag{18}$$

For given $\boldsymbol{\xi}$ and $\boldsymbol{\theta}_\beta$, the value of $d_j(t_{req} \mid \boldsymbol{\xi}, \boldsymbol{\theta}_\beta)$ yields a single value of the fragility function $P(\mathscr{F} \mid \cdot)$, namely, $P(\mathscr{F} \mid d_j(t_{req} \mid \boldsymbol{\xi}, \boldsymbol{\theta}_\beta))$ (Fig. 1). The aleatory uncertainty in $\boldsymbol{\xi}$ expressed by $\boldsymbol{\Xi}$ can be averaged out through the fragility function $P(\mathscr{F} \mid \cdot)$. This operation will yield the probabilities

$$p_j(\boldsymbol{\theta}_\beta) = E_{\boldsymbol{\Xi}}[P(\mathscr{F} \mid d_j(t_{req} \mid \boldsymbol{\Xi}, \boldsymbol{\theta}_\beta))] \tag{19}$$

In case where components of the parameter vector $\boldsymbol{\theta}_\beta$ are uncertain in the epistemic sense and this uncertainty is modeled by $\boldsymbol{\Theta}_\beta$, the function $p_j(\boldsymbol{\theta}_\beta)$ will generate the epistemic random variables

$$\widetilde{P}_j = p_j(\boldsymbol{\Theta}_\beta) \ \ (j = 1, 2, \dots, N) \tag{20}$$

They can be used to compose another epistemic random variable

$$\widetilde{P}_f = N^{-1} \sum_{j=1}^{N} \widetilde{P}_j \tag{21}$$

The variable $\widetilde{P}_f$ will quantify the epistemic uncertainty in the failure probability $P_f(t_{req})$ (Fig. 1).

The distribution of the random variable $\widetilde{P}_f$ expressed, say, by a density $\pi_0(p)$ can be interpreted as a prior distribution in the standard Bayesian updating procedure [19, 20]. The prior density $\pi_0(p)$ will reflect the uncertainties present in the char depth model (19). In principle, the function $\pi_0(p)$ can also reflect the epistemic uncertainties related to the computer fire model $\boldsymbol{y}(\boldsymbol{x}, t \mid \boldsymbol{\theta})$.

It is technically possible to update the density $\pi_0(p)$ by collecting a small-size sample of char depths $\{d'_1, d'_2, \dots, d'_n\}$. They can be recorded at the time t_{req} in a series of experiments which imitate the fire exposure of structure under analysis. The new evidence for the updating can be the sample of fragility function values given by $\{P(\mathscr{F} \mid d'_1), P(\mathscr{F} \mid d'_2), \dots, P(\mathscr{F} \mid d'_n)\}$. The updating will yield a posterior density $\pi_1(p)$. Both $\pi_0(p)$ and $\pi_1(p)$ can be used for calculating conservative percentiles related to the failure probability $P_f(t_{req})$. These percentiles can then be applied to verification of the fire safety criterion (3).

6. Numerical illustration

A simply supported timber beam spans 4.5 m and carries random variable load Z_1 and random permanent load Z_2. The original width and depth of the beam are assumed to be random. They are modelled by the respective variables Z_3 and Z_4. The strength of the beam is expressed by the random variable Z_5. Probability distributions of these random variables are listed in Table 1. The beam can be subjected to a fire represented by the sequence

$$\boldsymbol{y}_j = \{(y_{1j}(t_\tau), (y_{2j}(t_\tau)),\ \tau = 1, 2, \dots, 60;\ \Delta t = 1 \text{ min}\} \tag{22}$$

where $y_{1j}(t_\tau)$ is temperature of gases at the beam surface (°C); $y_{2j}(t_\tau)$ is oxygen concentration influencing beam charring (%). The components of $\boldsymbol{y}_j$ are graphically represented by Fig. 2. The conditional probability of failure of the beam, $P(\mathscr{F} \mid \boldsymbol{y}_j)$, is to be estimated for the given sequence $\boldsymbol{y}_j$ and the required time to failure $t_{req} = 60$ min.

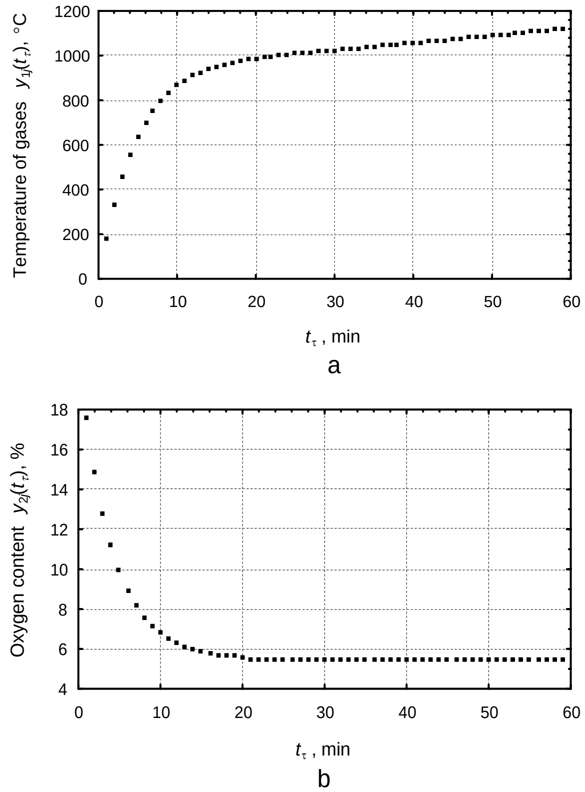

Fig. 2 Graphical representation of the realization $\boldsymbol{y}_j = \{(y_{1j}(t_\tau), y_{2j}(t_\tau)),\ \tau = 1, 2, \dots, 60\}$ of the signal $\boldsymbol{y}(t)$ ($\Delta t = 1$ min): a – sequence of temperatures $y_{1j}(t_\tau)$, b – sequence of oxygen concentration $y_{2j}(t_\tau)$

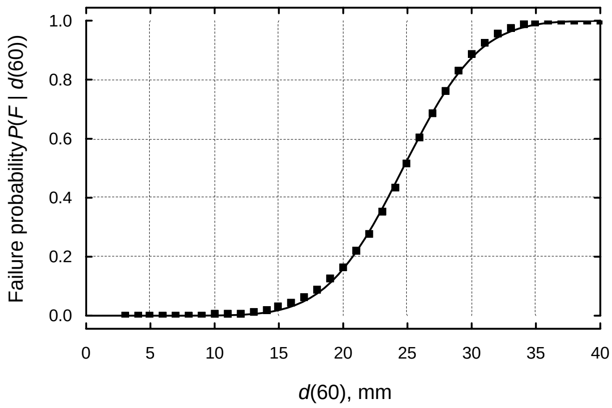

Fig. 3 Fragility function fitted to the estimated values of the conditional failure probability $P(\mathscr{F} \mid d(60))$

The load due to the beam weight was added to the permanent load. Negative effect of corner rounding in the residual section is ignored for simplicity. It is assumed that the beam sections will be exposed to fire on three sides.

The action effect $e_{fi}(\boldsymbol{z})$ and the resistance $r(\boldsymbol{z}, d(60))$ of the beam at mid-span are expressed in the form

Table 1

Aleatory random variables used in the problem (components of $\boldsymbol{Z}$ and $\boldsymbol{\Xi}$)

Name	Notation	Mean	Coeff. of variation	Distribution
Permanent load	Z_1	0.5 kN/m	0.07	Lognormal
Variable load	Z_2	2.0 kN/m	0.15	"
Width	Z_3	0.10 m	0.03	Normal
Depth	Z_4	0.25 m	0.03	"
Timber strength	Z_5	15 MPa	0.17	Lognormal
Wood density	Ξ_1	420 kg/m^3	0.07	"
Wood moisture content	Ξ_2	8.1 %	0.06	Normal
Volumetric oxygen concentration	Ξ_3	20 %	0.03	"
Initial ambient temperature	Ξ_4	20 °C	0.05	"

$$e_{fi}(\boldsymbol{z}) = 2.53\ (z_1 + z_2) \tag{23}$$

$$r(\boldsymbol{z}, d(60)) = 0.1667\ z_5(z_3 - 2d(60))(z_4 - d(60))^2 \tag{24}$$

Eq. (24) is a simplified expression of resistance used for the sake of brevity. More accurate models of the resistance of glue-laminated timber beams are presented by Ngamcharoen et al. [21] and Toratti et al. [22].

With Eqs. (23) and (24), the fragility function $P(\mathscr{F}\,|\,d(60))$ takes on the form

$$P(\mathscr{F}\,|\,d(60)) = P\Big(0.1667\ Z_5(Z_3 - 2d(60)) \times \times (Z_4 - d(60))^2 - 2.53(Z_1 + Z_2)\Big) \tag{25}$$

This function has one demand variable $d(60)$. Values of $P(\mathscr{F}\,|\,\cdot)$ estimated by means of Monte Carlo simulation for the char depth range $d(60) = 3$ - 40 mm are shown in Fig. 3. This figure also contains the normal distribution function $F(\cdot)$ fitted to the estimated values

$$P(\mathscr{F}\,|\,d(60)) = F(d(60)\ |\ \mu = 24.7,\ \sigma = 4.66) \tag{26}$$

where μ and σ are in mm.

The charring rate $\beta(\cdot)$ is modeled by the random function $\beta(t, \boldsymbol{y}(t), \boldsymbol{\Xi}\,|\,\boldsymbol{\Theta}_\beta)$. Probability distributions of its aleatory arguments $\boldsymbol{\Xi}$ and epistemic parameters $\boldsymbol{\Theta}_\beta$ are summarized in Tables 1 and 2, respectively. The empirical expression of $\beta(\cdot)$ was adopted from Hietaniemi [7, 8]

$$\beta(t_\tau, \mathrm{y}_j, \boldsymbol{\xi}|\boldsymbol{\theta}_\beta) = (\theta_{\beta 1} + (1-\theta_{\beta 1})(y_{2j}(t_\tau)/\xi_3)^{0.737}) \times \times \theta_{\beta 2}\ exp\{t_\tau/\theta_{\beta 3}\} \times \times \theta_{\beta 4}\left(\theta_{\beta 5}(y_{1j}(t_\tau) - \xi_4) + 0.75\sigma'(y_{1j}^4(t_\tau) - \xi_4^4)\right)^{\theta_{\beta 9}} \times \times (\xi_1 + \theta_{\beta 6})^{-1}(\theta_{\beta 7} + \theta_{\beta 8}\xi_2)^{-1} \tag{27}$$

where σ' = Stefan-Boltzmann constant [1, 23].

The estimation of the conditional damage probability $P(\mathscr{F}\,|\,\boldsymbol{y}_j)$ amounts to a propagation of uncertainties through the models (27), (13), and (26). Results of this propagation can be expressed by samples of the following quantities obtained by applying Monte Carlo simulation:

$$d_{mj}(\boldsymbol{\theta}_{\beta k}) = E_{\boldsymbol{\Xi}}[d_j(60|\boldsymbol{\Xi}, \boldsymbol{\theta}_{\beta k})] \approx N_\xi^{-1}\sum_{l=1}^{N_\xi} d_j(60|\boldsymbol{\xi}_l, \boldsymbol{\theta}_{\beta k}) \tag{28}$$

Table 2

Epistemic model parameters used in the problem (components of $\boldsymbol{\Theta}_\beta$)

Parameter	Original notation*	Epistemic probability distribution**
$\Theta_{\beta 1}$	ξ	U(5.5, 0.2255), %
$\Theta_{\beta 2}$	ψ_0	T(2.7, 3.6, 5.0)
$\Theta_{\beta 3}$	τ	T(90, 100, 110), min
$\Theta_{\beta 4}$	ϑ	T(1.026, 1.162, 1.387), kW/m^2
$\Theta_{\beta 5}$	h	T(11, 13, 15)
$\Theta_{\beta 6}$	ρ_0	N(465, 93), kg/m^3
$\Theta_{\beta 7}$	A	U(505, 1095), kJkg
$\Theta_{\beta 8}$	B	U(2430, 2550), kJkg
$\Theta_{\beta 9}$	p	N(0.5, 0.04)

* In Hietaniemi [7, 8]

** U = uniform; T = triangular; N = normal

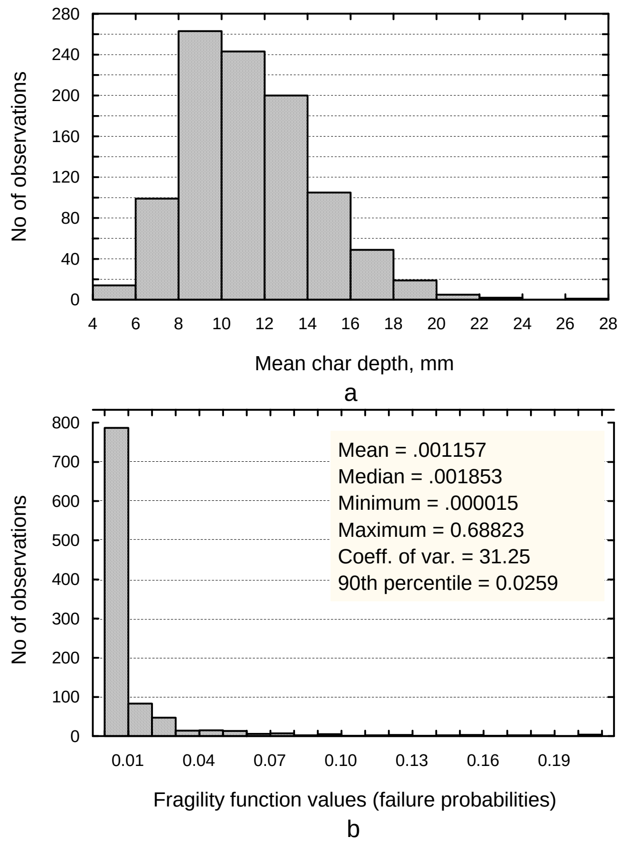

Fig. 4 Visualisation of the simulation results: a – histogram of the values of mean char depths $d_{mj}(\boldsymbol{\theta}_{\beta k})$, b – histogram of the fragility function values $p_j(\boldsymbol{\theta}_{\beta k})$

$$p_j(\boldsymbol{\theta}_{\beta k}) = E_{\Xi}[P(\mathscr{F} \mid d_j(60 \mid \Xi, \boldsymbol{\theta}_{\beta k}))] \approx$$
$$\approx N_{\xi}^{-1} \sum_{l=1}^{N_{\xi}} P(\mathscr{F} \mid d_j(60 \mid \boldsymbol{\xi}_l, \boldsymbol{\theta}_{\beta k})) \qquad (29)$$

where $\boldsymbol{\theta}_{\beta k}$ = value of $\boldsymbol{\Theta}_{\beta}$ sampled from the probability distributions given in Table 2 (k = 1, 2, … , 1000);ξ_l = value of Ξ sampled from the probability distributions given in Table 1 (k = 1, 2, … , N_{ξ}; N_{ξ} = 1000).

The histogram of the sample $\{d_{mj}(\boldsymbol{\theta}_{\beta k}),\ k = 1, 2, \ldots, 1000\}$ shown in Fig. 4 expresses the influence of the epistemic uncertainty in components of $\boldsymbol{\theta}_{\beta}$ on the accuracy of predicting the char depth for the fire characterized by the fixed sequence $\boldsymbol{y}_j$. The histogram of the sample $\{p_j(\boldsymbol{\theta}_{\beta k}),\ k = 1, 2, \ldots, 1000\}$ expresses the degree of epistemic uncertainty in the unknown value of the failure probability $P(\mathscr{F} \mid \boldsymbol{y}_j)$.

7. Conclusions

This paper presented a probabilistic approach to assessing the safety of timber structures exposed to fire. The probability that a timber structure will fail during the required time of fire resistance was applied as a measure inversely proportional to the fire safety. This probability can be estimated by developing a fragility function for a timber structure under analysis. The fragility function allows to relate results of a probabilistic computer simulation of potential fire to the failure probability of exposed structure.

The particular feature of timber structures is that the key demand variable of the fragility function is the depth of char front at the moment of required time to failure. Further demand variables can, if necessary, be the intensities of permanent and variable loads applied on the structure during the fire.

An application of the fragility function can be very helpful in cases where the problem of failure probability estimation involves both aleatory and epistemic uncertainties. In this case the fragility function can be used for propagating epistemic uncertainties. These uncertainties are usually related to mathematical models applied to the computer fire simulation and calculation of the char depth. They can be transformed through the fragility function into the epistemic uncertainty in the probability of failure.

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E. R. Vaidogas, V. Juocevičius

GAISRO VEIKIAMOS MEDINĖS KONSTRUKCIJOS PATIKIMUMO VERTINIMAS NAUDOJANT PAŽEIDŽIAMUMO FUNKCIJĄ

R e z i u m ė

Atsitiktiniai gaisrai pastatuose ir jų sukelti pažeidimai yra sunkiai nuspėjami ir neapibrėžti reiškiniai. Tiksliai prognozuoti gaisro veikiamos konstrukcijos suirimo neįmanoma. Galima tik vertinti jos patikimumą arba jam atvirkščią dydį – suirimo tikimybę. Pastarąją galima įvairiai matematiškai išreikšti. Šiame darbe suirimo tikimybė yra apibrėžiama naudojant medinės konstrukcijos pažeidžiamumo funkciją. Straipsnyje aprašytas gaisro veikiamos masyvios medinės konstrukcijos pažeidžiamumo funkcijos sudarymo metodas, leidžiantis įvertinti stochastinius ir episteminius neapibrėžtumus. Pažeidžiamumo funkcija leido susieti tikimybinio gaisro modeliavimo rezultatus su konstrukcijos pažaidos tikimybe. Pažeidžiamumo funkcijos formulavimas paremtas galimybe medinės konstrukcijos pažaidą išreikšti medienos apanglėjimo gyliu. Medienos apanglėjimo gylis laikomas pagrindiniu sijos pažeidžiamumo funkcijos argumentu. Konstrukcijos suirimo tikimybės įvertis skaičiuojamas kaip pažeidžiamumo funkcijos reikšmių vidurkis.

E. R. Vaidogas, V. Juocevičius

RELIABILITY OF A TIMBER STRUCTURE EXPOSED TO FIRE: ESTIMATION USING FRAGILITY FUNCTION

S u m m a r y

Natural fires occurring in buildings and damage to structures caused by these fires have always been uncertain phenomena. Failure of a structure exposed to a natural fire can not be predicted with certainty. The present paper describes an approach to an estimation of failure probability by developing a fragility function for a timber structure exposed to the hazard of fire. The fragility function is used to relate results of a probabilistic computer simulation of a potential fire to the probability that an exposed structure will fail during this fire. The developing of the fragility function utilizes the fact that fire damage to an unprotected timber structure can be expressed through the depth of char front. This depth is used as a key demand variable of the fragility function. The estimate of the failure probability is calculated as a mean of fragility function values.

Э. Р. Вайдогас, В. Юоцявичюс

НАДЕЖНОСТЬ ДЕРЕВЯННОЙ КОНСТРУКЦИИ ПОД ДЕЙСТВИЕМ ПОЖАРА: ОЦЕНКА ПРИ ПОМОЩИ ФУНКЦИИ ПОВРЕЖДЕНИЯ

Р е з ю м е

Непреднамеренные пожары в зданиях и повреждения причиненные ими всегда были сложно предсказуемыми и неопределёнными явлениями. Точно определить вероятность разрушения конструкций под аварийными влияниями пожара невозможно. В этой статье описан метод для создания функций повреждения массивной деревянной конструкции под действием пожара, позволяющий оценить стохастические и эпистемные неопределённости. Функция повреждения позволила соединить результаты вероятностного моделирования пожара с повреждением конструкции. Моделирование функций повреждения основывается на возможности выразить повреждение конструкции глубиной обугливания дерева. Эта глубина принята как главная переменная в компьютерном моделировании функции повреждения. Вероятность разрушения конструкции выражается и оценивается как среднее значение функции повреждения.

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