ISSN 1392 - 1207. MECHANIKA. 2009. Nr.1(75)

Effect of the temperature gradient on heat transfer and friction in laminar liquid film

S. Šinkūnas*, A. Kiela**
**Kaunas University of Technology, Donelaičio 20, 44239 Kaunas, Lithuania, E-mail: stasys.sinkunas@ktu.lt*
***Kaunas College, Pramonės 22, 50387 Kaunas, Lithuania, E-mail: algimantas.kiela@kauko.lt*

Nomenclature

a - thermal diffusivity, m^2/s; a_b - wavy thermal diffusivity, m^2/s; a_t - turbulent thermal diffusivity, m^2/s; c - specific heat, J/(kg·K); d - hydraulic diameter of the film, m; g - acceleration of gravity, m/s^2; Nu_d - Nusselt number, $\alpha d/\lambda$; Nu_M - modified Nusselt number, $(\alpha/\lambda)(\nu^2/g)^{1/3}$; Pr - Prandtl number, ν/a; q - heat flux density, W/m^2; R - cross curvature of the wetted surface (tube external radius), m; Re - Reynolds number, $4\Gamma/(\rho\nu)$; T - temperature, K; v^* - dynamic velocity, $(\tau_w/g)^{1/2}$; w - film velocity, m/s; y - distance from wetted surface, m; α - heat transfer coefficient, W/(m^2·K); Γ - wetting density, kg/(m·s); δ - liquid film thickness, m; φ - dimensionless film velocity, w/v^*; η - dimensionless distance from wetted surface, $v^* y/\nu$; η_δ - dimensionless film thickness, $v^*\delta/\nu$; ε_R - relative cross curvature of the film, δ/R; λ - thermal conductivity, W/(m·K); ν - kinematic viscosity, m^2/s; ϑ - temperature field; ρ - liquid density, kg/m^3; τ - shear stress, Pa;
Subscripts: b - wavy; f - film flow; g - gas or vapour; s - film surface; t - turbulent; w - wetted surface.

1. Introduction

In recent years the research in the field of two-phase flow heat transfer has been constantly increasing due to the rapid growth of the technology applications that require the transfer of high heat rates in a relatively small space and volume. Such applications vary from compact heat exchanger to cooling systems for computer. Two-phase flow heat transfer was the subject of numerous researchers [1-3] for the last decade. Unsteady transfer processes taking place in two-phase flow, which is obtained by spraying water into low potential flue gas were researched numerically [4]. It was shown that the change of two-phase system can be classified according to the peculiarities of transfer processes, picking out unsteady and equilibrium modes of the state change. Thin liquid films falling under the influence of gravity are widely encountered in a variety of industrial applications that involve gas-liquid two-phase flow [5, 6]. Flow in nuclear reactor cores, steam condenser, water tube boilers and vertical tube evaporators are some of the practical examples. In order to design these systems with greater efficiency and lower cost, a basic understanding of heat and momentum transfer processes occurring in falling films is needed.

Theoretical model [7] was derived to present the temperature distribution of falling liquid films flowing over a vertical heated/cooled plate with constant temperature. The temperature gradients for different flow rates and different liquid films were also discussed. The temperature distributions for liquid films of water, ethanol aqueous solutions and glycerol aqueous solutions were experimentally investigated with a sensitive thermal imaging system. It was found that the surface temperature of a film flowing over a vertical solid plate has a characteristic relationship with the film flow distance. A lower flow rate of the film or a higher temperature of the wall generally leads to a higher surface temperature in the film inception.

The paper [8] describes an experimental investigation of the hydrodynamics of an evaporating wetting film meniscus in a capillary tube where a temperature gradient is applied along the wall. The results showed that the ability of the evaporating meniscus to wet the capillary tube is degraded by the temperature gradient along the wall.

It has been shown in study [9] that the heat transfer coefficients obtained from using the 1D transient liquid crystal scheme are higher than those obtained from employing the 3D scheme when surface heat transfer is highly nonuniform such on a hot surface subject to jet impingement cooling. This is due to the fact that 1D method does not include the lateral heat flows induced by local temperature gradients.

The paper [10] has investigated the heat and momentum transfer of a water film falling over a tilted plate with solar radiant heating and water evaporation. The results revealed that the gradients of temperature and the mass fraction of water vapor in the gas layer, and the wind velocity played a key role in the heat and momentum transfer along the gas-water interface. The water film Reynolds number related to the film thickness markedly exerted an influence on the eddy viscosity and the turbulent Prandtl number of the water film.

Study [11] reviews experimental local heat transfer data for laminar and turbulent film heat transfer of downward condensing films under the influence of interfacial waviness and shear stress effects. The results demonstrate that the dimensionless film thickness, incorporating shear stress, provides a more appropriate length scale to estimate laminar-wavy film heat transfer as well as transition to turbulence.

Local reflux condensation heat transfer coefficients have been determined inside a vertical tube within water, ethanol and isopropanol as the test fluids in [12]. The heat transfer has found to be impeded by shear stress only in cases of a very thin film, i.e. in the smooth laminar range and it can well be correlated by a simple analytical model. In the laminar-wavy range, including developing turbulence, the heat transfer coefficients are found to increase with the shear stress, an effect which proved to be enhanced with rising *Pr* numbers.

Noninvasive measuring method based on luminescence indicators to determine the temperature distribution and the local film thickness simultaneously was developed [13]. Results are presented for the temperature distribution measurements in a laminar-wavy water film with Reynolds number of 125. The measured temperature distributions were used to calculate the local heat transfer coefficients and heat flux perpendicular to the wall for different points in the development of a solitary wave.

2. Stabilized heat transfer in a liquid film flow

For stabilized turbulent liquid film flow on a vertical surface shear stress can be expressed as

$$\tau = \rho(\nu + \nu_t)\frac{dw}{dy} \tag{1}$$

and dimensionless form is

$$\frac{\tau}{\tau_w} = \left(1 + \frac{\nu_t}{\nu}\right)\frac{d\varphi}{d\eta} \tag{2}$$

By taking into account a variation of liquid physical properties, Eq. (2) can be rewritten as follows

$$\frac{\tau}{\tau_w} = \frac{\rho}{\rho_f}\left(\frac{\nu}{\nu_f} + \frac{\nu_t}{\nu_f}\right)\frac{d\varphi}{d\eta} \tag{3}$$

Then, the dimensionless film velocity can be expressed as

$$\varphi = \int_0^\eta \frac{\tau/\tau_w}{\frac{\rho}{\rho_f}\left(\frac{\nu}{\nu_f} + \frac{\nu_t}{\nu_f}\right)} d\eta \tag{4}$$

In the case of liquid density variation, the following expression can be used

$$\tau = g\int_y^\delta (\rho - \rho_g)dy \tag{5}$$

Then, shear stress in the turbulent film can be defined by the following expression

$$\frac{\tau}{\tau_w} = \frac{\int_y^\delta (\rho - \rho_g)dy}{\int_0^\delta (\rho - \rho_g)dy} = 1 - \frac{\int_0^\eta \frac{\rho - \rho_g}{\rho_f} d\eta}{\int_0^{\eta_\delta} \frac{\rho - \rho_g}{\rho_f} d\eta} \tag{6}$$

In practice, the operation of real film heat exchangers is based on vertical tubes. Therefore, the equation for heat flux calculation across the turbulent film evaluating the surface cross curvature is as follows

$$q = -c\rho\left(1 + \varepsilon_R \frac{y}{\delta}\right)(a + a_b + a_t)\frac{dT}{dy} \tag{7}$$

In the case of plane film flow $(\varepsilon_R = 0)$, Eq. (7) can be rearranged as follows

$$q = -\frac{c\rho v^*}{Pr_f}\left(\frac{a}{a_f} + \frac{a_b}{a_f} + \frac{a_t}{a_f}\right)\frac{dT}{d\eta} \tag{8}$$

Let us denote that

$$\psi = \int_0^\eta \frac{\frac{c_f \rho_f}{c\rho}\frac{q}{q_w}}{\frac{a}{a_f} + \frac{a_b}{a_f} + \frac{a_t}{a_f}} d\eta \tag{9}$$

Then, from the Eq. (8), we can obtain the expression defining temperature field in the film

$$T_w - T = \vartheta = \frac{q_w Pr_f \psi}{c_f \rho_f v^*} \tag{10}$$

The temperature field intensity and heat flux density in the liquid film, when $\varepsilon_R = 0$ and $q_s = 0$ can be defined by the energy equation

$$c\rho w\frac{\partial T}{\partial x} + \frac{dq}{dy} = 0 \tag{11}$$

Twice integrating of Eq. (11) within the limits from 0 to y, q_w to q and from 0 to δ, q_w to 0 respectively, allows obtaining the ratio of heat flux densities in the film: when $T_w = const$

$$\frac{q}{q_w} = 1 - \frac{\int_0^\eta \frac{c\rho}{c_f \rho_f}\varphi\psi d\eta}{\int_0^{\eta_\delta} \frac{c\rho}{c_f \rho_f}\varphi\psi d\eta} \tag{12}$$

and when $q_w = const$

$$\frac{q}{q_w} = 1 - \frac{\int_0^\eta \frac{c\rho}{c_f \rho_f}\varphi d\eta}{\int_0^{\eta_\delta} \frac{c\rho}{c_f \rho_f}\varphi d\eta} \tag{13}$$

In accordance with Eq. (5), shear stress on the wall can be defined by the following expression

$$\tau_w = g\int_0^{\delta}\left(\rho-\rho_g\right)dy = \frac{\nu_f \rho_f g}{v^*}\int_0^{\eta_\delta}\frac{\rho-\rho_g}{\rho_f}d\eta \tag{14}$$

Then, assuming that $\tau_w = \rho_f v^{*2}$, the dynamic velocity of the film can be determined as follows

$$v^* = \left(g\nu_f\int_0^{\eta_\delta}\frac{\rho-\rho_g}{\rho_f}d\eta\right)^{1/3} \tag{15}$$

Dividing of Eq. (10) for temperature gradient $\Delta T = T_w - T_f$ and taking into account that $\alpha = q_w/\Delta T$, leads to the expression of relative temperature field in the film

$$\frac{\vartheta}{\Delta T} = \frac{Nu_{Mf}\psi}{\left(\int_0^{\eta_\delta}\frac{\rho-\rho_g}{\rho_f-\rho_g}d\eta\right)^{1/3}} \tag{16}$$

In this case, the following correlation can be obtained

$$Nu_{Mf} = \frac{\alpha}{\lambda_f}\left(\frac{\nu_f^2}{g}\frac{\rho_f}{\rho_f-\rho_g}\right)^{1/3} \tag{17}$$

It is evident that

$$\frac{\int_0^{\delta}\vartheta wc\rho dy}{\Delta T\int_0^{\delta}wc\rho dy} = \frac{\int_0^{\eta_\delta}\vartheta\varphi c\rho d\eta}{\Delta T\int_0^{\eta_\delta}\varphi c\rho d\eta} = 1 \tag{18}$$

Then, taking into account above-mentioned ratio, we obtain the following expression from Eq. (16)

$$Nu_{Mf} = \frac{\left(\int_0^{\eta_\delta}\frac{c\rho}{c_f\rho_f}\varphi d\eta\right)\left(\int_0^{\eta_\delta}\frac{\rho-\rho_g}{\rho_f-\rho_g}d\eta\right)^{1/3}}{\int_0^{\eta_\delta}\frac{c\rho}{c_f\rho_f}\varphi\psi d\eta} \tag{19}$$

In regard to the variation of liquid physical properties, the film Reynolds number can be defined as

$$Re_f = 4\int_0^{\eta_\delta}\frac{\rho}{\rho_f}\varphi d\eta \tag{20}$$

Heat transfer and shear stress calculations were performed for laminar flows of water, transformer oil, compressor oil, fuel oil and glycerine films respectively. The method of gradual approximation was applied for the calculations. The results of calculations are presented in Figs. 1and 2.

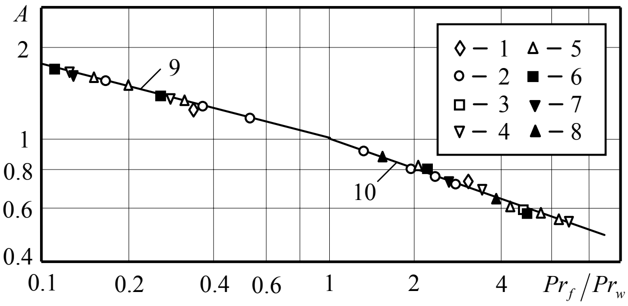

Fig. 1 Influence of the temperature gradient on dimensionless film thickness of various liquids in the case of laminar flow: *1*, *2*, *3*, *4*, *5* - water, transformer oil, compressor oil, glycerine, fuel oil respectively ($q_w = const$); *6*, *7*, *8* - compressor oil, glycerine, fuel oil respectively ($T_w = const$); *9*, *10* - calculation according Eq. (21), when $m = 0.25$ and $n = 0.32$ respectively; $A = \eta_\delta/\eta_{\delta_0}$

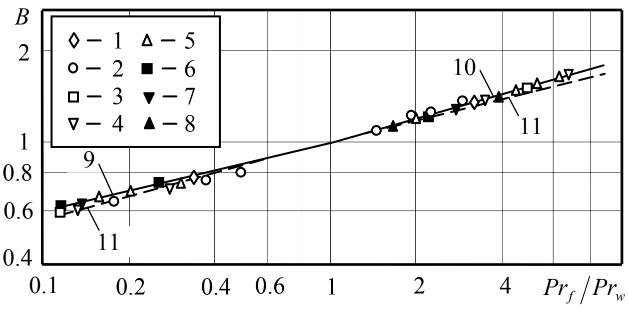

Fig. 2 Dependence of stabilized heat transfer on the temperature gradient in the case of laminar film flow for various liquids: *1*, *2*, *3*, *4* - water, transformer oil, compressor oil, glycerine, fuel oil respectively ($q_w = const$); *6*, *7*, *8* - compressor oil, glycerine, fuel oil respectively ($T_w = const$); *9*, *10*, *11* - calculation according Eq. (22), when $n = 0.23,\ 0.28,\ 0.25$ respectively; $B = Nu_M/Nu_{M_0}$

As we can see from Figs. 1 and 2, regardless of very different temperature dependent physical properties of used liquids and different boundary conditions on the wall, the calculations can be generalized in a good agreement with the following correlations

$$\frac{\eta_\delta}{\eta_{\delta_0}} = \left(\frac{Pr_f}{Pr_w}\right)^{-m} \tag{21}$$

$$\frac{Nu_M}{Nu_{M_0}} = \left(\frac{Pr_f}{Pr_w}\right)^{n} \tag{22}$$

where index 0 means heat transfer and shear stress under steady physical properties.

It evidently is seen from Figs. 1 and 2, that in the case of film heating $m \cong 0.32$ and $n \cong 0.28$ is respectively, but $m \cong 0.25$ and $n \cong 0.23$ is respectively, when the film is cooling.

Since the exponent n little depends on the heat flux direction, with sufficient accuracy it can be taken equal 0.25.

Therefore, in accordance with Eq. (21), we can obtain the equation for the dimensionless film thickness (friction) calculation in laminar plane flow

$$\eta_\delta = \left(\frac{3}{4} Re_f\right)^{1/2} \left(\frac{Pr_w}{Pr_f}\right)^m \tag{22}$$

In the case of temperature variation, liquid density varies very little. Then, in accordance with Eq. (15) the dynamic velocity of the film can be defined by the following equation

$$v^* = \left[g \nu_f \eta_\delta \left(1 - \frac{\rho_g}{\rho_f}\right)\right]^{1/3} \tag{23}$$

and the dimensionless form respectively

$$\eta_\delta = \frac{\delta^{3/2}}{\nu_f} \left(g \frac{\rho_f - \rho_g}{\rho_f} \right)^{1/2} \tag{24}$$

Consequently, the following correlation can be obtained

$$\frac{\delta}{\delta_0} = \left(\frac{\eta_\delta}{\eta_{\delta_0}}\right)^{2/3} = \left(\frac{Pr_w}{Pr_f}\right)^{\frac{2}{3}m} \tag{25}$$

Considering that, $Nu_M = 2.07 Re^{-1/3}$, $Nu_d = 7.5$ for boundary condition $T_w = const$ and $Nu_M = 2.27 Re^{-1/3}$, $Nu_d = 8.24$ for boundary condition $q_w = const$, we can obtain the following correlations for heat transfer calculations in laminar film: when $T_w = const$

$$Nu_{Mf} = 2.07 Re_f^{-1/3} \left(\frac{Pr_f}{Pr_w}\right)^{1/4} \tag{26}$$

or

$$Nu_{df} = 7.5 \left(\frac{Pr_f}{Pr_w}\right)^{1/4} \tag{27}$$

and when $q_w = const$

$$Nu_{Mf} = 2.27 Re_f^{-1/3} \left(\frac{Pr_f}{Pr_w}\right)^{1/4} \tag{28}$$

$$Nu_{df} = 8.24 \left(\frac{Pr_f}{Pr_w}\right)^{1/4} \tag{29}$$

For both cases, hydraulic diameter of the film in the Nu_d numbers can be determined like as for the isothermal film

$$d = 4\delta = 4\left(\frac{3}{4}\frac{\nu_f^2}{g} Re_f\right)^{1/3} \tag{29}$$

3. Conclusions

1. The main difficulties for the wavy and turbulent film flow are determining the thermal diffusivity coefficient a, the turbulent thermal diffusivity coefficient a_t and the turbulent kinematic viscosity ν_f under varying physical properties of the film. Therefore, heat transfer and friction calculations were carried out for laminar flows of water, transformer oil, compressor oil, fuel oil and glycerine films respectively.

2. The dependencies of stabilized heat transfer and friction on temperature gradient in the case of laminar film flow of various liquids with respect to variability of liquid physical properties were estimated analytically.

3. In the case of nonisothermality, transformation of the film thickness first of all is related to the variation of liquid viscosity. However, viscosity variation depends on the film temperature field, which is determined by the liquid thermal properties. Therefore, the influence of nonisothermality on the film thickness is more reasonable to evaluate using the ratio Pr_f / Pr_w.

References

1. **Miliauskas, G., Garmus, V.** Peculiarities of heated liquid evaporating droplets state change. -Mechanika. -Kaunas: Technologija, 2004, Nr.5(49), p.31-38.
2. **Ždankus, T.** Peculiarities of tube bundle heat transfer to vertical foam flow. -Mechanika. -Kaunas: Technologija, 2003, Nr.3(41), p.28-33.
3. **Miliauskas, G., Montvilas, R.** Influence of heating intensivity to semitransparent droplet state. -Mechanika. -Kaunas: Technologija, 1999, Nr.5(20), p.17-22.
4. **Miliauskas, G., Lukoševičius, V., Šavelskis, V.** Regulation of flue gas condition by water spraying. -Mechanika. -Kaunas: Technologija, 2002, Nr.4(36), p.30-36.
5. **Fujita, I., Hihara, E.** Heat and mass transfer coefficients of falling film absorption process. -Int. J. of Heat and Mass Transfer, 2005, v.48, p.2779-2786.
6. **Chakraborty, S., Som, S.K.** Heat transfer in an evaporating thin liquid film moving slowly along the walls of an inclined microchannel. -Int. J. of Heat and Mass Transfer, 2005, v.48, p.2801-2805.
7. **Zhang, F., Thang, D.L., Geng, J., Wang, Z.X., Zhang, Z.B.** Study on the temperature distribution of heated falling liquid films. -Physica D: Nonlinear Phenomena, 2008, v.237, p.867-872.
8. **Buffone, C., Sefiane, K., Christy, J.R.E.** Experimental investigation of the hydrodynamics and stability of

an evaporating wetting film placed in a temperature gradient. -Applied Thermal Engineering, 2004, v.24, p.1157-1170.

9. **Wang, T., Lin, M., Bunker, R.S.** Flow and heat transfer of confined impingement jets cooling using a 3-D transient liquid crystal scheme. -Int. J. of Heat and Mass Transfer, 2005, v.48, p.4887-4903.
10. **Song, B., Inaba, H., Horibe, A., Ozaki, K.** Heat, mass and momentum transfer of a water film flowing down a tilted plate exposed to solar irradiation. -Int. J. of Thermal Sciences, 1999, v.38, p.384-395.
11. **Peterson, P.F., Schrock, V.E., Kuhn, S.Z.** Recent experiments for laminar and turbulent film heat transfer in vertical tubes. -Nuclear Engineering and Design, 1997, v.175, p.157-166.
12. **Gross, U., Philipp, C.** Conjugated shear stress and Prandtl number effects on reflux condensation heat transfer inside a vertical tube. -Int. J. of Heat and Mass Transfer, 2006, v.49, p.144-153.
13. **Schagen, A., Modigell, M., Dietze, G., Kneer, R.** Simultaneous measurement of local film thickness and temperature distribution in wavy liquid films using a luminescence technique. -Int. J. of Heat and Mass Transfer, 2006, v.49, p.5049-5061.

S. Šinkūnas, A. Kiela

TEMPERATŪROS GRADIENTO POVEIKIS ŠILUMOS ATIDAVIMUI IR TRINČIAI LAMINARINĖJE SKYSČIO PLĖVELĖJE

R e z i u m ė

Teoriškai išnagrinėtas stabilizuotas šilumos atidavimas per skysčio plėvelę, esant maksimaliam šilumos srauto tankiui ant drėkinamojo paviršiaus. Taip būna tuomet, kai laistomas paviršius šildo arba aušina juo tekančią skysčio plėvelę. Teoriškai apskaičiuotas šilumos perdavimas ir trintis laminariškai tekant labai skirtingų skysčių (vandens, transformatorinės bei kompresorinės alyvos, glicerino ir mazuto) plėvelei. Pateikta laminarinės plėvelės šilumos atidavimo ir trinties, kintant skysčio fizikinėms savybėms, skaičiavimo metodika. Nustatytos priklausomybės tarp temperatūros gradiento ir stabilizuoto šilumos atidavimo bei trinties, esant laminariniam plėvelės tekėjimui.

S. Šinkūnas, A. Kiela

EFFECT OF THE TEMPERATURE GRADIENT ON HEAT TRANSFER AND FRICTION IN LAMINAR LIQUID FILM

S u m m a r y

Analytical study of the stabilized heat transfer across liquid film flow when the maximum heat flux density is observed on the wetted surface was carried out. This situation may feature either surface or liquid side heating. Prediction of heat transfer and friction are made for laminar film flow of very different physical properties liquids (water, transformer and compressor oil, glycerine and fuel oil). A method for the calculations of heat transfer and friction in laminar film with respect to variability of liquid physical properties was suggested. The dependencies of stabilized heat transfer and friction on temperature gradient for laminar film flow were estimated analytically.

С. Шинкунас, А. Кела

ВЛИЯНИЕ ТЕМПЕРАТУРНОГО НАПОРА НА ТЕПЛООБМЕН И ТРЕНИЕ В ЛАМИНАРНОЙ ПЛЕНКЕ ЖИДКОСТИ

Р е з ю м е

Проведен теоретический анализ стабилизированного теплообмена в пленке жидкости при максимальной плотности теплового потока на орошаемой стенке. Такой случай встречается на практике при нагреве или охлаждении пленки со стороны орошаемой стенки. В данной работе приводятся результаты теоретического расчета теплоотдачи и трения ряда жидкостей, сильно отличающихся по физическим свойствам (воды, трансформаторного и компрессорного масла, глицерина и мазута). Предложена методика для определения теплоотдачи и трения в ламинарной пленке с учетом переменности физических свойств различных жидкостей. Определены зависимости стабилизированной теплоотдачи и трения от температурного напора при ламинарном течении пленки.

Received December 01, 2008
Accepted January 15, 2009