ISSN 1392 - 1207. MECHANIKA. 2009. Nr.3(77)

Method for a direct calculation of stress-strain state parameters at normal right-angled sections of structural members given curvilinear stress diagrams

I. Židonis

Šiauliai University, Vilniaus g. 141, 76353 Šiauliai, Lithuania, E-mail: ipolitas.zidonis@gmail.com

1. Introduction

At present, it is very important to have a engineering method enabling us to use nonlinear strain-stress relationships [1-3]. In the case of concrete and reinforcement, such relationships of the Eurocode [4] are adopted in the regulation [5].

In paper [6], for the calculation of stress-strain state at normal right-angled sections of bar-shaped structural members, the author proposed a method of successive approximations (iterative method). Continuing this work, the author has prepared a vast amount of material for a *direct* calculation of the parameters of the aforesaid state, i.e. without having resort to the method successive approximations. The direct calculation is applicable when we know in advance the values of strains ε_ε and $\varepsilon_{0\varepsilon}$ of any layer of the material located at distance a_ε from axis $w-w$ (Fig. 1). Here symbol ε_ε denotes the actual strain, and symbol $\varepsilon_{0\varepsilon}$ denotes the strain corresponding to the hypothesis of plane sections. It is often maintained that $\varepsilon_\varepsilon = \varepsilon_{0\varepsilon}$. For instance, when calculating the cracking moment or when calculating the stress-strain state of reinforced concrete members having cracks in the tension zone in the sections between the cracks $\varepsilon_\varepsilon = \varepsilon_{ct,lim}$; or when calculating the breaking moment or the reinforcement area $\varepsilon_\varepsilon = \varepsilon_y$. The direct calculation method that has been developed by the author is applicable to various cases when the members are *non-layered*. The members may be without cracks or with cracks in the tension zone, they may be of a rectangular cross-section or have flanges. Both for the tension zone and for the compression zone, various stress diagrams may be assumed: curvilinear, triangular, rectangular, etc. Stresses of the tension zone of the core material may also be ignored, for instance, when calculating the breaking moment or when calculating the area of the reinforcement. It is possible to apply this method for the calculation of the reinforced concrete members with the reinforcement concentrated not only in the tension zone and in the compression zone, the method is applicable also for the case when reinforcement of the member is located at any height. It should be noted that the equation of bending moments is applicable to any axis $a-a$ (Fig. 1), located at distance a_a from axis $w-w$. Because of the limited scope of the paper, we present material only for a *direct* calculation of the members with rectangular cross-section, having no cracks, given the universal 5th degree curvilinear stress diagrams. More of the already prepared research material is planned to be published as separate papers.

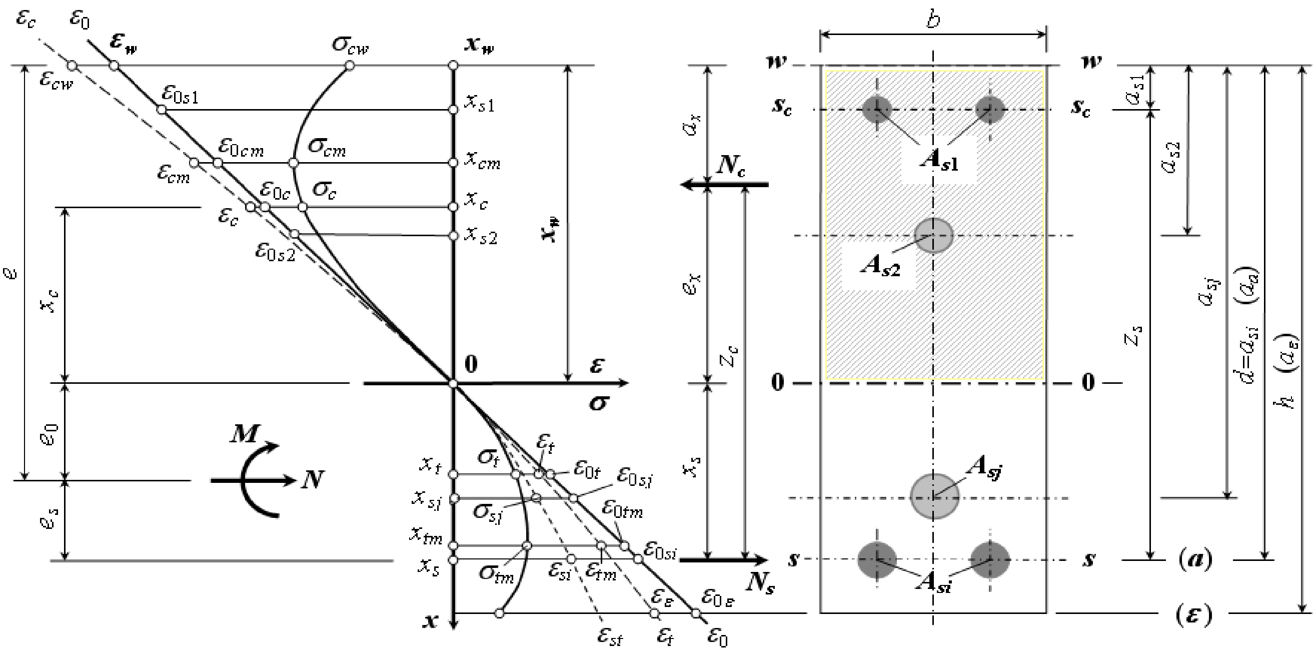

Fig. 1 Cross-section of the member and stress-strain diagrams

2. The essence of the method and its formulae

Static balance Eqs. (29) and (32) that have been presented in paper [6], in the present paper, are used for the case of a nonlayered member with a rectangular cross-section (Fig. 1). Therefore, $b_i = b$, $E_i = E$. The member may be strengthened or weakened by any resilient material, e.g. by using reinforcement. We take into account through the employment of parameters k_{si}, α_{esi}, A_{si} and $\nu_{Si} = const$ (the symbols are the same as in [6]). When we add that $\Sigma N_i = N$, $\Sigma M_i = M$, $a_{iu} = 0$ and $a_{siu} = a_{si}$, we receive the following Eqs. (1) and (2):

$$\left(k_t\omega_{nt} - k_c\omega_{nc}\right)x_w^2 + \left[2k_t\omega_{nt}d_u + \frac{\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}}{b} + \frac{\Sigma\left(P_i\nu_{Si}/\nu_{pi}\right) + N}{bE\varepsilon_\varepsilon / k_\varepsilon}\right]x_w + k_t\omega_{nt}d_u^2 + \frac{\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}a_{si}}{b} + \frac{\Sigma\left(P_i\nu_{Si}/\nu_{pi}\right) + N}{bE\varepsilon_\varepsilon / k_\varepsilon}a_\varepsilon = 0 \tag{1}$$

$$\left[k_t\left(\omega_{nt} - \omega_{mt}\right) - k_c\left(\omega_{nc} - \omega_{mc}\right)\right]x_w^3 + \left[k_t\left(\omega_{nt}a_a + 2\omega_{nt}d_u - 3\omega_{mt}d_u\right) - k_c\omega_{nc}a_a\right]x_w^2 + \left\{k_t\left(2\omega_{nt}a_a d_u + \omega_{nt}d_u^2 - 3\omega_{mt}d_u^2\right) + \frac{\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}\left(a_a - a_{si}\right)}{b} + \frac{\Sigma\left(P_i\nu_{Si}/\nu_{pi}\right)\left(a_a - a_{si}\right) + N\left(a_a - e\right) + M}{bE\varepsilon_\varepsilon / k_\varepsilon}\right\}x_w + k_t\left(\omega_{nt}a_a d_u^2 - \omega_{mt}d_u^3\right) + \frac{\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}\left(a_a - a_{si}\right)a_{si}}{b} + \frac{\Sigma\left(P_i\nu_{Si}/\nu_{pi}\right)\left(a_a - a_{si}\right) + N\left(a_a - e\right) + M}{bE\varepsilon_\varepsilon / k_\varepsilon}a_\varepsilon = 0 \tag{2}$$

Symbols used but not explained in the present paper are the same as in papers [6] and [7]. The following symbols have been changed: ω has been changed into ω_n and ϖ has been changed into ω_m (ω_n is attributed to equations of projections, and ω_m is attributed to equations of moments) and instead of indexes 1 we use indexes c, instead of index 2 we use index t (c denotes compression, and t denotes tension). Other symbols of parameters used in Eqs. (1) and (2) are as follows:

E and E_{si} denote the elasticity moduli of the layers of the main material in cross-section (e.g. layers of the concrete) and reinforcement (weakening) of the cross-section; $\overline{E}_{si}$ is secant elasticity (strain) moduli of the reinforcement; ε_{pi} is prestrain of reinforcement; σ_{pi} is prestress of reinforcement; ε_{si} is strain* of reinforcement caused by external forces; σ_{si} is stress* of reinforcement caused by external forces; ε_ε denotes for any strain* at any selected distance a_ε from axis $w-w$; $\varepsilon_{0\varepsilon}$ denotes the same*, corresponding to the hypothesis of plane sections; ε_{0m} denotes strain*, corresponding to the extreme σ_m strain*, when the hypothesis of plane sections is used; ε_{0cm} denotes ε_{0m} of compressed material*; ε_{0tm} denotes ε_{0m} of tensioned material; symbols of other dimensions are shown in the Fig. 1 and [6]

$$\varepsilon_{Si} = \varepsilon_{pi} + \varepsilon_{si} \tag{3}$$

$$\sigma_{Si} = \sigma_{pi} + \sigma_{si} \tag{4}$$

$$\nu_{Si} = \sigma_{Si} / \overline{E}_{si}\varepsilon_{Si} \tag{5}$$

$$\nu_{pi} = \sigma_{pi} / \overline{E}_{si}\varepsilon_{pi} \tag{6}$$

$$k_\varepsilon = \varepsilon_\varepsilon / \varepsilon_{0\varepsilon} \tag{7}$$

$$k_{si} = \varepsilon_{si} / \varepsilon_{0si} \tag{8}$$

$$\alpha_{esi} = E_{si} / E \tag{9}$$

$$P_i = \sigma_{pi}A_{si} = \overline{E}_{si}A_{si}\varepsilon_{pi} = \nu_{pi}E_{si}A_{si}\varepsilon_{pi} \tag{10}$$

$$Z_{si} = k_{si}\alpha_{esi}A_{si}\nu_{Si} \tag{11}$$

$$Z_{bsi} = k_{si}\alpha_{esi}A_{si}\nu_{Si} / b \tag{12}$$

$$Z_E = bE / k_\varepsilon \tag{13}$$

$$Z_{pn} = \frac{\Sigma\left(P_i\nu_{Si}/\nu_{pi}\right) + N}{Z_E} \tag{14}$$

$$Z_{pn\varepsilon} = Z_{pn} / \varepsilon_\varepsilon \tag{15}$$

$$Z_n = \Sigma Z_{bsi} + Z_{pn\varepsilon} \quad (16$$

$$Z_{na} = \Sigma Z_{bsi}a_{si} + Z_{pn\varepsilon}a_\varepsilon \tag{17}$$

$$Z_{pm} = \frac{\Sigma\left(P_i\nu_{Si}/\nu_{pi}\right)\left(a_a - a_{si}\right) + N\left(a_a - e\right)}{Z_E} \tag{18}$$

$$Z_{pm\varepsilon} = \left(Z_{pm} + \frac{M}{Z_E}\right)\frac{1}{\varepsilon_\varepsilon} \tag{19}$$

$$Z_m = \Sigma Z_{bsi}\left(a_a - a_{si}\right) + Z_{pm\varepsilon} \tag{20}$$

$$Z_{ma} = \Sigma Z_{bsi}\left(a_a - a_{si}\right)a_{si} + Z_{pm\varepsilon}a_\varepsilon \tag{21}$$

When we insert the values of parameters into Eqs. (1) and (2), we receive static balance equations of the projections of the forces (22) and (24) and static balance equations of bending moments (23) and (25)

$$\left(k_t\omega_{nt} - k_c\omega_{nc}\right)x_w^2 + \left(2k_t\omega_{nt}d_u + \frac{\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}}{b} + \frac{Z_{pn}}{\varepsilon_\varepsilon}\right)x_w + k_t\omega_{nt}d_u^2 + \frac{\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}a_{si}}{b} + \frac{Z_{pn}}{\varepsilon_\varepsilon}a_\varepsilon = 0 \tag{22}$$

* for the compression material negative values are taken

$$\left[k_t(\omega_{nt}-\omega_{mt})-k_c(\omega_{nc}-\omega_{mc})\right]x_w^3+$$
$$+\left\{k_t\left[\omega_{nt}(2d_u+a_a)-3\omega_{mt}d_u\right]-k_c\omega_{nc}a_a\right\}x_w^2+$$
$$+\left\{\begin{array}{l} k_t\left[\omega_{nt}(d_u+2a_a)-3\omega_{mt}d_u\right]d_u+ \\ +\dfrac{\Sigma k_{si}\alpha_{esi}A_{si}v_{Si}}{b}(a_a-a_{si})+\left(Z_{pm}+\dfrac{M}{Z_E}\right)\dfrac{1}{\varepsilon_\varepsilon}\end{array}\right\}x_w+$$
$$+k_t(\omega_{nt}a_a-\omega_{mt}d_u)d_u^2+$$
$$+\frac{\Sigma k_{si}\alpha_{esi}A_{si}v_{Si}}{b}(a_a-a_{si})a_{si}+$$
$$+\left(Z_{pm}+\frac{M}{Z_E}\right)\frac{a_\varepsilon}{\varepsilon_\varepsilon}=0 \qquad (23)$$

$$(k_t\omega_{nt}-k_c\omega_{nc})x_w^2+(2k_t\omega_{nt}d_u+Z_n)x_w+$$
$$+k_t\omega_{nt}d_u^2+Z_{na}=0 \qquad (24)$$

$$\left[k_t(\omega_{nt}-\omega_{mt})-k_c(\omega_{nc}-\omega_{mc})\right]x_w^3+$$
$$+\left\{k_t\left[\omega_{nt}(2d_u+a_a)-3\omega_{mt}d_u\right]-k_c\omega_{nc}a_a\right\}x_w^2+$$
$$+\left\{k_t\left[\omega_{nt}(d_u+2a_a)-3\omega_{mt}d_u\right]d_u+Z_m\right\}x_w+$$
$$+k_t(\omega_{nt}a_a-\omega_{mt}d_u)d_u^2+Z_{ma}=0 \qquad (25)$$

When the tension zone has no cracks, then in Eqs. (22-25)

$$d_u=x_{th}-x_w=h \qquad (26)$$

In this paper, we analyze the case when the stress-strain diagram of the compression zone of the material of the beam (e.g. concrete of classes C08/10–C90/105) may be defined by the formulae presented in paper [7]

$$\sigma_c=E_c\varepsilon_c\left(1+c_1\eta+c_2\eta^2+c_3\eta^3+c_4\eta^4\right)=$$
$$=v_cE_c\varepsilon_c=v_c\sigma_{ce} \qquad (27)$$

$$v_c=1+c_1\eta+c_2\eta^2+c_3\eta^3+c_4\eta^4 \qquad (28)$$

where $E_c=tan\,\beta$; $\sigma_{ce1}=E_c\varepsilon_{c1}$; $\sigma_{c1}=f_{cm}$; $v_{c1}=\sigma_{c1}/\sigma_{ce1}$. Values of coefficients c_1, c_2, c_3, c_4 and symbols are the same as in paper [7].

When Eq. (27) is applied not to concrete but to some other compressed or tensioned material, then $\varepsilon_{c1}=\varepsilon_m$, $\sigma_{c1}=\sigma_m$, $\sigma_{ce1}=\sigma_{em}$, $v_{c1}=v_m=\sigma_m/\sigma_{em}$, etc.

$$\eta=\varepsilon_c/\varepsilon_{c1} \qquad (29)$$

The stress-strain diagram of compression zone of concrete of classes C25/30-C90/105 in some cases may be defined by a simpler formula presented in paper [8]. The author will present the material prepared for this case in his next paper.

2.1. Parameters of the compression zone

For the parameters of compression zone we use the following symbols:

$$\eta_{0\varepsilon c}=\varepsilon_{0\varepsilon}/\varepsilon_{0cm}=const \qquad (30)$$

$$\frac{x_{cm}}{a_\varepsilon+x_w}=\frac{\varepsilon_{0cm}}{\varepsilon_{0\varepsilon}}=\frac{\varepsilon_{cm}/k_{cm}}{\varepsilon_\varepsilon/k_\varepsilon}=\frac{1}{\eta_{0\varepsilon c}} \qquad (31)$$

$$x_{cm}=a_\varepsilon+x_w/\eta_{0\varepsilon c} \qquad (32)$$

$$\eta_c=\eta_{cw}=\frac{\varepsilon_w}{\varepsilon_{0cm}}=\frac{x_w}{x_{cm}}=\frac{x_w}{a_\varepsilon+x_w}\eta_{0\varepsilon c} \qquad (33)$$

$$\kappa_{1c}=c_{1c}\eta_{cw},\ \kappa_{2c}=c_{2c}\eta_{cw}^2,\ \kappa_{3c}=c_{3c}\eta_{cw}^3,\ \kappa_{4c}=c_{4c}\eta_{cw}^4 \qquad (34)$$

$$u_{1c}=c_{1c}\eta_{0\varepsilon c},\ u_{2c}=c_{2c}\eta_{0\varepsilon c}^2,\ u_{2c}=c_{2c}\eta_{0\varepsilon c}^2,\ u_{4c}=c_{4c}\eta_{0\varepsilon c}^4 \qquad (35)$$

$$\left.\begin{array}{l} n_{0c}=30a_\varepsilon^4 \\ n_{1c}=(120+20u_{1c})a_\varepsilon^3 \\ n_{2c}=(180+60u_{1c}+15u_{2c})a_\varepsilon^2 \\ n_{3c}=(120+60u_{1c}+30u_{2c}+12u_{3c})a_\varepsilon \\ n_{4c}=30+20u_{1c}+15u_{2c}+12u_{3c}+10u_{4c}\end{array}\right\} \qquad (36)$$

$$v_{4x}=1/60(x_w+a_\varepsilon)^4 \qquad (37)$$

$$\left.\begin{array}{l} m_{0c}=140a_\varepsilon^4 \\ m_{1c}=(560+105u_{1c})a_\varepsilon^3 \\ m_{2c}=(840+315u_{1c}+84u_{2c})a_\varepsilon^2 \\ m_{3c}=(560+315u_{1c}+168u_{2c}+70u_{3c})a_\varepsilon \\ m_{4c}=140+105u_{1c}+84u_{2c}+70u_{3c}+60u_{4c}\end{array}\right\} \qquad (38)$$

$$u_{4x}=v_{4x}/7 \qquad (39)$$

When we insert the values of parameters of the compression zone into ((20), (21) [7]), we receive the following

$$\omega_{nc}=\frac{1}{2}+\frac{\kappa_{1c}}{3}+\frac{\kappa_{2c}}{4}+\frac{\kappa_{3c}}{5}+\frac{\kappa_{4c}}{6}=$$
$$=\frac{1}{2}+\frac{c_{1c}}{3}\eta_{cw}+\frac{c_{2c}}{4}\eta_{cw}^2+\frac{c_{3c}}{5}\eta_{cw}^3+\frac{c_{4c}}{6}\eta_{cw}^4=$$
$$=\left(n_{0c}+n_{1c}x_w+n_{2c}x_w^2+n_{3c}x_w^3+n_{4c}x_w^4\right)v_{4x} \qquad (40)$$

$$\omega_{mc}=\frac{1}{3}+\frac{\kappa_{1c}}{4}+\frac{\kappa_{2c}}{5}+\frac{\kappa_{3c}}{6}+\frac{\kappa_{4c}}{7}=$$
$$=\frac{1}{3}+\frac{c_{1c}}{4}\eta_{cw}+\frac{c_{2c}}{5}\eta_{cw}^2+\frac{c_{3c}}{6}\eta_{cw}^3+\frac{c_{4c}}{7}\eta_{cw}^4=$$
$$=\left(m_{0c}+m_{1c}x_w+m_{2c}x_w^2+m_{3c}x_w^3+m_{4c}x_w^4\right)u_{4x} \qquad (41)$$

2.2. Parameters of the tension zone

Here for the tension zone we assume a curvilinear diagram of stresses that may be described by Eq. (27).

Parameters are analogous to the parameters of compression zone (please note that the value of x_w is negative):

$$x_{th}=h+x_w \qquad (42)$$

$$\eta_{0\varepsilon t}=\varepsilon_{0\varepsilon}/\varepsilon_{0tm}=const \qquad (43)$$

$$x_{tm}=(a_\varepsilon+x_w)/\eta_{0\varepsilon t} \qquad (44)$$

$$\frac{x_{tm}}{a_\varepsilon + x_w} = \frac{\varepsilon_{0tm}}{\varepsilon_{0\varepsilon}} = \frac{\varepsilon_{tm}/k_{tm}}{\varepsilon_\varepsilon/k_\varepsilon} = \frac{1}{\eta_{0\varepsilon t}} \tag{45}$$

$$\eta_t = \eta_{th} = \frac{x_{th}}{x_{tm}} = \frac{x_{th}}{a_\varepsilon + x_w}\eta_{0\varepsilon t} = \frac{h + x_w}{a_\varepsilon + x_w}\eta_{0\varepsilon t} \tag{46}$$

$$\kappa_{1t} = c_{1c}\eta_{th},\ \kappa_{2t} = c_{2c}\eta_{th}^2,\ \kappa_{3t} = c_{3c}\eta_{th}^3,\ \kappa_{4t} = c_{4c}\eta_{th}^4 \tag{47}$$

$$u_{1t} = c_{1t}\eta_{0\varepsilon t},\ u_{2t} = c_{2t}\eta_{0\varepsilon t}^2,\ u_{3t} = c_{3t}\eta_{0\varepsilon t}^3,\ u_{4t} = c_{4t}\eta_{0\varepsilon t}^4 \tag{48}$$

$$\left.\begin{aligned}
n_{0t} &= 30a_\varepsilon^4 + 20u_{1t}ha_\varepsilon^3 + 15u_{2t}h^2a_\varepsilon^2 + 12u_{3t}h^3a_\varepsilon + 10u_{4t}h^4 \\
n_{1t} &= 120a_\varepsilon^3 + 20u_{1t}(3h + a_\varepsilon)a_\varepsilon^2 + 30u_{2t}h(h + a_\varepsilon)a_\varepsilon + 12u_{3t}h^2(h + 3a_\varepsilon) + 40u_{4t}h^3 \\
n_{2t} &= 180a_\varepsilon^2 + 60u_{1t}(h + a_\varepsilon)a_\varepsilon + 15u_{2t}(h^2 + 4ha_\varepsilon + a_\varepsilon^2) + 36u_{3t}h(h + a_\varepsilon) + 60u_{4t}h^2 \\
n_{3t} &= 120a_\varepsilon + 20u_{1t}(h + 3a_\varepsilon) + 30u_{2t}(h + a_\varepsilon) + 12u_{3t}(3h + a_\varepsilon) + 40u_{4t}h \\
n_{4t} &= 30 + 20u_{1t} + 15u_{2t} + 12u_{3t} + 10u_{4t}
\end{aligned}\right\} \tag{49}$$

$$\left.\begin{aligned}
m_{0t} &= 140a_\varepsilon^4 + 105u_{1t}ha_\varepsilon^3 + 84u_{2t}h^2a_\varepsilon^2 + 70u_{3t}h^3a_\varepsilon + 60u_{4t}h^4 \\
m_{1t} &= 560a_\varepsilon^3 + 105u_{1t}(3h + a_\varepsilon)a_\varepsilon^2 + 168u_{2t}h(h + a_\varepsilon)a_\varepsilon + 70u_{3t}h^2(h + 3a_\varepsilon) + 240u_{4t}h^3 \\
m_{2t} &= 840a_\varepsilon^2 + 315u_{1t}(h + a_\varepsilon)a_\varepsilon + 84u_{2t}(h^2 + 4ha_\varepsilon + a_\varepsilon^2) + 210u_{3t}h(h + a_\varepsilon) + 360u_{4t}h^2 \\
m_{3t} &= 560a_\varepsilon + 105u_{1t}(h + 3a_\varepsilon) + 168u_{2t}(h + a_\varepsilon) + 70u_{3t}(3h + a_\varepsilon) + 240u_{4t}h \\
m_{4t} &= 140 + 105u_{1t} + 84u_{2t} + 70u_{3t} + 60u_{4t}
\end{aligned}\right\} \tag{50}$$

When we insert the values of the parameters of tension zone into ((20), (21) [7]), we receive

$$\begin{aligned}
\omega_{nt} &= \frac{1}{2} + \frac{\kappa_{1t}}{3} + \frac{\kappa_{2t}}{4} + \frac{\kappa_{3t}}{5} + \frac{\kappa_{4t}}{6} = \\
&= \frac{1}{2} + \frac{c_{1t}}{3}\eta_{th} + \frac{c_{2t}}{4}\eta_{th}^2 + \frac{c_{3t}}{5}\eta_{th}^3 + \frac{c_{4t}}{6}\eta_{th}^4 = \\
&= \left(n_{0t} + n_{1t}x_w + n_{2t}x_w^2 + n_{3t}x_w^3 + n_{4t}x_w^4\right)v_{4x}
\end{aligned} \tag{51}$$

$$\begin{aligned}
\omega_{mt} &= \frac{1}{3} + \frac{\kappa_{1t}}{4} + \frac{\kappa_{2t}}{5} + \frac{\kappa_{3t}}{6} + \frac{\kappa_{4t}}{7} = \\
&= \frac{1}{3} + \frac{c_{1t}}{4}\eta_{th} + \frac{c_{2t}}{5}\eta_{th}^2 + \frac{c_{3t}}{6}\eta_{th}^3 + \frac{c_{4t}}{7}\eta_{th}^4 = \\
&= \left(m_{0t} + m_{1t}x_w + m_{2t}x_w^2 + m_{3t}x_w^3 + m_{4t}x_w^4\right)u_{4x}
\end{aligned} \tag{52}$$

It should be noted that the values of coefficients n_{ic}, n_{it}, m_{ic} and m_{it} do not depend on x_w, but depend on ε_ε.

For nonlayered members without cracks, when we insert respective values of the parameters into projection Eqs. (22) or (24) and moments Eqs. (23) or (25), we get the static balance equations of projections (53) and moments (54). Here for both zones (the compression zone and the tension zone) we use curvilinear 5th degree stress diagrams.

$$\begin{aligned}
&a_{n0} + a_{n1}x_w + a_{n2}x_w^2 + a_{n3}x_w^3 + \\
&+ a_{n4}x_w^4 + a_{n5}x_w^5 + a_{n6}x_w^6 = 0
\end{aligned} \tag{53}$$

$$\begin{aligned}
&a_{m0} + a_{m1}x_w + a_{m2}x_w^2 + a_{m3}x_w^3 + \\
&+ a_{m4}x_w^4 + a_{m5}x_w^5 + a_{m6}x_w^6 + a_{m7}x_w^7 = 0
\end{aligned} \tag{54}$$

where

$$a_{n0} = k_t n_{0t}h^2 + 60a_\varepsilon^4\left(\frac{\Sigma k_{si}\alpha_{esi}A_{si}v_{Si}a_{si}}{b} + a_\varepsilon\frac{Z_{pn}}{\varepsilon_\varepsilon}\right) \tag{55}$$

$$a_{n1} = k_t\left(2n_{0t}h + n_{1t}h^2\right) + 60a_\varepsilon^3\left[\frac{\Sigma k_{si}\alpha_{esi}A_{si}v_{Si}(4a_{si} + a_\varepsilon)}{b} + 5a_\varepsilon\frac{Z_{pn}}{\varepsilon_\varepsilon}\right] \tag{56}$$

$$a_{n2} = k_t\left(n_{0t} + 2n_{1t}h + n_{2t}h^2\right) - k_c n_{0c} + 120a_\varepsilon^2\left[\frac{\Sigma k_{si}\alpha_{esi}A_{si}v_{Si}(3a_{si} + 2a_\varepsilon)}{b} + 5a_\varepsilon\frac{Z_{pn}}{\varepsilon_\varepsilon}\right] \tag{57}$$

$$a_{n3} = k_t\left(n_{1t} + 2n_{2t}h + n_{3t}h^2\right) - k_c n_{1c} + 120a_\varepsilon\left[\frac{\Sigma k_{si}\alpha_{esi}A_{si}v_{Si}(2a_{si} + 3a_\varepsilon)}{b} + 5a_\varepsilon\frac{Z_{pn}}{\varepsilon_\varepsilon}\right] \tag{58}$$

$$a_{n4} = k_t\left(n_{2t} + 2n_{3t}h + n_{4t}h^2\right) - k_c n_{2c} + 60\left[\frac{\Sigma k_{si}\alpha_{esi}A_{si}v_{Si}(a_{si} + 4a_\varepsilon)}{b} + 5a_\varepsilon\frac{Z_{pn}}{\varepsilon_\varepsilon}\right] \tag{59}$$

$$a_{n5} = k_t\left(n_{3t} + 2n_{4t}h\right) - k_c n_{3c} + 60\left(\frac{\Sigma k_{si}\alpha_{esi}A_{si}v_{Si}}{b} + \frac{Z_{pn}}{\varepsilon_\varepsilon}\right) \tag{60}$$

$$a_{n6} = k_t n_{4t} - k_c n_{4c} \tag{61}$$

$$a_{m0}=k_t\left(7n_{0t}h^2a_a-m_{0t}h^3\right)+420a_\varepsilon^4\left[\frac{\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}a_{si}\left(a_a-a_{si}\right)}{b}+a_\varepsilon\left(Z_{pm}+\frac{M}{Z_E}\right)\frac{1}{\varepsilon_\varepsilon}\right] \tag{62}$$

$$a_{m1}=k_t\left\{7\left[n_{0t}\left(2a_a+h\right)h+n_{1t}h^2a_a\right]-3m_{0t}h^2-m_{1t}h^3\right\}+ \\ +420a_\varepsilon^3\left[\frac{\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}\left(4a_{si}+a_\varepsilon\right)\left(a_a-a_{si}\right)}{b}+5a_\varepsilon\left(Z_{pm}+\frac{M}{Z_E}\right)\frac{1}{\varepsilon_\varepsilon}\right] \tag{63}$$

$$a_{m2}=k_t\left\{7\left[n_{0t}\left(a_a+2h\right)+n_{1t}\left(2a_a+h\right)h+n_{2t}h^2a_a\right]-3m_{0t}h-3m_{1t}h^2-m_{2t}h^3\right\}- \\ -7k_cn_{0c}a_a+840a_\varepsilon^2\left[\frac{\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}\left(3a_{si}+2a_\varepsilon\right)\left(a_a-a_{si}\right)}{b}+5a_\varepsilon\left(Z_{pm}+\frac{M}{Z_E}\right)\frac{1}{\varepsilon_\varepsilon}\right] \tag{64}$$

$$a_{m3}=k_t\left\{7\left[n_{0t}+n_{1t}\left(a_a+2h\right)+n_{2t}\left(2a_a+h\right)h+n_{3t}h^2a_a\right]-m_{0t}-3m_{1t}h-3m_{2t}h^2-m_{3t}h^3\right\}- \\ -k_c\left[7\left(n_{0c}+n_{1c}a_a\right)-m_{0c}\right]+840a_\varepsilon\left[\frac{\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}\left(2a_{si}+3a_\varepsilon\right)\left(a_a-a_{si}\right)}{b}+5a_\varepsilon\left(Z_{pm}+\frac{M}{Z_E}\right)\frac{1}{\varepsilon_\varepsilon}\right] \quad 65)$$

$$a_{m4}=k_t\left\{7\left[n_{1t}+n_{2t}\left(a_a+2h\right)+n_{3t}\left(2a_a+h\right)h+n_{4t}h^2a_a\right]-m_{1t}-3m_{2t}h-3m_{3t}h^2-m_{4t}h^3\right\}- \\ -k_c\left[7\left(n_{1c}+n_{2c}a_a\right)-m_{1c}\right]+420\left[\frac{\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}\left(a_{si}+4a_\varepsilon\right)\left(a_a-a_{si}\right)}{b}+5a_\varepsilon\left(Z_{pm}+\frac{M}{Z_E}\right)\frac{1}{\varepsilon_\varepsilon}\right] \tag{66}$$

$$a_{n5}=k_t\left\{7\left[n_{2t}+n_{3t}\left(a_a+2h\right)+n_{4t}\left(2a_a+h\right)h\right]-m_{2t}-3m_{3t}h-3m_{4t}h^2\right\}- \\ -k_c\left[7\left(n_{2c}+n_{3c}a_a\right)-m_{2c}\right]+420\left[\frac{\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}\left(a_a-a_{si}\right)}{b}+\left(Z_{pm}+\frac{M}{Z_E}\right)\frac{1}{\varepsilon_\varepsilon}\right] \tag{67}$$

$$a_{m6}=k_t\left\{7\left[n_{3t}+n_{4t}\left(a_a+2h\right)\right]-m_{3t}-3m_{4t}h\right\}-k_c\left[7\left(n_{3c}+n_{4c}a_a\right)-m_{3c}\right] \tag{68}$$

$$a_{m7}=k_t\left(7n_{4t}-m_{4t}\right)-k_c\left(7n_{4c}-m_{4c}\right) \tag{69}$$

When the calculation is based on the method of successive approximations, then the new value of strain ε_ε is established from the moment Eqs. (54) or (23).

When the stresses of the tension zone are disregarded, then in the static equilibrium equations $k_t=0$.

3. Examples of the method use

Calculation of the cracking moment of a reinforced concrete beam.

Below we demonstrate how it is possible to use the formulae proposed in the present paper for the calculation of the cracking moment M_{cr} of a flexural member, the cross-section of which is shown in Fig.1. Only the tension zone of the beam is reinforced, i.e. $A_{si}=A_s$, $a_{si}=a_s$, $\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}a_{si}=k_s\alpha_{es}A_s\nu_Sa_s$, etc. The data are from [7].

For example, dimensions of the rectangular cross-section are shown in Fig. 1: $b=0.20$ m, $h=0.50$ m, $d=0.46$ m. Strength class of concrete C25/30: $\sigma_{c1}=f_{cm}=-33$ MPa, $E_{cm}=22\cdot\left(\frac{f_{cm}}{10}\right)^{0.3}=31.476$ GPa, $\varepsilon_{0cm}=\varepsilon_{c1}=-2.0694$ ‰, $\varepsilon_{cu1}=-3.5$ ‰, $f_{ctm}=2.5650$ MPa. Reinforcement S400: $f_s=f_{sk}=400$ MPa, $E_s==200$ GPa, $A_s=14.681\cdot10^{-4}$ m^2, $\rho_l=\frac{14.681}{20\cdot46}\cdot100==1.596\,\%$. $E=E_c=1.1E_{cm}=1.1\cdot31.476=34.623$ GPa, $\alpha_{es}=E_s/E_c=200/34.623=5.7765$.

$$Z_{bs}=\frac{k_s\alpha_{es}A_s\nu_S}{b}=\frac{1\cdot5.7765\cdot14.681\cdot10^{-4}\cdot1}{0.20}=0.04240\,\text{m}.$$

$\nu_{c1}=\frac{\sigma_{c1}}{E_c\varepsilon_{c1}}=\frac{-33}{34.623\cdot(-2.0694)}=0.4606$. $c_{1c}=-0.6311$, $c_{2c}=0.1059$, $c_{3c}=-0.01559$, $c_{4c}=0.001389$.

Let us assume that: $k_t=k_c=k_s=k_\varepsilon=1$, $a_\varepsilon==0.50$ m, $a_a=a_s=d=0.46$ m, $\varepsilon_\varepsilon=\varepsilon_{0\varepsilon}=\varepsilon_{ctm,\lim}==0.14817$ ‰.

Let us assume, for instance, a diagram of stresses of the tension zone of concrete that would be similar in quality to the diagram of stresses of the compression zone of concrete, i.e. $\varepsilon_{t1}=\varepsilon_{0tm}=\frac{2.5650}{0.4606\cdot34.623}=0.16084$ ‰, and the threshold concrete extension (breaking) strain $\varepsilon_{0\varepsilon}=\varepsilon_\varepsilon=\varepsilon_{ctm,\lim}=2\cdot2.5650/34.623=0.14817$ ‰, i.e. $\nu_{t1}=\sigma_{t1}/E_c\varepsilon_{t1}=\frac{2.565}{34.623\cdot0.16084}=0.4606=\nu_{c1}$. For the compression zone, $\eta_{0\varepsilon c}=\frac{\varepsilon_{0\varepsilon}}{\varepsilon_{0cm}}=\frac{0.14817}{-2.0694}=-0.07160$, for the tension zone $\eta_{0\varepsilon t}=\varepsilon_{0\varepsilon}/\varepsilon_{0tm}=\frac{0.14817}{0.16084}=0.92123$.

When we insert the values of parameters into Eq. (53), we receive $x_w=-0.2474$ m.

When we know the position of the neutral axis ($x_w=-0.2474$ m) and the strain ε_ε ($\varepsilon_\varepsilon=0.14817$ ‰), then the values of other parameters can be calculated with ease.

The value of the cracking moment M_{cr} can be calculated using Eq. (54): $M_{cr} = 40.97$ kN·m.

When we insert the values of parameters $d_u = h$ $\omega_{nc} = 0.4854$, $\omega_{mc} = 0.3224$, $\omega_{nt} = 0.3264$ and $\omega_{mt} = 0.2041$ and $d_u = h = 0.50$ m, $x_w = -0.2474$ m, $Z_m = 0.9746 \cdot 10^{-6} M$ (N·m)$^{-1}$ and $Z_{ma} = 0.4873 \cdot 10^{-6} M$ N^{-1} into Eq. (25), we receive $M_{cr} = 40.95$ kN·m.

Calculation of x_w from the Eq. (54) of moments.

If we know the values of, for instance, $M = 40.97$ kN·m and $\varepsilon_{0\varepsilon} = \varepsilon_\varepsilon = 2 \cdot \frac{2.5650}{34.623} = 0.14817$ ‰, then from Eq. (54) we can calculate the value $x_w = -0.2477$ m.

4. Conclusions

The method and the formulae presented in the paper can be applied directly (without the successive approximation cycles) for the calculation of stress-strain state parameters according to curvilinear material stress diagrams at normal sections of beam members with rectangular cross-sections. The formulae are applicable to the members without cracks. They are also applicable for the calculation of the members having cracks for their sections between the cracks. When the stresses of the tension zone are ignored, the formulae may be used even for the sections that are located near the crack. The direct calculation is possible when we know the strains ε_ε and $\varepsilon_{0\varepsilon}$ of some of the layers. In other cases, the calculations have to be repeated.

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I. Židonis

METODAS STRYPINIŲ STAČIAKAMPIO SKERSPJŪVIO ELEMENTŲ ĮTEMPIŲ-DEFORMACIJŲ BŪVIO PARAMETRAMS STATMENUOSE PJŪVIUOSE TIESIOGIAI APSKAIČIUOTI PAGAL KREIVINES ĮTEMPIŲ DIAGRAMAS

R e z i u m ė

Šis darbas yra ankstesnių darbų tęsinys. Straipsnyje pateikta metodika ir formulės, leidžiančios skaičiavimus atlikti tiesiogiai (be nuoseklaus artėjimo ciklų). Panaudojamos kreivinės medžiagų įtempių diagramos. Formulės sudarytos strypinių stačiakampio skerspjūvio elementų įtempių-deformacijų būvio parametrams apskaičiuoti ašiai statmenuose pjūviuose. Jos taikytinos elementams be plyšių (pavyzdžiui, plyšimo momentui apskaičiuoti) ir elementų su plyšiais pjūviams tarp plyšių (armuotų elementų įtempių-deformacijų būviui apskaičiuoti, armatūros deformacijos nukrypimui nuo plokščiųjų pjūvių nustatyti). Įtempių galima ir nepaisyti. Kai elementų tempiamos zonos įtempių nepaisoma, formulės tinka net ir pjūviams ties plyšiu (irimo momentui arba armavimui apskaičiuoti). Tiesiogiai skaičiuoti galima tuomet, kai žinomos kurio nors sluoksnio deformacijų ε_ε ir $\varepsilon_{0\varepsilon}$ reikšmės. Kitais atvejais skaičiavimus reikia kartoti.

Čia pateikta tik dalis jau atlikto darbo. Panaudotos penktojo laipsnio kreivinės įtempių diagramos. Kituose straipsniuose numatoma paskelbti formules, kuriose galima imti įvairių formų įtempių diagramas, elementai gali būti su lentynomis ir su plyšiais.

I. Židonis

METHOD FOR A DIRECT CALCULATION OF STRESS-STRAIN STATE PARAMETERS AT NORMAL RIGHT-ANGLED SECTIONS OF STRUCTURAL MEMBERS GIVEN CURVILINEAR STRESS DIAGRAMS

S u m m a r y

The present paper is a continuation of previous papers. The paper presents a methodology and formulae for direct calculations (without successive approximations cycles). Curvilinear material stress diagrams are employed. The formulae have been devised for the calculation of stress-strain state parameters at normal sections of structural members. They are applicable for both crack-less members (e.g. for the calculation of the cracking moment) and for the calculation of sections between cracks of the members with cracks (for the calculation of the stress-strain parameters of reinforced members, to establish the

deviation of the deformation of the reinforcement from the plane sections). It is also possible to disregard the stresses. When the stresses of the tensile zone of the members are disregarded, the formulae may be also used for the sections near the crack (for the calculation of the breaking moment or the reinforcement). The direct calculation is possible when we know the strain ε_ε of any of the layers. In other cases, the calculations have to be repeated.

The present paper presents only a part of the work done. A case is presented when curvilinear stress diagrams of the fifth degree are used. Further papers will present the formulae employing simpler third degree curvilinear stress diagrams or the ones employing noncurvilinear stress diagrams or the ones devised for the calculation of parameters at the sections near the crack or when the members are with flanges.

И. Жидонис

МЕТОД ПРЯМОГО РАСЧЕТА ПАРАМЕТРОВ НАПРЯЖЕННО-ДЕФОРМИРОВАННОГО СОСТОЯНИЯ ПО НОРМАЛЬНЫМ СЕЧЕНИЯМ СТЕРЖНЕВЫХ ЭЛЕМЕНТОВ ПРЯМОУГОЛЬНОГО СЕЧЕНИЯ ПРИ КРИВОЛИНЕЙНЫХ ДИАГРАММАХ НАПРЯЖЕНИЙ

Р е з ю м е

Работа является продолжением предыдущих работ. В статье представлены методика и формулы прямого расчета (без циклов последовательного приближения) параметров напряженно-деформированного состояния по нормальным сечениям стержневых элементов прямоугольного сечения при криволинейных диаграммах напряжений материалов пятой степени. Формулы предназначены для элементов без трещин (например, для расчета момента трещинообразования) и для сечений между трещинами в элементах с трещинами (для расчета параметров напряженно-деформированного состояния армированных элементов с определением отклонения деформаций арматуры от плоских сечений). Напряжения растянутой зоны могут и не учитываться. В этих случаях формулы пригодны также и для расчета в сечениях по трещинам (для определения момента разрушения или расчета армирования). Прямой расчет возможен в тех случаях, когда известны деформации и любого одного слоя. В других случаях расчет необходимо повторять.

Здесь представлена только часть уже выполненной работы. В следующих статьях предполагается опубликовать формулы, в которых можно принимать диаграммы напряжений различных форм, элементы могут иметь полки и трещины.

Received March 23, 2009
Accepted May 15, 2009

DOI: 10.5755/j02.mech.15236