Global optimization of grillages using genetic algorithms

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1. Introduction

Optimization is an inherent part of all engineering practice. In the construction of buildings that means, all parts of buildings from foundations to roofs should be designed and built optimally and thrifty as much as the conditions of safety and comfort allow. We note that many problems of engineering (and also of physics, technology, economics) are reduced to global minimization of multi-modal functions. Such problems are difficult in the sense of the algorithmic complexity theory and global optimization algorithms are computationally very intensive. The global optimization algorithms are reviewed in Törn and Žilinskas [1], Horst et al. [2], Dagys et al. [3], Atkočiūnas et al. [4]. In this paper we shall concentrate on the optimal design of grillage-type foundations, which are the most popular and effective scheme of foundations, especially in case of weak grounds. From the mathematical point of view these problems are very attractive, because here the lower bound of global solution is known in advance. In the case when all piles have equal characteristics, the bound is obtained simply dividing the total sum of active forces on the grillage by the number of piles. It is very important, because even with the use of sophisticated global optimization algorithms and parallel computers only the small-scale problems (usually possessing few tens of design parameters) can be solved to the utmost, i.e., to the global solution. Thus, in optimization of grillages always it is possible to judge on the rationality of obtained solutions.

Grillage consists of separate beams. Separate beam may be supported by piles, may reside on other beams, or a mixed scheme may be the case. The optimal scheme of grillage should possess, depending on the given carrying capacities of piles, the minimum possible number of piles. Theoretically, reactive forces in all the piles should approach the limit magnitudes of reactions for those piles (limit pile reactions may differ from beam to beam provided different characteristics, i.e., diameters, lengths, profiles of piles are given). This goal can be achieved by choosing appropriate pile positions. Designer may arrive at the optimal pile placement schema by engineering tests algorithms only in case of simple geometries and simple loadings. Otherwise mathematical optimization procedures are evident necessities.

Practically, the goal is much more difficult to achieve if designer due to some considerations introduces into the grillage scheme the so-called 'immovable supports' that have to retain their positions and do not participate in optimization process. Pursuing the mathematical transparency, this case is not considered in the paper. The other preclusion for achieving the global solution is the required minimum allowed distance between adjacent supports; this requirement emerges due to the technological features of the pile driver. In optimization procedures it is treated as the constraint (for local optimization) or penalty (global optimization).

Local optimization. The first works in the grillage optimization were based on the local optimization of a single beam (Belevičius, [5-8]). All grillage is divided into separate beams, the "upper beams" resting on other – the "lower beams". First, all the beams are analyzed and optimized separately. Joints and intersections of the upper and lower beams are idealized as an immovable supports for upper beams. Reactive forces in these fictitious supports are obtained during the analysis stage of the upper beams. Joints for the lower beams are idealized as a concentrated loads of the same magnitude but of opposite sign to the reactive forces in fictitious supports. If more than 2 beams meet at the joint, all the beams are considered to be the "uppers" except for one the "lower". Distinguishing between the upper and lower beams can be done automatically or by the designer's wish. Since the obtained fictitious reactions/concentrated loads depend on the pile positions, all calculations are iterated in order to level with proper accuracy the forces at joints (or stiffnesses, if desired). The solution for each separate beam requires three steps: finite element analysis, sensitivity analysis, and optimal redesign with linear programming.

The main shortcoming of this technique is that the search inevitably leads to a local solution which in most cases does not satisfy the designer. A halfway solution of the problem is to start optimization procedure from near-optimum initial scheme. However, the special expert system which analyses geometry of the beam, loadings and carrying capacities of piles and yields the quasi-optimal initial pile placement scheme becomes the most sophisticated part of all project and still is not capable to render rational initial solutions for a sophisticated grillages.

Global optimization. First of all two-dimensional grillage is "unfolded" to a one-dimensional beam, and the supports are allowed to range through this space freely. The optimization routine yields the distribution of supports in this space. The "black box" finite element program transforms the one-dimensional beam back into the real grillage with a given positions of supports and returns the value of objective function – the maximum of reactive forces at supports. In case when two or more supports approaches each other to a distance less than minimum allowed or the distribution of supports is invalid, i.e. yielding the grillage-mechanism, the finite element solution is skipped and penalty is given instead of maximum reactive force. The black box routine provides also the sensitivity results of objective function to capacitate the local search around the promising point.

The Branch-and-Bound (BB) global optimization algorithm which allows rejecting the not promising subdomains from the whole problem domain was used for the optimization. Any BB algorithm consists of two main steps:

the branching rule which allows selecting a subdomain and dividing it into several smaller parts, and computation of lower bounds of objective function for each new subdomain. If the bound is larger than the best known approximation of the value, then this subdomain is rejected from the further searches. Since all global optimization algorithms are computationally very demanding, the parallel computations were employed using the 'master – slave' template to parallelize the algorithm (Baravykaitė [8]).

Using this approach the optimization problems possessing until 40 design parameters were solved. The main drawback of this solution scheme is connected with a lengthy, unacceptable for the engineering practice solution time. For example, the problem with 10 design parameters on a cluster of 10 dual processors PC of Vilnius Gediminas Technical University was solved in 60 min., while the problem with 15 design parameters requires 5 hours (Čiegis [9]). Later the solution of the problem using GAs has clearly shown that the landscape of objective function is very complex, with a numerous local minimums. The global search algorithm inevitably spends time exploring sub-domains around these points.

The paper is organized as follows. In section 2 we formulate the optimization problem. In section 3 the genetic algorithms (GA) and the coding of the problem for GA are described. Section 4 provides numerical examples and comparison of results obtained using GA and other global optimization results. Some final conclusions are given in section 5.

2. Problem formulation

The finite element method is used for the evaluation of objective function. The girders of grillage are idealized as beam elements with given cross-section and material characteristics, and the piles – as the supports with specified displacements (where zero displacements are the most common case), or supports with specified stiffness characteristics. Supports of the first type are rather non-realistic representations and sometimes yield misleading analysis results. For example, when multiple supports are needed to carry large concentrated load, this kind of supports will lead to a logjam. If odd number of supports is placed under load, the central support will be located just beneath the load and will take all the force. In case of even number of supports the "saw-teeth" like distribution of reactions is observed, and the more supports will be installed, the larger in absolute value reactions will arise.

The optimization problem is stated as follows (see, eg. Belevičius et al. [5])

$$\min_{s.t. \; x \in D} P(x) \qquad (1)$$

where $P(x)$ is the objective function, D is the feasible shape of structure, which is defined by the type of certain supports, the given number and layout of different cross-sections as well as different materials in the structure.

P is defined by the maximum difference between vertical reactive force at a support and allowable reaction for this support, thus allowing us to achieve different reactions at supports on different beams, or even at particular supports on the same beam

$$P(x) = \max_{1 \le i \le N_s} \left| R_i - f_i R_{allowable} \right| \qquad (2)$$

here N_s denotes the number of supports, $R_{allowable}$ is allowable reaction, f_i are factors to this reaction and R_i are reactive forces in each support.

Evidently the minimization problem of reactive forces in piles can not be represented in closed form algebra, is nonlinear and nonconvex. The value of objective function is supplied by an independent finite element program which is connected to the optimization algorithm as a black box.

Finite element matrices and sensitivity analysis. The problem has to be solved in statics and in linear stage

$$[K]\{u\} = \{F\} \qquad (3)$$

Here $[K]$ is the stiffness matrix of grillage, $\{u\}$ are the displacements of grillage nodes, and $\{F\}$ are the loadings. The reactive forces at a rigid supports are obtained using equation

$$R_i = \sum_j K_{ij} u_j , \; i = 1, 2, ..., N_s \qquad (4)$$

where a part of nodal displacements (displacements of free nodes) are already obtained via Eq. (3), and the displacements of nodes representing the rigid supports are specified (usually – zero). If the supports have finite stiffnesses k_i

$$R_i \approx k_i u_i , \; i = 1, 2, ..., N_s \qquad (5)$$

The sensitivity analysis which is required for the local search around the certain optimization solution is performed using the pseudo-load approach; thus the expensive and not accurate numerical calculation of derivatives can be avoided. Denoting the support positions by $x_i, i = 1, 2, ..., N_s$

$$R_{i,x_i} = [K]_{,x_i}\{u\} + [K]\{u\}_{,x_i} \qquad (6)$$

Here the derivative of stiffness matrix is obtained analytically, while the derivative of displacements supposes solution of the general sensitivity equation

$$[K]\{u\}_{,x_i} = \{F\}_{,x_i} - [K]_{,x_i}\{u\} \qquad (7)$$

The derivatives of load vector are obtained also in a closed form, analytically.

A simple two-node beam element with 6 dof's at a node (three displacements and three rotations about local element axes) is employed in the analysis. The element stiffness matrix can be found in many finite element textbooks, for example, in Zienkiewicz [10]. More details about finite element matrices are provided in Belevičius [7].

Program. The finite element mesh of frame is prepared automatically by the special pre-processor, introducing nodes at the immovable support places (if any), jumps of material and cross-sections properties, etc. The number of movable supports is obtained also by program

so that the total magnitude of loading could be absorbed by the piles of given characteristics. The frame is remeshed either in each step of redesign (for local optimization), either for each guess of BB algorithm or for each individual of genetic algorithms (GA) population (for global optimization).

3. GAs for optimization of grillages

In the last decades, a lot of attention has been given to the application of GA in various optimization problems. It can be expected, that the GAs which are stochastic (probabilistic) global optimization methods simulating the evolution laws of the nature (Goldberg [11]) may be promising in this type of large scale optimization problems due to the following reasons:

1. the convergence behavior of stochastic algorithms only depends on the objective function evaluations, i.e. the expensive sensitivity information is not needed;
2. the theoretical analysis of stochastic search methods indicates that they can be executed in polynomial time, on the average, while deterministic methods take exponential number of function evaluations;
3. there is no guarantee that the stochastic algorithms will find the better solution within a predefined time or number of iterations. Also, they are likely to yield an approximate solution. However, this drawback is not very important for real-world applications where the theoretical optimum solution is usually not required.

Algorithm. The typical genetic algorithm is schematically shown in Fig. 1.
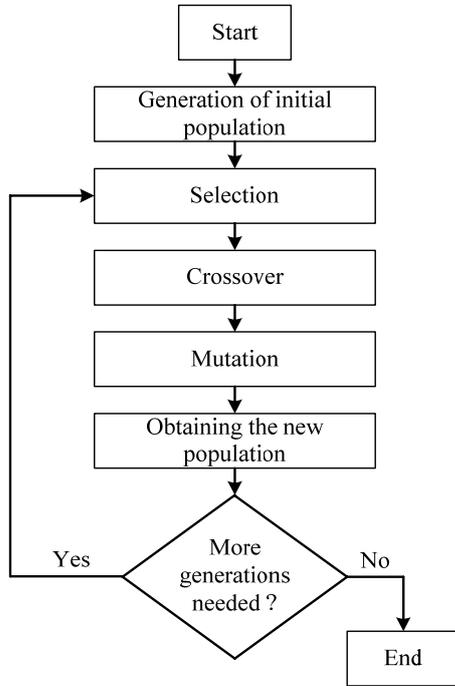
Fig. 1 Scheme of genetic algorithm

In the implementation of GA for grillage optimization, the individual of a population is one particular variant of the grillage with a set of supports of given characteristics. The number of supports is obtained in advance by the pre-processor of finite element program. This theoretical number of supports is the same for all individuals of the population. The initial population of individuals is generated randomly: the probability to gain the numerical values "1" or "0" for all genes of the chromosome is 0.5 (see subsection below). The population size remains constant during all optimization process. The elitist selection strategy, where several best individuals of the population are always chosen to survive during the selection, proved to be more effective than the classical GAs for the optimization of grillages. These best individuals bypass the crossover stage. The remaining candidates for survival are chosen by the roulette principle and undergo the crossover. During this stage, the one-point crossover operator is applied to the two chosen individuals at the random gene (thus, there is a small probability to avoid the crossover in case when the last gene is elected as the crossover point). In the mutation phase of algorithm, the mutation operator is applied to all the genes of genotype with an equal (usually small) probability.

All the genetic parameters of algorithm (population size, number of elite individuals, probabilities of the crossover and mutation, number of generations) must be thoroughly adjusted to the particular problem; they are provided in the next chapter.

Coding of an individual. Again, the optimization routine works with one-dimensional construct of the grillage, therefore the position of a particular pile is implicitly described by the only coordinate. First of all, the possible positions of piles are obtained dividing the overall length of one-dimensional construct by the intended number of those positions. It should be noted, that this number of positions must equal to the 2^N, where N is the number of bits for coding of pile coordinate. The more bits will be allocated for coding, the more possible positions of piles will be at hand, and therefore the solution of problem can be closer to the global one. However, at the same time the length of one population individual increases. From the engineering point of view, the N providing the distance between adjacent piles of about 0.5 m is usually sufficient. The whole individual is coded connecting coordinates of all piles into one string of bits

$$\underbrace{a_1 a_2 \ldots a_N}_{Node\_1} \underbrace{a_1 a_2 \ldots a_N}_{Node\_2} \ldots \underbrace{a_1 a_2 \ldots a_N}_{Node\_K} , \ a_i \in \left\{ \begin{array}{c} 0 \\ 1 \end{array} \right\} \quad (9)$$

here K is the total number of piles, N is the intended number of bits for coding of coordinate of each pile.

For the sake of transparency let us illustrate the coding of an individual by the following simple example. Let the whole length of grillage is 3.5 m, and the theoretical number of piles is 4. In case we allocate 3 bits for coding of one pile coordinate, 8 possible positions of piles will be available, and the distance between two adjacent piles will be 0.5 m. This one-dimensional construct of grillage with possible pile positions and the corresponding strings of bits are shown in Fig. 2.
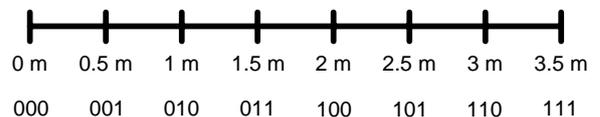

Fig. 2 Coding of possible pile positions

For example, the problem solution yields the string 000010101110. The pile placement scheme corresponding to this string is rendered in Fig. 3.
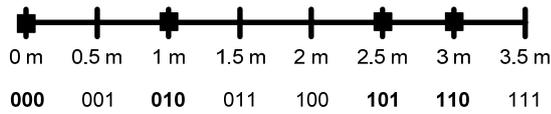

Fig. 3 Possible pile placement in one-dimensional construct

The finite element program with the meshing and transformation modules is connected to the genetic optimization routine as the black box; its only aim is to return the maximum reactive force or the penalty, if the constraints of problems are violated. The sensitivity information is not needed here.

4. Numerical results

To compare possible optimization strategies, two support placement schemes for relatively simple grillages

a

b
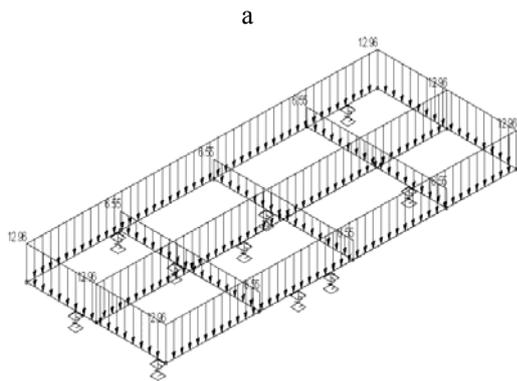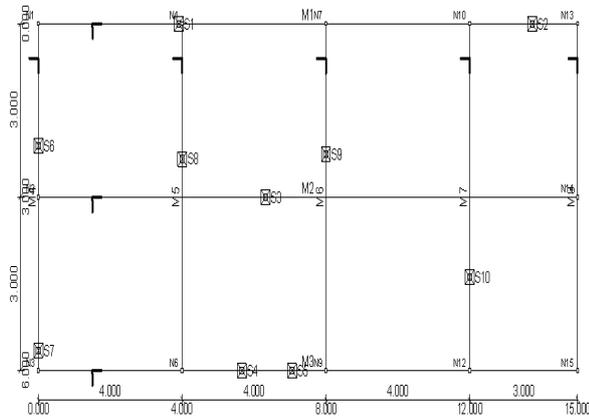
Fig. 4 10-pile grillage: a – geometry, b – load cases and the obtained pile placement scheme

for which the solutions with other algorithms are available, were reobtained. The code requires only input on geometry of grillage, loading, pile and beam material characteristics (i.e. carrying capacity or stiffnesses). Also the allowable vertical reaction and minimum allowable distance between two adjacent supports should be known.

Despite of simple geometry of both schemes, these grillages expose extreme sensitivity to the positions of supports. Small changes of supports' coordinates may raise significant perturbations in reactive forces. Also, several very different support distribution schemes may demonstrate close magnitudes of the objective function.

Example 1. Grillage of rectangular shape loaded with two sets of distributed vertical loadings (Fig. 4, a). Construction of grillage consists of standard prefab reinforced concrete girders. The main determinant data for support scheme are the maximum allowable vertical reaction, the minimum allowable distance between two adjacent supports, and the vertical stiffness of support: 200, 0.20, and 1.e15, accordingly. The theoretical number of supports is 10.

The genetic parameters adjusted to this problem are the following: the population size – 20 individuals (2 – elite individuals), the probabilities of crossover and mutation – 99% and 1%, accordingly; the number of generations – 200. 10 bits were allotted for coding of support position, thus in the one-dimensional construct of grillage obtaining 1024 possible different positions for each support. Since GA is stochastic algorithm, the problem was solved 30 times; each random numerical experiment requires approximately 23 minutes. The best obtained objective function values are shown in Fig. 5.

Thus, the best solutions were obtained in the 29th, 14th and 8th experiments: the maximum reactive force is
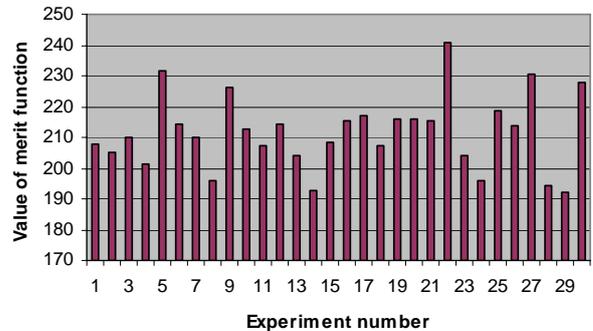

Fig. 5 Example 1: the best obtained objective function values

Table 1

Numerical results of optimization experiments (example 1)

Number of pile	Coordinates (29th exp.)	Coordinates (14th exp.)
1	36.7676	62.1094
2	19.0430	57.7881
3	15.6006	15.7471
4	55.8838	24.5361
5	8.27637	21.7529
6	22.4121	35.0830
7	47.0215	70.9717
8	74.4873	1.02539
9	25.6348	32.0801
10	3.80859	52.8809

192.4, 192.8, and 195.7, accordingly, while the theoretical global solution is 183.8. The positions of supports for the two best obtained solutions belong to the very different topologies of grillage as illustrated in Table 1. The pile placement scheme corresponding to the 29th experiment is shown under the given loadings in Fig. 4, b.

These results of GA are compared to the previous results obtained using different algorithms in Table 2; the solution time and computer performances are provided also.

Example 2. Grillage consists of two rectangular frames under distributed loadings (Fig. 6, a). Theoretical number of supports for the main limiting factors 150, 0.10, 1.e10 (as in Example 1) is 15, whereas the theoretical global solution is 143.0. The tuned genetic parameters are (in sequence of Example 1) 30 (2 elite) individuals, 99.1%, 0.4%, 300 generations and 11 bits for coding of support position (thus having 2048 possible positions for each support in the grillage). Again, 30 independent numerical experiments were performed. The best obtained objective function values are shown in Fig. 7.

Table 2
Comparison of results obtained using different algorithms

Example 1: 10 supports Theoretical global solution 183.8		
Algorithm	Results	Platform and solution time
BB	207.5	Cluster of 10 dual Intel processor PC, 60 min [10]
BB	190.2	Pentium 2.5 GHz PC, 378 min [12]
GA	192.4	Pentium 1.6 GHz PC, 23 min (for one experiment)
Example 2: 15 supports Theoretical global solution 143.0		
BB	161.5	Cluster of 10 dual Intel processor PC, 300 min [10]
BB	161.1	Pentium 2.5 GHz PC, 300 min [Žilinskas, 2007]
GA	157.7	Pentium 1.6 GHz PC, 24 min (for one experiment)

a   b

Fig. 6 15-pile grillage: a – geometry, b – load cases and the obtained pile placement scheme
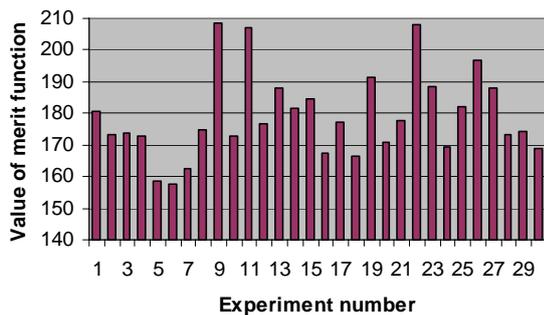
Fig. 7 Example 2: the best obtained objective function values

For this example the best solutions were obtained in the 6th, 5th and 7th experiments: 157.7, 158.6, and 162.7, respectively. The coordinates of supports in one-dimensional space for the two best obtained solutions are shown in Table 3. The best pile placement scheme is shown also graphically in Fig. 6, b.

Table 3
Numerical results of optimization experiments (example 2)

Number of pile	Coordinates (6th exp.)	Coordinates (5th exp.)
1	19.7764	44.8164
2	9.24902	38.8760
3	27.5967	23.9873
4	0.977539	26.3184
5	34.8154	52.1104
6	67.6758	9.54980
7	43.8389	28.4990
8	51.1328	36.2441
9	72.7139	75.0449
10	54.0654	42.0342
11	74.6689	66.5479
12	25.1904	2.40625
13	37.2969	60.9834
14	64.8936	68.3525
15	21.2803	22.2578

5. Conclusions

Different optimization techniques were compared for the global optimization problem of pile placement schemes in grillage-type foundations. The global solutions were not obtained using any of discussed algorithms; however, both BB and GA global optimization techniques render rational solutions. The required computer resources to achieve solutions of approximately the same accuracy level for BB algorithms are much higher than for GA.

GA compared with other global optimizers yields the reasonable solution in shorter time. The solution results depend on the genetic parameters (population size, mutation probability, crossover operator); investigation of reasonable ranges of these parameters always assures a better solution. One promising way to enhance the optimization process is to use the sensitivity information while mutating the genotype of an individual; the mutation process in GAs plays similar role as the local search around the BB guess.

Since the GAs are stochastic algorithms, always a number of numerical experiments should be performed. On the other hand, it may be very advantageous for the engineering practice, because the designer might choose a beneficial solution (i.e. certain topology of pile placement scheme) from a number of different topology solutions having close magnitudes of objective function. In case the advanced computer system is at designer disposal, the parallelization of solution process is straightforward, assigning one numerical experiment for one node.

Acknowledgement

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