Investigation of low cycle fatigue crack opening by finite element method

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1. Introduction

The strength calculation of different constructions is troubled by various defects, which condition the formation of cracks as well as the process of fracture. Even in case of applying the top level technologies it is problematic to obtain materials free of defects; therefore the laws of fracture mechanics are studied by imitating the crack. Thus the value of damage is defined by the state of stresses and strains near the crack, by the length and the shape of the crack, the rate of crack increase, and by the size of crack opening. In order to find out these values, experimental, analytical, and numerical methods are applied. This article presents the analytical, experimental and finite elements methods (FEM) of the crack opening calculations.

The analytical method calculation and its results in case of elastic-plastic deformation, by using the strain concentration coefficients is represented in reference [1]. Recently FEM has been commonly applied for solving the problems of mechanical, thermal, hydraulic, electromagnetic and other physical systems, and also for modeling dynamic processes. This method is relatively low-cost; besides, results are obtained faster than during eksperimental testing. The method allows quite precise calculation of stresses and strains states (fields) in the area of the crack tip, by using three-dimensional models. Therefore, applying the finite element method and using the results of analytical and experimental calculations, we gain much more thorough information on the rates and criteria of the body fracture; besides, we can form an opinion about reliability of the two methods.

This article presents the comparison between the results obtained by the analytical, FEM and experimental methods, while studying the size of the crack opening and its contour curve in case of cyclical symmetrical loading of a specimen, and depending on the cycle number and loading level. The experiment employs a grade 45 steel specimen with a rectangular cross-section working part, and with the central crack. The analytical method has been applied for calculations using the equations proposed by G.R.Irvin and N.A.Makhutov, and the programmable systems COSMOSWork and ANSYS have been used for the finite elements model. In addition, the influence of the specimen geometry on the crack opening has been taken into consideration.

2. Calculation models of the analytical and FE methods

For the analytical method the below presented crack opening calculation equations are used, in case of elastic plastic loading, proposed by G. R. Irvin [2], H. Neuber [3], N. I. Muskhelishvilli [4] and N. A. Makhutov

[5]. The calculations are made both in case of plane state of stresses and in case of plane state of strains; these states are investigated by tension of an infinite plate with a crack by nominal strains σ'_n in gross cross-section, when the length of the crack is $2l$ (Fig. 1). In case of cyclical loading of the plate, the following equations of elastic-plastic crack contour displacement are obtained:

$$\overline{v}_k = 4\overline{K}_{I\varepsilon_k} \left(\frac{r}{2\pi} \right)^{\frac{p_{\varepsilon_k}}{2}} \left[f\left(\frac{r}{l} \right) \right]^{p_{\varepsilon_k}} \tag{1}$$

in case of plane state of stresses, and

$$\overline{v}_k = 4\overline{K}_{I\varepsilon_k} \left(\frac{r}{2\pi} \right)^{\frac{p_{\varepsilon_k}}{2}} \left(1 - v_k^2 \right)^{p_{\varepsilon_k}} \left[f\left(\frac{r}{l} \right) \right]^{p_{\varepsilon_k}} \tag{2}$$

in case of plain state of strains, where in Eqs. (1) and (2) $f\left(\dfrac{r}{l} \right)$ is a correction function

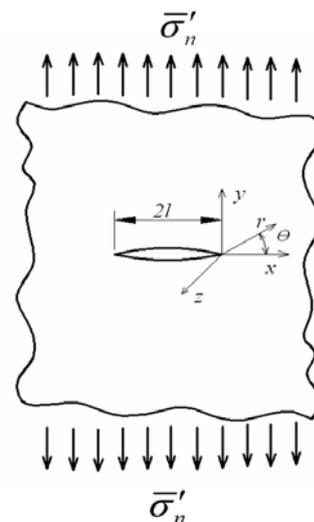$$f\left(\frac{r}{l} \right) = \frac{1 + r/l}{\sqrt{1 + r/2l}} \tag{3}$$

Fig. 1 The scheme of loading of infinite plate with a crack

Here $\overline{K}_{1k}$ is stress intensity coefficient for the loading scheme in Fig.1; it is

$$\overline{K}_{1k} = \overline{S}_{ink} \sqrt{\pi l} \tag{4}$$

In case of elastic-plastic strains in the area of cracks, elastic stress intensity coefficients $\overline{K}_{Ik}$ have to be changed as follows

$$\overline{K}_{I\varepsilon_k} = \overline{K}_{Ik}^{p_{\varepsilon k}} \qquad \text{when } \overline{S}_{ink} \leq 1 \qquad (5)$$

$$\overline{K}_{I\varepsilon_k} = \overline{K}_{I}^{p_{\varepsilon k}} \overline{S}_{ink}^{\frac{1-m_k}{m_k(1+m_k)}}, \quad \text{when } \overline{S}_{ink} > 1 \qquad (6)$$

here

$$p_{\varepsilon k} = \frac{2 - n(1 - m_k)(1 - \overline{S}_{ink})}{1 + m_k} \qquad (7)$$

m_k is curves power low approximation index

$$m_k = \frac{lg\overline{S}_k}{lg\left[\dfrac{A_{1,2}}{\overline{s}_T}\left(\overline{e}_0 - \dfrac{\overline{s}_T}{2}\right)F(k) + \overline{S}_k\right]} \qquad (8)$$
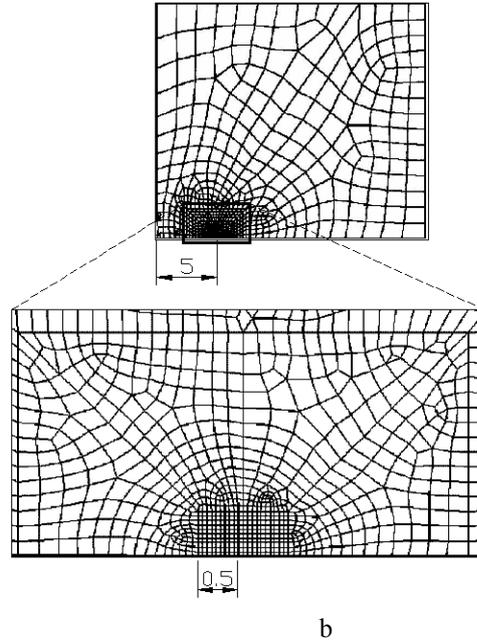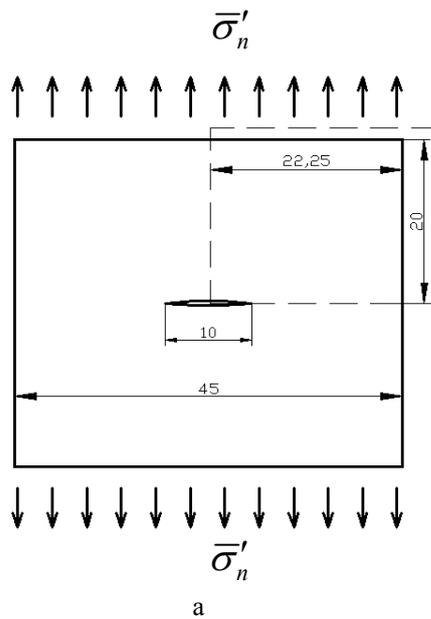

Fig. 2 Two-dimensional (2D) model with crack: a - model of plate with a crack for the FEM; b - the FEM mesh on the whole plate and in the zone of the stress concentration (measures indicated in mm)

On the basis of finite elements method a number of universal and specialized computer programs have been created. With the help of the most universal ones, i.e. ANSYS, ALGOR, ABAQUS, COSMOS and others, the target systems are researched in a very rapid and reliable way [6-8]. In this study we use the finite element programmable systems COSMOSWorks and ANSYS, with the aim to compare the results obtained by using these programs that are based on FEM, with the results received by analytical and experimental methods, i.e. to present the comparison between the results obtained by the analytical, the FE and the experimental methods, while studying the size of the crack opening.

The two-dimensional (2D) model with a crack was formed with the help of programmable system ANSYS, because, it allows to show plane state of stress and plane state of strain in the model (Fig. 2, b). As the plate has two symmetrical axes it is very convenient to investigate a quarter of the plate; in this way it takes less calculation time.

The mesh finite elements is shown in Fig. 2, b. Its density was selected by checking the dependence of the crack opening on the size of finite elements. The value of the element in the crack tip equals 50 micrometers. When using the plane model, opening of the crack contour was obtained in two cases: plane state of stresses and plain state of strains. In order to compare the opening results in the plate of specific thickness with a crack, with the results of

the plane stress and plane strain state opening, the 3D model (Fig. 3, a) was formed with the help of the programmable system COSMOSWorks. The results spatial model would enable to observe the real opening of the
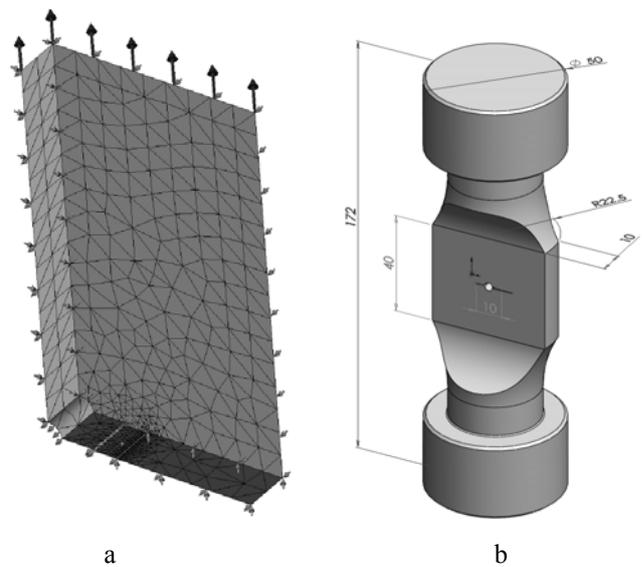

Fig. 3 Three-dimensional (3D) model: a – model of the plate of specific thickness with a crack and FE mesh; b – model coinciding with an experimental specimen

crack. As the specimen has three symmetrical planes, only one eighth of the specimen was used for calculations as demonstrated in Fig. 3, a.

Thus the results of he crack opening of this model imitate the opening of real crack contour. The experiment employs a grade 45 steel specimen with the central crack (Fig. 3, b). During cyclic symmetrical loading of the specimen, its contour opening was measured with the help of the microscope after different number of cycles and at various loading levels.

3. The influence of model geometry on crack opening

During the calculations, question was raised if the model of the plate of specific thickness with a crack (Fig. 3, a) does not distort the loading geometry of the experimental specimen, and if the calculation results of the model imitating the real specimen coincide with the above-mentioned. This led to the calculations made at different loading levels and various numbers of cycles, using both rectangular models of specific thickness and models having real forms of specimen.

As we see in Fig. 4, the crack in case b is opened more as the force is concentrated near the specimen axis, while the specimen is not long enough for the loading to spread evenly in gross cross-section; however, the specimen cannot be longer as it has cyclical symmetrical loading, and the compression would cause buckling. Therefore the crack opening results differ when different models are used, i.e. the crack opens about 10% more when the model is identical to the real specimen.

4. Comparison of the calculation results obtained by the analytical, FE and experimental methods

With the help of the above-presented analytical method (Eqs. (1)-(8)) we calculated the crack opening contour and with the finite element programmable systems ANSYS and COSMOSWorks the curves of crack opening contour; under identical conditions of loading. Results obtained by the analytical, FE, and experimental methods are show in Figs. 5 - 9.

As we see in Figs. 5, 6 in case of lower loading level the results obtained by the experiment slightly differ from those obtained by the FE method; however, if we have higher loading level as in Figs. 7, 8 and 9, the results of the experimental and FE methods coincide very well. The results obtained by the analytical method are significantly higher both in case of lower and higher loading levels; however, the curves coincide at the crack tip. The reason for this lies in the fact that the curve contour obtained by the analytical method is too straight and thus do not repeat the real parabolic contour of the elastic-plastic crack, while the FE method performs this perfectly.
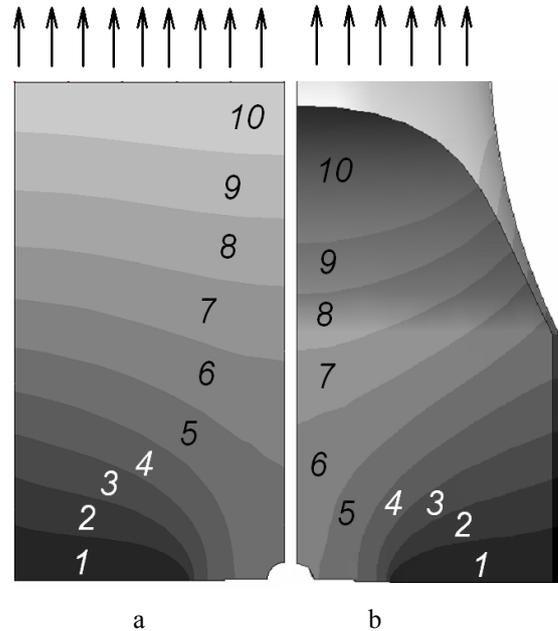

Fig. 4 Comparison of isolines of displacement along y axis, in case of identical loading conditions: a − of the plate of specific thickness; b − of the model imitating the real specimen (the numbers indicate a field coinciding with a specific interval of the displacement)
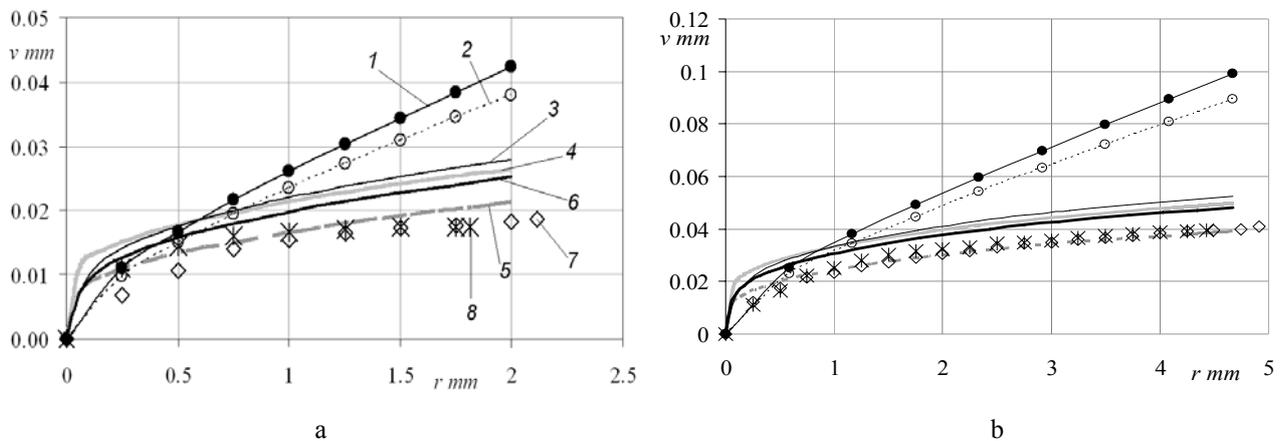

Fig. 5 Curves of the crack opening contour when the loading level is $\bar{\sigma}_{in} = 0.8$: a- when $k = 400$ cycles; b - when $k = 10000$ cycles. Curves titles: 1 - analytical method, plane state of stress, 2 - analytical method, plane state of strain, 3 - FE method, imitation of the real specimen, 4 - FE method, plane state of stress, 5 - FE method, plane state of strain, 6 - FE method, the rectangular plate of specific thickness; 7 and 8 - experimental results of the right and left sides of the crack. The numbering of curves types in Fig. 5, a correspond curves in Figs. 5, b, 6, 7, 8 and 9
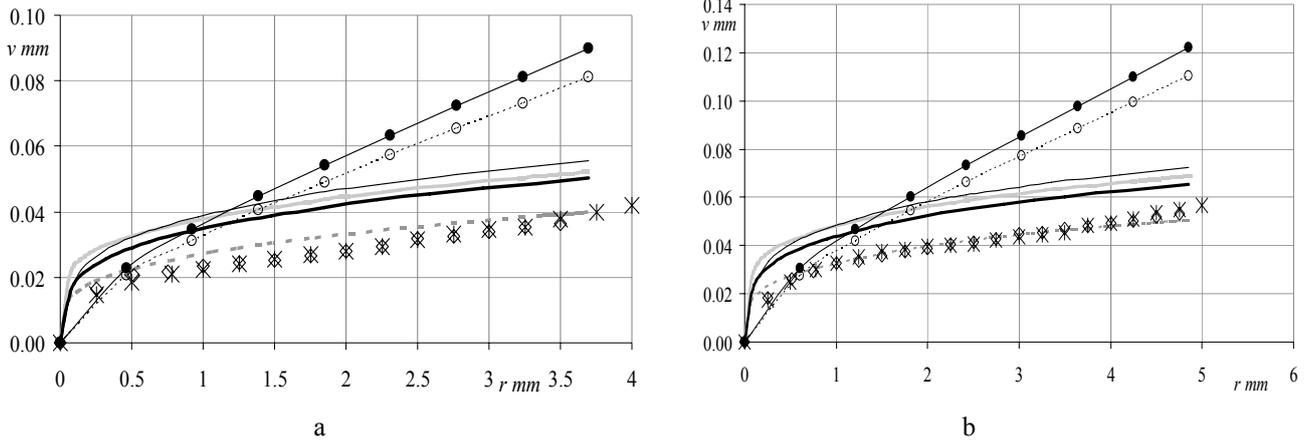
Fig. 6 Curves of the crack opening contour, when the loading $\overline{\sigma}_{in} = 0.9$: a - k = 4000 cycles; b - k = 5000 cycles
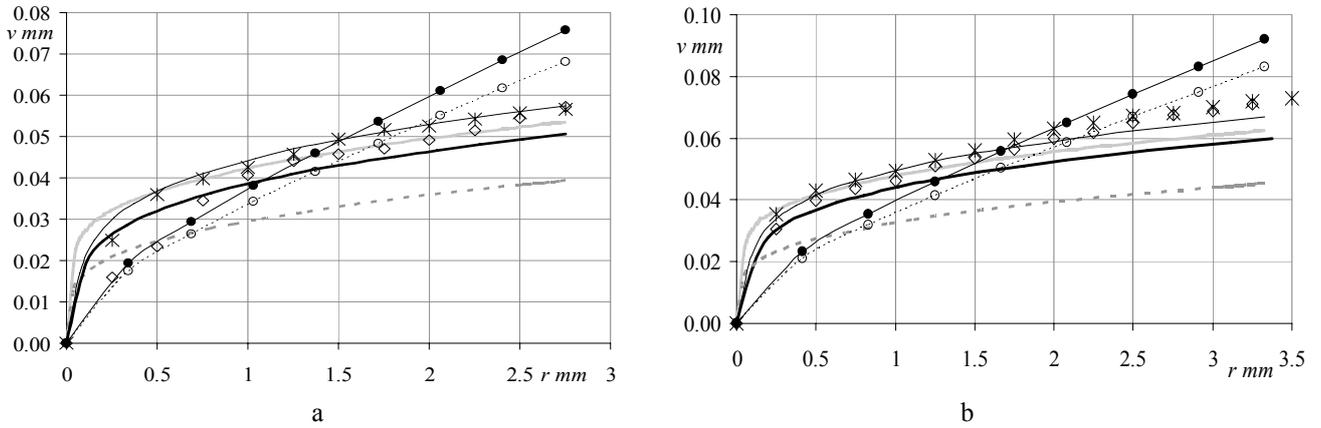

Fig. 7 Curves of the crack opening contour, when the loading $\overline{\sigma}_{in} = 1$: a – k = 400 cycles; b – k = 600 cycles
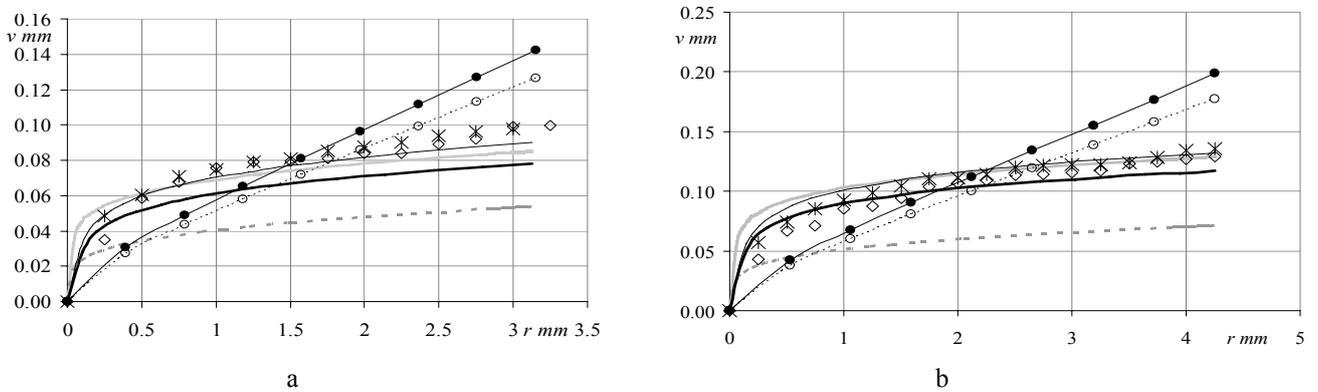

Fig. 8 Curves of the crack opening contour, when the loading $\overline{\sigma}_{in} = 1.1$: a – k = 100 cycles; b – k = 160 cycles
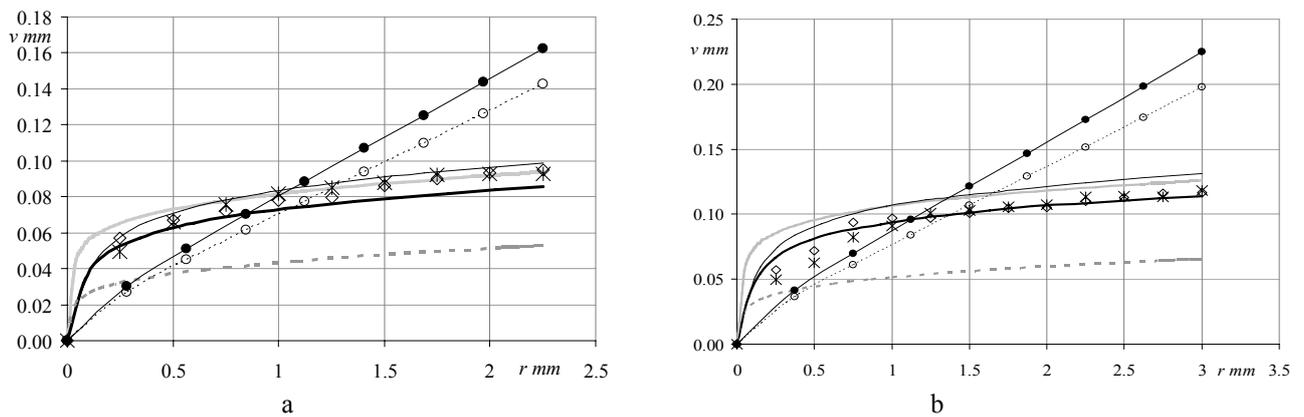

Fig. 9 Curves of the crack opening contour, when the loading $\overline{\sigma}_{in} = 1.2$: a – k = 20 cycles; b – k = 60 cycles

5. Conclusions

The results of the crack opening obtained by FEM accurately coincide with the experiment in case of higher loading levels ($\overline{\sigma}_{in}$=1; 1.1; 1.2), and are different when we have lower loading levels ($\overline{\sigma}_{in}$=0,8; 0,9).

The results obtained by the analytical method show good coincidence with the experimental and FE methods only at the crack tip (0.5-1.5 mm from the crack tip), as the curve contour obtained by the analytical method is too straight and thus do not repeat the real parabolic elastic plastic crack contour.

The results of the crack opening obtained by FE method by imitating a real specimen (by repeating its real geometry) demonstrated better coincidence with the experiment results and they are about 10% higher than those obtained from a rectangular plate.

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PLYŠIO ATSIVĖRIMO TYRIMAS BAIGTINIŲ ELEMENTŲ METODU ESANT MAŽACIKLIAM APKROVIMUI

R e z i u m ė

Darbe lyginami analitiniu, baigtinių elementų ir eksperimentiniu metodais gauti rezultatai, t. y. plyšio atsivėrimo dydis ir jo kontūro kreivė cikliškai simetriškai apkrautame bandinyje, priklausomai nuo ciklų skaičiaus ir apkrovimo lygio. Eksperimente naudojamas plieno 45 sta-čekampės formos bandinys su centriniu plyšiu. Analitinės išraiškos buvo gautos pasinaudojus Irvino ir Machutovo priklausomybėmis, o BEM skaičiavimams buvo naudojami programų sistemos COSMOSWorks ir ANSYS. Darbe atsižvelgta į modelio geometrijos įtaką plyšio atsivėrimui. Rezultatai parodė, kad eksperimentai ir BEM gauti duomenys gerai sutapo, o naudojant analitines priklausomybes geras rezultatas gaunamas tik plyšio viršūnėje.

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INVESTIGATION OF LOW CYCLE FATIGUE CRACK OPENING BY FINITE ELEMENT METHOD

S u m m a r y

This research presents the comparison the results obtained by the analytical, the finite element and the experimental methods, while studying the size of crack opening and its contour curve in case of cyclic symmetrical loading of a specimen depending on the cycle number and loading level. The experiment employs a grade 45 steel specimen with a working part of rectangular cross-section, and with the central crack. The analytical method has been applied for calculations using the equations proposed by Irvin and Makhutov, and the programmable systems COSMOSWorks and ANSYS have been used for FEM. In addition, the influence of the specimen geometry on the crack opening has been taken into consideration. The results demonstrate quite adequate coincidence between the results of experimental and finite element methods, while the analytical calculations are accurate only at the crack tip.

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ИССЛЕДОВАНИЕ ОТКРЫТИЯ УСТАЛОСТНОЙ ТРЕЩИНЫ МЕТОДОМ КОНЕЧНЫХ ЭЛЕМЕНТОВ ПРИ МАЛОЦИКЛОВОМ НАГРУЖЕНИИ

Р е з ю м е

В настоящей работе представлено сопоставление результатов, полученных аналитическим, экспериментальным и методом конечных элементов открытия величины и формы трещины при циклическом симметричном нагружении образца, в зависимости от числа цикла и уровня нагрузки. При исследовании использовался образец с центральной трещиной изготовленный из стали 45 с прямоугольным поперечным сечением рабочей части. Аналитические вычисления проведены используя уравнения Ирвина и Махутова, а для расчётов МКЕ были использованы программные пакеты COSMOSWorks и ANSYS. Кроме того, было учтено влияниие геометрии образца на открытие трещины. Проведенное исследование показало весьма адекватное совпадение экспериментальных и МКЕ результатов, в то время как результаты аналитическово расчета с упомянутыми выше совпадают только у вершины трещины.