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MATLAB implementation in direct probability design of optimal steel trusses

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1. Introduction

The aim of structures design is to secure their safe work in particular exploitation period with a view of economy. Structures design, regulated by design standards [1, 2], is based on the method of limit states and operates with deterministic material physical properties and external action values. Partial reliability ratios and combinations of them secure the reliability of the structures. Direct probability design (DPD) [3-7], directly allows to control safety bounds of the structures by using probability theory and statistical calculations. The application of energy extremum principle is natural for both mechanical and mathematical view while formulating and solving deterministic [8, 9] and stochastic [10, 11] problems of structures analysis and optimization.

DPD of optimal steel trusses is analyzed in this paper by using equilibrium finite elements [12] and mathematical programming means [13]. The variations of material physical-mechanical characteristics, element cross-sections geometrical characteristics and external actions are estimated in design process as random values approximated by normal distribution law. Mathematical model allows designing trusses from one load case, but since these trusses work only in elastic stage the problem and algorithm easily can be reconstructed for trusses design from several load cases. Trusses are designed from steel profiles (HE, IPE, TUB) considering to the dispersion of profiles discrete characteristics and directly estimating reliability requirements of strength and stability for bar elements.

Solution algorithm of obtained DPD mathematical model for optimal trusses is realized in MathWorks MATLAB environment analysis and optimization system JWM SAOSYS Toolbox v0.40 (Structural Analysis and Optimization System) created by the authors' for structures modeling by finite elements. MATLAB characterizes by convenient usage, numerous functional and supple technological facilities. Jointly with optimization problems solving key modulus Optimization Toolbox it became effective tool for experimental systems design.

Numerical example of bridge-truss DPD subjected to static loading is presented. Analysis of material physical-mechanical properties, variations of external loading and the influence of element limit reliability indices on truss volume is performed.

Owning the data of statistical control, evaluating the dispersion of random values more exactly and complexly applying mathematical programming theory for the solution of optimization problems we can not only guarantee sought reliability of the structure elements but also to create more economic projects.

2. Mathematical model of optimal trusses

Optimal trusses DPD considering the reliability of strength and stability reserve is carried out by solving nonlinear mathematical programming optimization problem:

find $$min\ \boldsymbol{L}^T \boldsymbol{A_0} \quad (1)$$

subject to: $$\left[\boldsymbol{A}\right]\boldsymbol{N} = \boldsymbol{\mu_F}\ , \quad (2)$$

$$\boldsymbol{\beta}\left(A_0, N\right) \ge \left| \begin{matrix} \boldsymbol{\beta_{0,t}}, & N \ge 0; \\ \boldsymbol{\beta_{0,c}}, & N < 0. \end{matrix} \right. \quad (3)$$

$$\boldsymbol{A_0} \ge \boldsymbol{A_{0,min}}\ . \quad (4)$$

Mathematical model of optimization problem (1-4) consists of: objective function (truss minimal volume criterion) (1); truss static equilibrium equations (2); truss elements reliability indices nonlinear conditions (element designing conditions) (3); structural restrictions (4). Here: $\boldsymbol{L}$ is the length vector of truss elements; $\boldsymbol{A_0}$ is the areas vector of elements cross-sections (optimized parameter); $[\boldsymbol{A}]$ is the ratios matrix of truss equilibrium equations; $\boldsymbol{N}$ is the elements axial forces vector; $\boldsymbol{\mu_F}$ is the mean values vector of truss external forces; $\boldsymbol{\beta}(\cdot)$ is the elements reliability indices vector-valued function; $\boldsymbol{\beta_{0,t}}$, $\boldsymbol{\beta_{0,c}}$ are the limit reliability indices vectors of the elements under tension and compression respectively. Unknowns of the problem (1-4) are: $\boldsymbol{A_0} \equiv \boldsymbol{\mu}_{A_0}$, $\boldsymbol{N} \equiv \boldsymbol{\mu}_N$.

3. Reliability of element strength reserve

Statistical probability dispersion is a characteristic for material properties (elastic modules E_i and steel design strengths under yield bound $R_{y,i}$), cross-sections geometry (cross-sections areas A_i and inertia moments I_i) and external actions F_i for random values. Dispersion of the named properties can be enough well approximated by *Gauss-Laplace's* distribution law characterized by mean value μ_X and dispersion σ_X^2

$$\boldsymbol{X} = \left\{\boldsymbol{E}, \boldsymbol{R}_y, \boldsymbol{A}, \boldsymbol{I}, \boldsymbol{F}\right\}; \quad X_i \in N\left(\mu_{X_i}; \sigma_{X_i}^2\right), \quad (5)$$
$$i = 1, 2, 3, ...$$

The truss consist of a set K of bar elements. The strength reserve Z_k of every element under tension or compression is equal to the difference between the element axial capacity $N_{0,k}$ and action effect N_k

$$Z_k = N_{0,k} - N_k\ , \quad k \in K \quad (6)$$

Limit state function of element strength reserve Z_k with varying arguments we can write in the following form

$$Z_k = z\left(A_{0,k}, I_k, l_k, R_{y,k}, E_k, N_k\right) = z(\boldsymbol{x}) = \begin{vmatrix} A_{0,k}R_{y,k} - N_k, & N_k \geq 0; \\ \tilde{\varphi}\left(E_k, R_{y,k}, l_k, I_k, A_{0,k}\right)A_{0,k}R_{y,k} + N_k, & N_k < 0 \end{vmatrix} \quad (7)$$

where buckling ratio function of the element under central compression is

$$\tilde{\varphi}\left(E, R_y, l, I, A\right) = \varphi\left(E, R_y, \lambda(l, I, A)\right) \quad (8)$$

$$I = min\left\{I_y, I_z\right\} \quad (9)$$

The influence of the variation of beam elements lengths l_k on the element strength reserve is not considered in this paper.

Normal distribution is characteristic to the function of element strength reserve Z_k

$$Z_k \in N\left(\mu_{Z_k}; \sigma^2_{Z_k}\right) \quad (10)$$

which is dependent on the arguments distributed under normal law (5). Mean strength reserve μ_{Z_k} and standard deviation of strength reserve σ_{Z_k} describe the normal distribution and can be defined as follows

$$\mu_{Z_k} = z\left(A_{0,k}, I_k, l_k, \mu_{R_{y,k}}, \mu_{E_k}, N_k\right) = z\left(\boldsymbol{\mu_x}\right) \quad (11)$$

$$\sigma_{Z_k} = \left[\left(\frac{\partial z(\boldsymbol{\mu_x})}{\partial A_{0,k}}\sigma_{A_{0,k}}\right)^2 + \left(\frac{\partial z(\boldsymbol{\mu_x})}{\partial I_k}\sigma_{I_k}\right)^2 + \left(\frac{\partial z(\boldsymbol{\mu_x})}{\partial R_{y,k}}\sigma_{R_{y,k}}\right)^2 + \left(\frac{\partial z(\boldsymbol{\mu_x})}{\partial E_k}\sigma_{E_k}\right)^2 + \left(\frac{\partial z(\boldsymbol{\mu_x})}{\partial N_k}\sigma_{N_k}\right)^2\right]^{1/2} \quad (12)$$

The definition of the real standard deviation is realized as follows

$$\sigma_{Z_k} = \left[\left({}^{Z_k}\delta_{A_{0,k}} A_{0,k}\nu_{A_k}\right)^2 + \left({}^{Z_k}\delta_{I_k} I_k \nu_{I_k}\right)^2 + \left({}^{Z_k}\delta_{R_{y,k}}\sigma_{R_{y,k}}\right)^2 + \left({}^{Z_k}\delta_{E_k}\sigma_{E_k}\right)^2 + \sigma^2_{N_k}\right]^{1/2} \quad (13)$$

here ν_{A_k}, ν_{I_k} are variation ratios of cross-section area and inertia moment respectively. The partial derivatives values ${}^{Z_k}\delta_x$ at the points $\boldsymbol{\mu_x}$ is calculated by such formulas:

$${}^{Z_k}\delta_{A_{0,k}} = \begin{vmatrix} \mu_{R_{y,k}}, & N_k \geq 0; \\ {}^{\tilde{\varphi}}\delta_{A_{0,k}} A_{0,k}\mu_{R_{y,k}} + \tilde{\varphi}_k \mu_{R_{y,k}}, & N_k < 0 \end{vmatrix} \quad (14)$$

$${}^{Z_k}\delta_{I_k} = \begin{vmatrix} 0, & N_k \geq 0; \\ {}^{\tilde{\varphi}}\delta_{I_k} A_{0,k}\mu_{R_{y,k}}, & N_k < 0 \end{vmatrix} \quad (15)$$

$${}^{Z_k}\delta_{R_{y,k}} = \begin{matrix} A_{0,k}, & N_k \geq 0; \\ {}^{\tilde{\varphi}}\delta_{R_{y,k}} A_{0,k}\mu_{R_{y,k}} + \tilde{\varphi}_k A_{0,k}, & N_k < 0 \end{matrix} \quad (16)$$

$${}^{Z_k}\delta_{E_k} = \begin{matrix} 0, & N_k \geq 0; \\ {}^{\tilde{\varphi}}\delta_{E_k} A_{0,k}\mu_{R_{y,k}}, & N_k < 0 \end{matrix} \quad (17)$$

$$\tilde{\varphi}_k = \tilde{\varphi}_k\left(\mu_{E_k}, \mu_{R_{y,k}}, l_k, I_k, A_{0,k}\right) \quad (18)$$

where the definition of the partial derivatives values ${}^{\tilde{\varphi}}\delta_x$ of buckling ratio by numerical differentiation method is discussed in the other section.

According to the normal distribution characteristics μ_{Z_k} and σ_{Z_k} of the strength reserve Z_k (Fig. 1) we can define and control the probability α_k of the limit state event described by the reliability index β_k of the strength reserve

$$\alpha_k = 1 - \Phi\left(\beta_k\right) \quad (19)$$

$$\beta_k = \frac{\mu_{Z_k}}{\sigma_{Z_k}} \quad (20)$$

The condition of element design we can write as follows

$$\beta_k \geq \begin{vmatrix} \beta_{0,t,k}, & N_k \geq 0; \\ \beta_{0,c,k}, & N_k < 0. \end{vmatrix} \quad k \in K \quad (21)$$

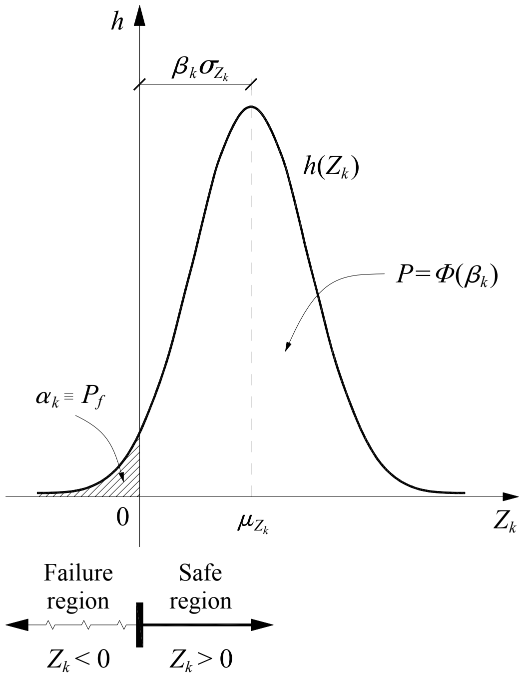

Fig. 1 Distribution of element Z_k, failure and safe regions

Before designing of the structure we set strength reserve Z_k limit reliabilities $P_{0,t,k}$, $P_{0,c,k}$ for every element. Applying tabels $P = \Phi(\beta)$ [1, 4] we define strength reserve limit realibility indices $\beta_{0,t,k}$, $\beta_{0,c,k}$ of the elements under tension and compression.

4. Statistical indices of element internal forces

Elements axial forces vector $\boldsymbol{N}$ (the variable of the problem) is equal to the mean axial forces vector $\boldsymbol{N} \equiv \boldsymbol{\mu_N}$ in optimization problem (1)-(4). Elements axial forces unknown standard deviations $\boldsymbol{\sigma_N}$ (12) are in the elements axial conditions (3). Truss analysis problem is needed to define these deviations.

The full system of equations of truss analysis problem is

$$\begin{cases} [\boldsymbol{A}]\boldsymbol{N} = \boldsymbol{F}, \\ [\boldsymbol{A}]^T \boldsymbol{u} - [\boldsymbol{D}(\boldsymbol{p})]\boldsymbol{N} = \boldsymbol{0} \end{cases} \tag{22}$$

here $\boldsymbol{p} = \{\boldsymbol{E}, \boldsymbol{A_0}\}$ are materials elastic modules and elements cross-sections areas vector ($n_p \times 1$). We can solve this system of equations (22) with respect to $\boldsymbol{N}$. We will get then such an elastic internal forces vector solving formula, which is expressed by influence matrix $[\boldsymbol{Q}(\boldsymbol{p})]$

$$\begin{aligned} \boldsymbol{N}(\boldsymbol{F}, \boldsymbol{p}) &= [\boldsymbol{D}(\boldsymbol{p})]^{-1}[\boldsymbol{A}]^T\left([\boldsymbol{A}][\boldsymbol{D}(\boldsymbol{p})]^{-1}[\boldsymbol{A}]^T\right)^{-1}\boldsymbol{F} = \\ &= [\boldsymbol{Q}(\boldsymbol{p})]\boldsymbol{F} \end{aligned} \tag{23}$$

Matrix-function of internal forces influence of structure elements has the following form

$$\underset{(n_e \times m)}{[\boldsymbol{Q}(\boldsymbol{p})]} = \begin{bmatrix} q_{1,1}(\boldsymbol{p}) & q_{1,2}(\boldsymbol{p}) & \dots & q_{1,m}(\boldsymbol{p}) \\ q_{2,1}(\boldsymbol{p}) & q_{2,2}(\boldsymbol{p}) & \dots & q_{2,m}(\boldsymbol{p}) \\ \dots & \dots & \dots & \dots \\ q_{k,1}(\boldsymbol{p}) & q_{k,2}(\boldsymbol{p}) & \dots & q_{k,m}(\boldsymbol{p}) \\ \dots & \dots & \dots & \dots \\ q_{n_e,1}(\boldsymbol{p}) & q_{n_e,2}(\boldsymbol{p}) & \dots & q_{n_e,m}(\boldsymbol{p}) \end{bmatrix} \tag{24}$$

Axial force of separate element, expressed by internal forces influence functions, we can write as follows

$$N_k(\boldsymbol{F}, \boldsymbol{p}) = q_{k,1}(\boldsymbol{p})F_1 + q_{k,2}(\boldsymbol{p})F_2 + \dots + q_{k,m}(\boldsymbol{p})F_m \tag{25}$$

Since for function $N_k(\boldsymbol{F}, \boldsymbol{p})$ arguments is characteristic normal distribution law, the following denotation is true

$$\begin{cases} \boldsymbol{N} \in \{N(\boldsymbol{\mu}_N; diag(\boldsymbol{\sigma_N})\boldsymbol{\sigma_N})\}, \\ N_k \in N(\mu_{N_k}; \sigma^2_{N_k}), \ k \in K \end{cases} \tag{26}$$

The standard deviation of axial force is calculated as follows

$$\sigma_{N_k} = \left[\sum_{i=1}^{m}\left(\frac{\partial N_k(\boldsymbol{\mu_F}, \boldsymbol{\mu_p})}{\partial F_i}\sigma_{F_i}\right)^2 + \sum_{j=1}^{n_p}\left(\frac{\partial N_k(\boldsymbol{\mu_F}, \boldsymbol{\mu_p})}{\partial p_j}\sigma_{p_j}\right)^2\right]^{1/2} \tag{27}$$

Performing differential calculus we can write:

$$\frac{\partial N_k(\boldsymbol{\mu_F}, \boldsymbol{\mu_p})}{\partial F_i} = q_{k,i}(\boldsymbol{\mu_p}) = q_{k,i} \tag{28}$$

$$\frac{\partial N_k(\boldsymbol{\mu_F}, \boldsymbol{\mu_p})}{\partial p_j} = \frac{\partial q_{k,1}(\boldsymbol{\mu_p})}{\partial p_j}\mu_{F_1} + \dots + \frac{\partial q_{k,m}(\boldsymbol{\mu_p})}{\partial p_j}\mu_{F_m} \tag{29}$$

Finally standard deviation of element axial force N_k is calculated as follows

$$\begin{aligned} &\boldsymbol{\sigma_N} : \sigma_{N_k} = \\ &= \sqrt{\sum_{i=1}^{m}\left(q_{k,i}\sigma_{F_i}\right)^2 + \sum_{j=1}^{n_p}\left(\frac{\partial\left[q_{k,1..m}(\boldsymbol{\mu_p})\right]}{\partial p_j}\boldsymbol{\mu_F}\sigma_{p_j}\right)^2}, \\ &k \in K \end{aligned} \tag{30}$$

Calculations of partial derivatives are needed to define standard deviations vector $\boldsymbol{\sigma_N}$ of the structure axial forces. Searches of such derivative analytical shapes are senseless or impossible while designing structures of various complexities. Therefore, Richardson's finite differences numerical extrapolation method is applied to calculate the derivatives values. Function derivative of one argument at point x_0 is calculated as follows

$$\begin{aligned} f'(x_0) = \frac{1}{12h}\big[f(x_0 - 2h) - 8f(x_0 - h) + \\ +8f(x_0 + h) - f(x_0 + 2h)\big] + O(h^4) \end{aligned} \tag{31}$$

where h is set function variation.

5. Assortments: discrete characteristics fields of profiles in structure optimization

Design of steel structures is inseparable from the discrete sets of profiles assortments. Analyzing the distribution of discrete characteristics of *I-A* cross-sections (Fig. 2) we can notice, that there is no homologous relation among these characteristics. Therefore, the allowable point of cross-sections geometrical characteristics ${}^{k}\boldsymbol{G} = \{A_{0,k}, I_k\}$ of the discrete field $\mathscr{D}_{I\text{-}A}$ has to be found while optimizing. Thus, for the whole structure we can write

$$[\boldsymbol{G}] = [\boldsymbol{A_0}, \boldsymbol{G_1}] \tag{32}$$

The mathematical model (1)-(4) of truss optimization problem has only one optimized parameter $\boldsymbol{A_0}$ (one problem variable), which is the vector of optimized leading geometry. Moments of inertia of optimized cross-sections I_k compose the driven (controlled) geometry vector $\boldsymbol{G_1}$. Leading geometry $\boldsymbol{A_0}$ is optimized by iterative solving of optimization problem (1)-(4), while driven geometry $\boldsymbol{G_1}$ is corrected with reference to conditions of the optimization problem constraints (3) and admissible field bounds of assortments $I_{max}(A)$, $I_{min}(A)$.

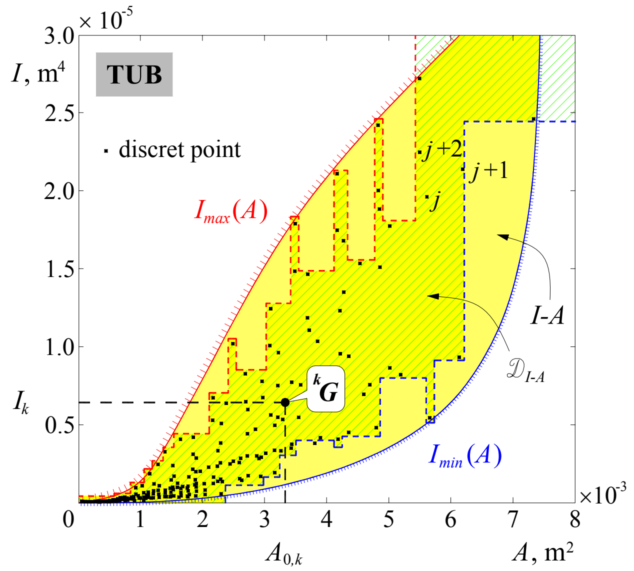

Fig. 2 Discrete values field $\mathscr{D}_{I\text{-}A}$ and admissible I-A characteristics field of TUB profiles assortment

6. Trusses design algorithm and its programmable realization

The algorithm is realized in complex MATLAB and authors' created JWM SAOSYS *Toolbox v0.40* system of structures modeling, analysis and optimal design. Further we will describe the main parts of truss stochastic design algorithm (Fig. 3) and its realization.

Truss modeling. Parameterize and model the truss by elastic equilibrium LINK1 finite elements under tension or compression [12]. Create the input file (Batch and Data File) of initial data of the structure model (SAOSYS preprocessor).

Preparing of design environment. After reading input data file, the routine P1 of SAOSYS preprocessor creates the database DB of the structure (MATLAB structural data field). The routine P2, which controls profiles assortments, reads and prepares steel profiles HE, IPE, TUB assortments from SRT database. The routine P3 collects the finite elements library FELIB of SAOSYS system. Lastly, the routine P4 creates and initiates the ensemble FE of finite elements, which compose the structure.

Preparative calculations. The routine P5 creates the ratios matrix [A] of the truss equilibrium equations, the mean values vector $\boldsymbol{\mu}_F$ of the external loads and the standard deviations vector $\boldsymbol{\sigma}_F$. Prepare the edge values vectors $\boldsymbol{A}_{0,min}$, $\boldsymbol{A}_{0,max}$, $\boldsymbol{G}_{1,min}$, $\boldsymbol{G}_{1,max}$ of the optimal geometry sections areas $A_{0,k}$ and inertia moments I_k with reference to SRT database of profiles assortments. Prepare the total lengths vector $\boldsymbol{L}$ and the lengths vector $\boldsymbol{L}_{max}$ of the longest elements in elements groups of the structure. Prepare the indices vectors $\boldsymbol{\beta}_{0,t}$, $\boldsymbol{\beta}_{0,c}$ of a limit reliability of the truss elements under tension and compression respectively. Create the material characteristics mean values and standard deviations vectors: $\boldsymbol{\mu}_E$, $\boldsymbol{\mu}_{Ry}$, $\boldsymbol{\sigma}_E$, $\boldsymbol{\sigma}_{Ry}$. Create the variation ratios vectors $\boldsymbol{v}_{A_0}$, $\boldsymbol{v}_{G_1}$ of the optimal cross-sections areas inertia moments respectively. Prepare the partial ratios vectors $\boldsymbol{\eta}_{A_0}$, $\boldsymbol{\eta}_{G_1}$ for the solution of partial derivatives with finite differences.

Solving of the optimization problem. Solving of the optimization problem (1)-(4) we perform by an iterative approximation way and begin with the biggest vectors of the optimal cross-sections areas $\boldsymbol{A}_0 = \boldsymbol{A}_{0,max}$ and inertia moments $\boldsymbol{G}_1 = \boldsymbol{G}_{1,max}$.

Step 1: calculate finite variations of the cross-sections areas and inertia moments for numerical solution of functions partial derivatives

$$\boldsymbol{\delta}_{A_0} = diag\left(\boldsymbol{\eta}_{A_0}\right)\boldsymbol{A}_0\,; \quad \boldsymbol{\delta}_{G_1} = diag\left(\boldsymbol{\eta}_{G_1}\right)\boldsymbol{G}_1 \qquad (33)$$

Calculate the standard deviations vectors of the cross-sections areas and inertia moments

$$\boldsymbol{\sigma}_{A_0} = diag\left(\boldsymbol{v}_{A_0}\right)\boldsymbol{A}_0\,; \quad \boldsymbol{\sigma}_{G_1} = diag\left(\boldsymbol{v}_{G_1}\right)\boldsymbol{G}_1 \qquad (34)$$

Step 2: calculate the influence matrix [$\boldsymbol{Q}(\boldsymbol{\mu}_p)$] (23) of the mean internal forces of the truss elements. Calculate the standard deviations vector $\boldsymbol{\sigma}_N$ of the truss elements internal forces by *Richardson's* method numerically differentiating the matrix [$\boldsymbol{Q}(\boldsymbol{p})$] according all the members of the vector $\boldsymbol{p}$ and applying the formula (30).

Step 3: to perform one iteration the routine P6 solves the prepared optimization problem (1)-(4) of nonlinear mathematical programming. If the optimization problem was solved successfully – optimal solution was found – we have a new vector $\boldsymbol{A}_0^*$ of the optimal cross-sections areas and the vector $\boldsymbol{N}$ of the truss elements axial forces mean values.

If the solution of the optimization problem was not successful (an admissible point, optimal solution were not found) – increase the areas vector $\boldsymbol{A}_0$ of the optimal cross-sections

$$A_{0,k} = \begin{cases} \dfrac{1}{2}\left(A'_{0,k} + A_{0,k}\right), & \dfrac{\left|A'_{0,k} - A_{0,k}\right|}{A'_{0,k}} \geq \varepsilon; \\ \xi A_{0,k}\,, & \text{otherwise} \end{cases} \qquad k \in K \quad (35)$$

here: $A'_{0,k} \equiv A^{*,prev}_{0,k}$ is optimal area of the optimal cross-section in the previous iteration; ε is relative threshold (10^{-3} %) of recurring increase of $A_{0,k}$; ξ is the partial ratio of direct increase of $A_{0,k}$. Correct the inertia moments vector $\boldsymbol{G}_1$ and return to the Step 1.

Step 4: the routine P7 performs the correction of the cross-sections driven geometry vector $\boldsymbol{G}_1$ (the inertia moments vector).

Step 5: Calculate the volume V of the structure with the new $\boldsymbol{A}_0^*$. Perform this iterative process till convergence conditions of the problem will not be satisfied:

$$max\left\{\frac{\left|A'_{0,k} - A^*_{0,k}\right|}{A'_{0,k}},\, k \in K\right\} \leq \varepsilon_{A_0}\,; \quad \frac{\left|V' - V\right|}{V'} \leq \varepsilon_V \qquad (36)$$

here: V' is the volume of the previous iteration structure; ε_{A_0}, ε_V are the convergence tolerance criterions (0.1 %) of the cross-sections areas and structure volume respectively.

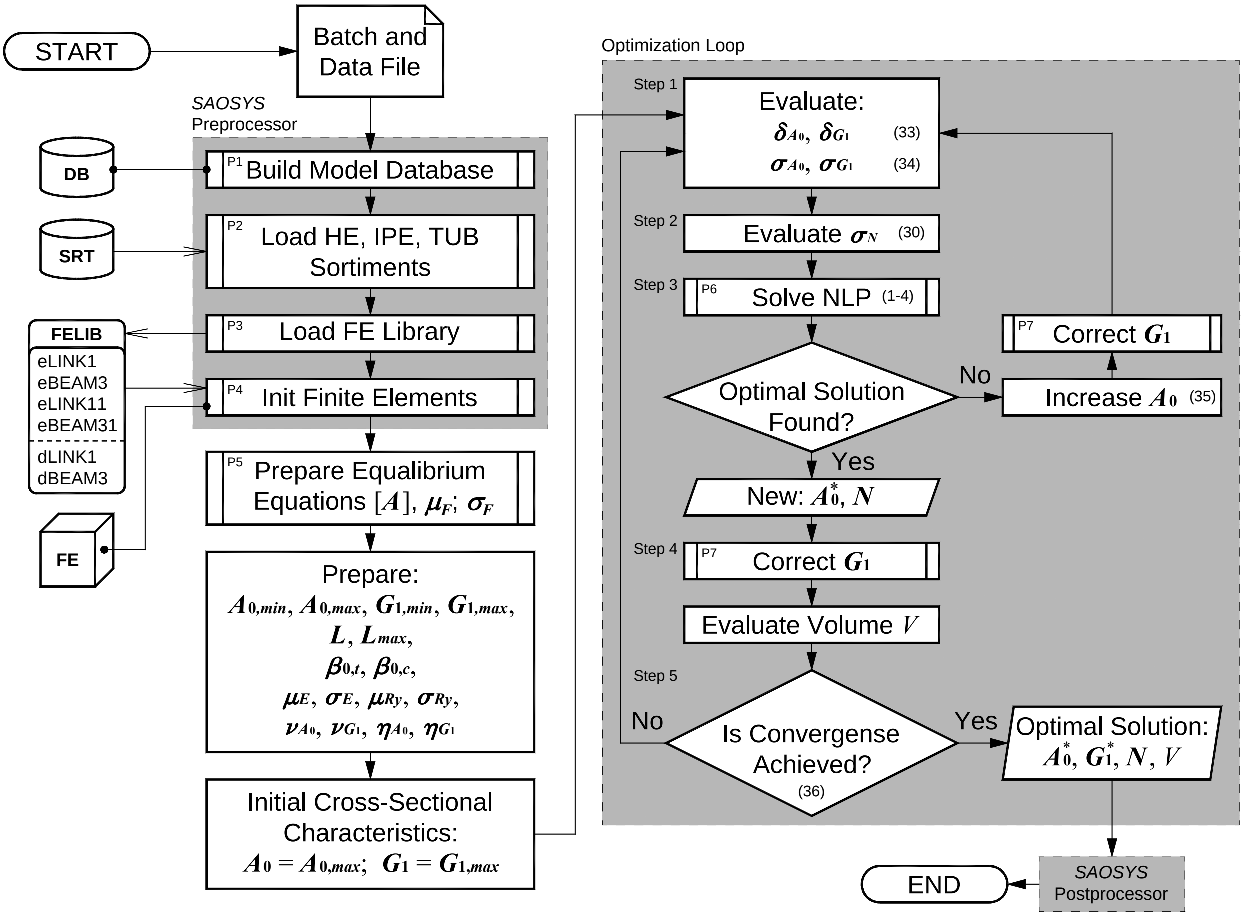

Fig. 3 The algorithm SAOSYS-TrussDPD of a direct probability design of optimal trusses

Correction procedure of the cross-section geometry $\boldsymbol{G}_1$. The concept of element groups set R we make out while designing the truss. We optimize the geometrical characteristics $\{A_{0,r}, G_{1,r}\}$ of single elements groups $r \in R$. These characteristics compose the vectors pair $\boldsymbol{A}_0$ and $\boldsymbol{G}_1$. Since we operate with element groups, the entirety of elements, which enter into the r group, we make out by intersection of sets – $K \cap R_r$. The vector $\boldsymbol{G}_1$ of optimal cross-sections limit inertia moments – is the vector, which satisfies the element design conditions (3) and the bounds of the admissible discrete fields $\mathscr{D}_{I\text{-}A}$ of profiles assortments

$$\boldsymbol{G}_1 \equiv \boldsymbol{I}_{lim} = \boldsymbol{I}_{lim}\left(\boldsymbol{A}_0^*, \boldsymbol{N}, \boldsymbol{L}_{max}, \mathscr{D}_{I-A}\right) \tag{37}$$

Define the limit inertia moments $\boldsymbol{I}_{lim} = \{I_{lim,r}, r \in R\}$ of cross-sections (elements under compression – stability conditions) by solving such an equation of reliability indices for every element

$$\tilde{\beta}_k\left(A_{0,r}^*, N_k, L_{max,r}, I_{lim,r,k}\right) = \beta_{0,c,k}\,, \quad k \in K \cap R_r, \quad r \in R \tag{38}$$

Finally we define the limit inertia moment of cross-section of elements group-set R_r

$$I_{lim,r} = max\left\{I_{lim,r,k},\ k \in K \cap R_r\right\} \tag{39}$$

also controlling and correcting satisfaction of the limit inertia moments $I_{lim,r}$ in the bounds conditions of discrete admissible fields of profiles assortments

$$^{\mathscr{D}}I_{min,r}\left(A_{0,r}^*\right) \le I_{lim,r} \le {}^{\mathscr{D}}I_{max,r}\left(A_{0,r}^*\right) \tag{40}$$

Solve the equation (38) by numerical method, sequentially increasing the argument $I_{lim,r,k}$ value of the function $\tilde{\beta}_k(\cdot)$ by the step t and controlling the inequality $\tilde{\beta}_k$-$\beta_{0,c,k}$ (Fig. 4). Automatically it performs such a function of MATLAB-SAOSYS

```
x = ConFunArgValSearch(hF, y, vInt, tol, <N>)
```

where: `x` is the found value ($I_{lim,r}$) of function argument; `hF` – function handle ($@\beta_k(I_{lim,r}) = \tilde{\beta}_k(\cdot)$); `y` is function result ($\beta_{0,c,k}$); `vInt` is the vector of search interval $\{I_{min,r}, I_{max,r}\}$; `tol` is search tolerance; `N` is granulation of search interval (optional parameter).

Define the discrete bounds $^{\mathscr{D}}I_{min,r}$, $^{\mathscr{D}}I_{max,r}$ of assortments profiles (Fig. 5) with reference to binary-bared search of discrete values, which is performed by such a function

```
vp = BinBarSearch(vD, x, b)
```

where: `vp` is the indices vector of discrete points got into the bar; `vD` is the vector ($^{\mathscr{D}}\boldsymbol{A}_{HE \vee IPE \vee TUB}$) of discrete values aligned in the increasing order; `x` is the real value ($A^*_{0,r}$); `b` is the width of search bar. Define discrete bounds according to the vector `vp` and perform the return of no admissible points $\left\{A^*_{0,r}, I_{lim,r}\right\}$ (Fig. 5 the points *1* and *3*) to the admissible zone (the points *4* and *5*).

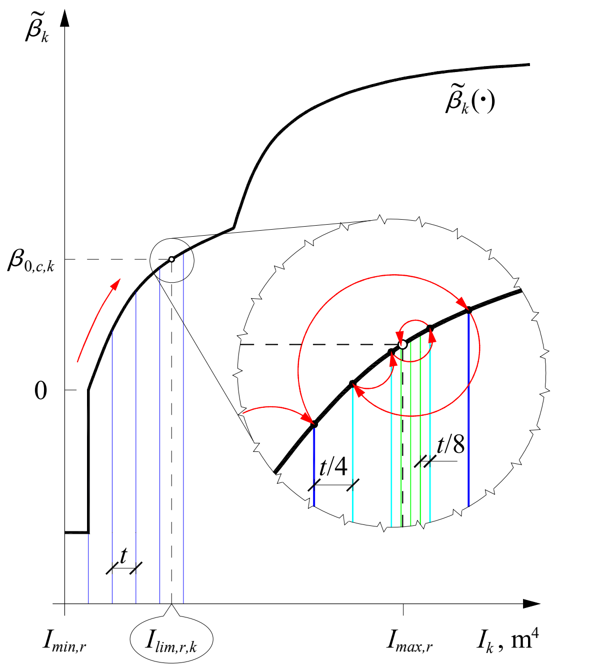

Fig. 4 The limit $I_{lim,r,k}$ definition of the element under compression with reference to equation (38)

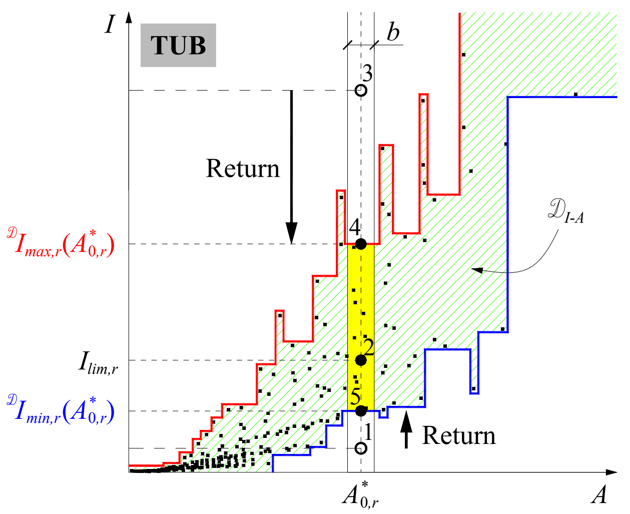

Fig. 5 Binary-bared search: return to the admissible field

Partial derivatives by the Richardson's numerical extrapolation method. To calculate the partial derivatives (31) of functions such a function is created

```
d = RichFDE(hFun, cvX0, np, h)
```

where: `d` is the value of calculated derivative; `hFun` is function handle; `cvX0` is the point-vector of derivative calculation; `np` is the number of function argument with reference to which differentiation is performed; `h` is argument finite difference.

7. Numerical example

Design structure. The bridge-truss subjected to one load case (Fig. 6) is designed. Material of the elements – steel S275: μ_E = 210 GPa; σ_E = 25.200 GPa; μ_{Ry} = = 275 MPa; σ_{Ry} = 8.333 MPa. The truss modeled by equilibrium LINK1 finite elements consist of: 14 nodes; 30 finite elements; 3 design parameters $R_{1\text{-}3}$ (elements cross-sections). Truss flanges (R_1, R_2) are designed by IPE, and grid (R_3) – by TUB profiles. Profiles variations ratios of cross-sections areas and inertia moments respectively are: ν_A = 10%; ν_I =5%. The truss is subjected by nodal loads: μ_{F1} = 55 kN; σ_{F1}= 10.061 kN; μ_{F2} = 90 kN; σ_{F2} = 16.463 kN.

Limit reliability index of the elements under tension is set $\beta_{0,t}$ = 1.64 (probability of failure $P_{f,0,t}$ = 0.0505); elements under compression – $\beta_{0,c}$ = 3.00 ($P_{f,0,c}$ = 0.00135).

Results. DPD of bridge-truss was performed by iterations. 10 approximation iterations were performed at all (Fig. 7). Calculated optimal theoretical cross-sections and the closest found profiles for them are presented in the table (Table). Designed structure volume is V = 0.210 m^3. To find the values of discrete profiles it is necessary to perform discrete optimization of truss DPD, the realization of which is intended in the future.

Table

Calculated optimal theoretical cross-sections (*) and the closest profiles to these cross-sections

	Profile	$A_{0,r}$, m^2	I_r, m^4
R_1:	*	**$2.636\cdot10^{-3}$**	**$1.009\cdot10^{-6}$**
	IPE O180	$2.710\cdot10^{-3}$	$1.173\cdot10^{-6}$
	IPE 180	$2.395\cdot10^{-3}$	$1.009\cdot10^{-6}$
	IPE 200	$2.848\cdot10^{-3}$	$1.424\cdot10^{-6}$
R_2:	*	**$5.911\cdot10^{-3}$**	**$7.881\cdot10^{-6}$**
	IPE 330	$6.261\cdot10^{-3}$	$7.881\cdot10^{-6}$
	IPE O300	$6.283\cdot10^{-3}$	$7.457\cdot10^{-6}$
	IPE A330	$5.474\cdot10^{-3}$	$6.852\cdot10^{-6}$
R_3:	*	**$4.476\cdot10^{-4}$**	**$1.034\cdot10^{-8}$**
	TUB 20×45×4.0	$4.560\cdot10^{-4}$	$2.467\cdot10^{-8}$
	TUB 25×40×4.0	$4.560\cdot10^{-4}$	$3.898\cdot10^{-8}$
	TUB 30×40×3.5	$4.410\cdot10^{-4}$	$5.083\cdot10^{-8}$

Consumption diagram of truss elements reliability indices $\boldsymbol{\Delta\beta} = \boldsymbol{\beta}_0 - \boldsymbol{\beta}$ (Fig. 8) shows, that truss elements E{3, 8, 9, 22, 25} are designed in the state of the limit reliability index $\boldsymbol{\beta}_0$. Reliability indices of the truss bottom flange are distributed in the interval $\beta_{1\text{-}7}$ = [1.640; 3.703], top flange – $\beta_{8\text{-}14}$ = [3.000; 5.100], grid – $\beta_{15\text{-}30}$ = [1.640; 8.022].

The influence of limit reliability index on the truss volume $V(\boldsymbol{\beta}_0)$ (Fig. 9) is performed, and the influence of standard deviations of external loads to the truss volume is realized through the loads reliability ratio γ_F

$$\sigma_F \cong \mu_F \frac{\gamma_F - 1}{1.64} \tag{41}$$

Also the influence of standard deviation of the steel yield strength σ_{Ry} and steel elastic modulus σ_E on the structure volume is performed and shown in the diagram (Fig. 10).

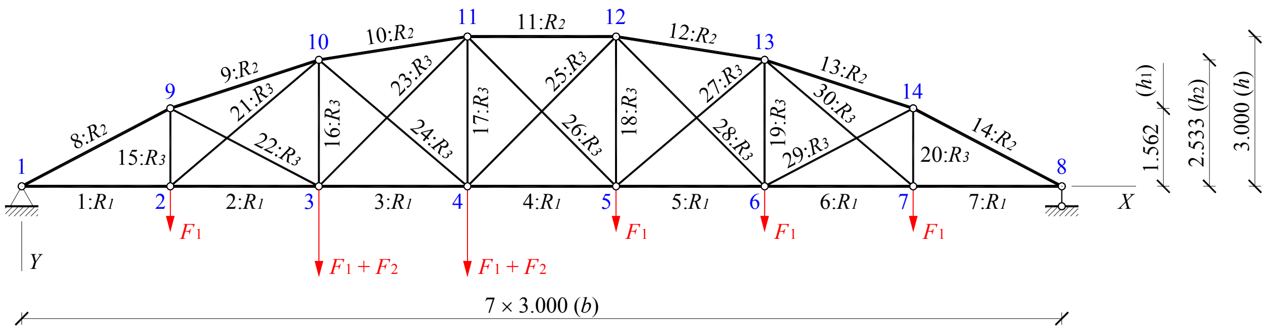

Fig. 6 Truss modeled with LINK1 finite elements

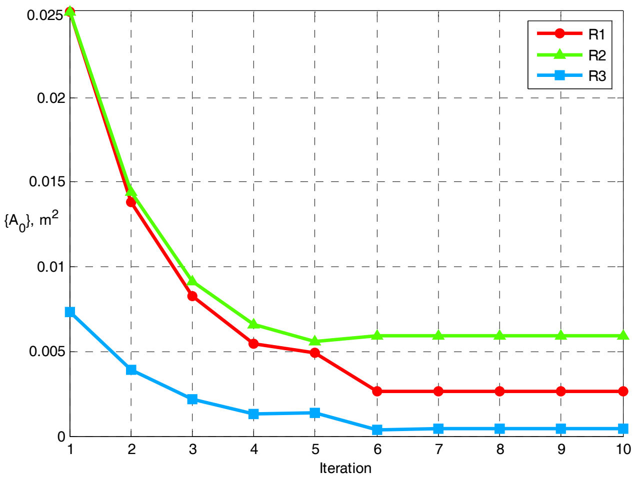

Fig. 7 Variation dynamics of iterative solution of the optimal A_0

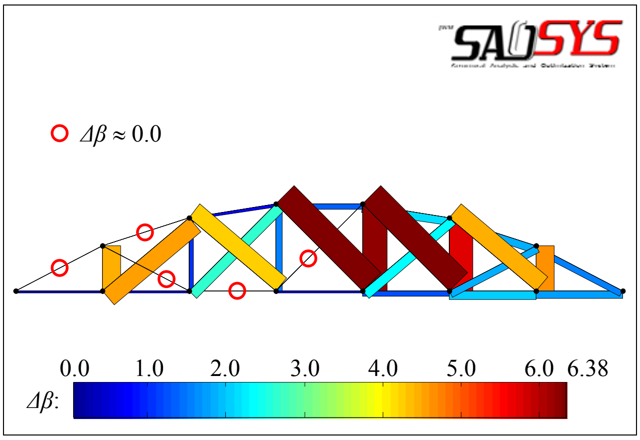

Fig. 8 Outgo diagram of truss elements reliability index

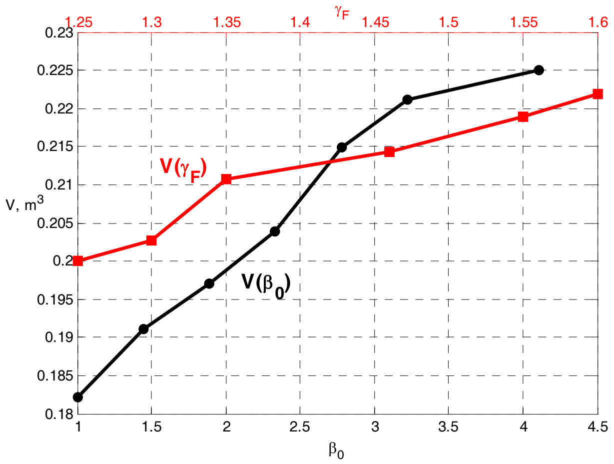

Fig. 9 Truss volume dependencies on the limit reliability index β_0 and loads reliability ratio γ_F

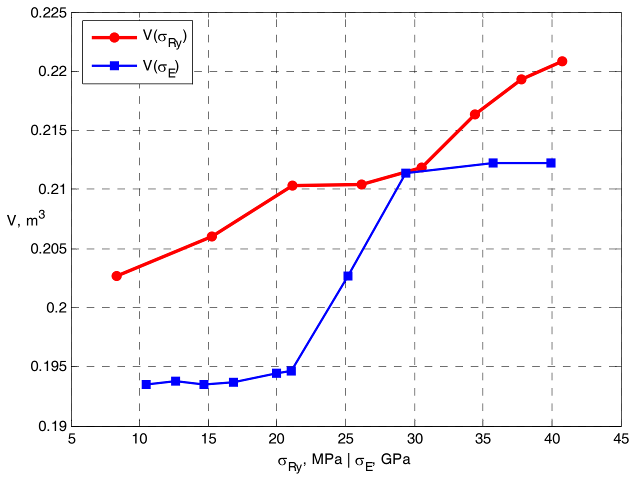

Fig. 10 Truss volume dependency on steel σ_{Ry}, σ_E standard deviations

8. Conclusions

1. Direct probability design of optimal trusses is an optimization problem of nonlinear mathematical programming, which can be solved in approximation way.

2. Applying the principle of admissible fields of assortments profiles characteristics (optimized leading $\boldsymbol{G}_0$ and controlled driven geometry $\boldsymbol{G}_1$) we can directly estimate the distribution of profiles discrete characteristics.

3. SAOSYS system created by the authors' and its modulus of trusses direct probability design TrussDPD allows to model and design any plane trusses from assortments profiles evaluating elements strength and stability reliabilities separately in the cases of tension or compression deformation.

4. The volume of the bridge-truss analyzed in numerical example responds fairly sensitively to the variations of the limit reliability index β_0, loads and material physical-mechanical properties.

5. With reference to the data of statistical control and applying mathematical programming theory we can not only guarantee the safety reliability of the structure elements but also create more economic projects.

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OPTIMALIŲ PLIENINIŲ SANTVARŲ TIESIOGINIS TIKIMYBINIS PROJEKTAVIMAS MATLAB APLINKOJE

R e z i u m ė

Straipsnyje nagrinėjamas optimalių plieninių santvarų tiesioginis tikimybinis projektavimas taikant matematinį programavimą. Projektuojant atsižvelgiama į medžiagų fizikinių-mechaninių savybių, elementų skerspjūvių geometrinių charakteristikų bei išorinių poveikių variacijas, taip pat į elementų stiprumo atsargos ir stabilumo patikimumo reikalavimus. Sudarytas optimalių santvarų projektavimo netiesinio matematinio programavimo uždavinio matematinis modelis. Pasiūlytas santvaros elementų projektavimo iš sortimentinių profiliuočių, tiesiogiai vertinant jų diskretinių charakteristikų sklaidą, algoritmas. Projektavimo algoritmas realizuotas MATLAB aplinkoje, autorių sukurtoje JWM SAOSYS Toolbox v0.40 konstrukcijų modeliavimo baigtiniais elementais, analizės ir optimalaus projektavimo sistemoje. Pateiktas tiltinės santvaros, veikiamos vienkartės apkrovos, tiesioginio tikimybinio optimalaus projektavimo pavyzdys.

V. Jankovski, J. Atkočiūnas

MATLAB IMPLEMENTATION IN DIRECT PROBABILITY DESIGN OF OPTIMAL STEEL TRUSSES

S u m m a r y

Direct probability design (DPD) of optimal steel trusses using mathematical programming means is discussed in this paper. The variations of material physical and mechanical properties, elements cross-sections geometry characteristics and external actions are considered while designing. Strength and stiffness requirements of truss elements are estimated. The mathematical model of nonlinear mathematical programming problem of optimal trusses design is created. Solution algorithm of truss elements (picked from profile assortments) design, directly evaluating dispersion of profiles discrete characteristics, is proposed. The finite elements structures analysis and optimization software JWM SAOSYS Toolbox v0.40, which is created in MATLAB environment, realizes design algorithm. Numerical example of bridge-truss subjected by the static load DPD is solved.

В. Янковски, Ю. Station

ПРЯМОЕ ВЕРОЯТНОСТНОЕ ПРОЕКТИРОВАНИЕ ОПТИМАЛЬНЫХ МЕТАЛЛИЧЕСКИХ ФЕРМ В СИСТЕМЕ MATLAB

Р е з ю м е

Рассматривается прямое вероятностное проектирование оптимальных металлических ферм. Фермы проектируются по условиям прочности и устойчивости элементов с заданной надежностью, при этом учитываются вариации физикомеханических свойств материалов, геометрических характеристик профилей и нагрузок. Построенная математическая модель задачи нелинейного программирования с условиями вероятностного оптимального проектирования и разработанный алгоритм решения учитывают и разброс дискретных характеристик профилей в ассортиментах. Алгоритм реализован в комплексной среде технических вычислений MATLAB, созданной авторами прикладноинструментальной системой JWM SAOSYS Toolbox v0.40 конечноэлементного моделирования, анализа и оптимального проектирования строительных конструкций. Приводится численный пример проектирования мостовой фермы минимального объема.

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