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Determination of stresses and strains in two-layer mechanically inhomogeneous pipe subjected to internal pressure at elastic plastic loading

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1. Introduction

Two layer metal plastic structures with range shape cross-section are frequently used [1 - 3]. The external plastic rust preventive cowers are applied in metal pressure vessels and pipelines. Plastic tubes with the reinforcing external metal layer are widely used [4]. Stresses distribution in multilayer structures subjected to internal pressure has been successfully determined by FEM [2]. However, it is difficult to use when it is necessary to determine how design parameters affect the stress values and strength of the structure.

Mechanically inhomogeneous two-layer pipe may be divided into two pipes with different mechanical properties: internal pipe 1 with modulus of elasticity E_1, Poisson's ratio ν_1, limit of elasticity σ_{e1}, power index of material hardening in elastic plastic zone m_{01} and external pipe 2 with E_2, ν_2, σ_{e2} and m_{02} (Fig. 1). Due to acting internal pressure p radial stretch and contact pressure p_c on the surface materials H (hard) and M (mild) appears. When $E_1 > E_2$ the structure is denote H-M and in opposite case – M-H. Dependencies for stresses and strain determination of two-layer pipe at elastic loading are presented in work [5].

These dependencies were obtained by using the relative parameters of the two-layer pipe (Fig. 1): $s_1 = \delta_1/r_1$, $s_2 = \delta_2/r_1$, $s = \delta/r_1 = s_1 + s_2$, $r/r_1 = 1 + s_1$, $r_2/r_1 = 1 + s_1 + s_2$, $\xi = \rho/r_1 - 1$.

Then the relative stresses of layer 1, when $0 \le \xi \le s_1$

$$\left.\begin{matrix}\sigma_{r1}^{L}(\xi)/p \\ \sigma_{\theta 1}^{L}(\xi)/p\end{matrix}\right\} = \frac{1 - C_p(1+s_1)^2}{s_1(2+s_1)} \mp \frac{1 - C_p}{s_1(2+s_1)}\left(\frac{1+s_1}{1+\xi}\right)^2 {}^{*} \qquad (1)$$

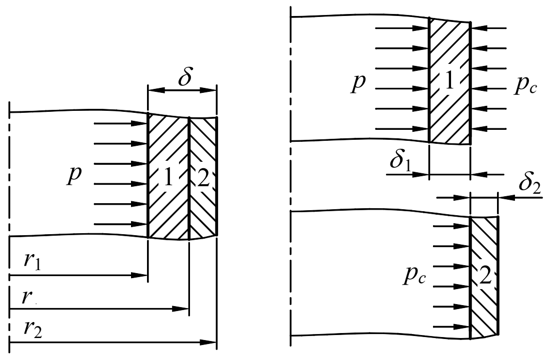

Fig. 1 Scheme of mechanically inhomogeneous two-layer pipe: a – scheme of the pipe; b – distribution of pressures in internal layer 1 and external layer 2

For external layer 2 ($s_1 < \xi \le s$)

$$\left.\begin{matrix}\sigma_{r2}^{L}(\xi)/p \\ \sigma_{\theta 2}^{L}(\xi)/p\end{matrix}\right\} = C_p(1+s_1)^2 \frac{(1+\xi)^2 \mp (1+s_1+s_2)^2}{s_2(2+2s_1+s_2)(1+\xi)^2} \qquad (2)$$

Contact pressure on the contact surface of 1 and 2 materials $p_c = p\,C_p$. The coefficient

$$C_p = \frac{2}{s_1(2+s_1)\left[C_1 - \nu_1 + (C_2 + \nu_2)E_1/E_2\right]} \qquad (3)$$

where $C_1 = \dfrac{(1+s_1)^2 + 1}{s_1(2+s_1)}$, $C_2 = \dfrac{(1+s_1+s_2)^2 + (1+s_1)^2}{s_2(2+2s_1+s_2)}$.

The analytical method for stress strain state determination of mechanically inhomogeneous two-layer pipe subjected to internal pressure at elastic plastic loading till now is not created. This analytical method is presented in this work. For estimating durability of pipes the maximum values of strains in overloaded zones must be known [6].

2. Stress strain state of two-layer pipe determination at elastic plastic loading

Stress intensity of the pipe subjected by internal pressure when $\sigma_z = 0$ is

$$\sigma_i = \sqrt{\sigma_\theta^2 + \sigma_r^2 - \sigma_\theta\sigma_r} \qquad (4)$$

The relative value of stress intensity of internal layer at elastic loading

$$\frac{\sigma_{i1}(\xi)}{p} = \frac{\sqrt{\left[1 - C_p(1+s_1)^2\right]^2 + 3(1-C_p)^2\left(\dfrac{1+s_1}{1+\xi}\right)^4}}{s_1(2+s_1)} \qquad (5)$$

may be determined by Eqs. (1) and (4) and of external layer

$$\frac{\sigma_{i2}(\xi)}{p} = \frac{C_p(1+s_1)^2\sqrt{(1+\xi)^4 + 3(1+s_1+s_2)^4}}{s_2(2+2s_1+s_2)(1+\xi)^2} \qquad (6)$$

– by Eqs. (2) and (4).

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* upper index L denotes Lame's equations
lower indexes 1 and 2 denotes the internal and external layer

Maximum external pressure when material is deformed elastically may be determined by the expression

$$p_{max}^{e} = \frac{\sigma_e}{\sigma_i(\xi)/p} \;^{**} \tag{7}$$

when σ_e is limit of elasticity; $\xi = 0$ for layer 1 and $\xi = s_1$ for layer 2.

Elastic plastic strains in this zone of the pipe appear when internal pressure $p > p_{max}^{e}$. Usually elastic plastic loading begins in layer 1 [5]. Stresses and strains are determined by assuming these assumptions:

- ratio of strains in the thickness of layer 1 at elastic and elastic plastic loading is the same [7];
- expression σ_r / p is valued at elastic and elastic plastic loading.

Then from the first assumption by estimating $e_{i1}(\xi_{p1}) = \sigma_{e1} / E_1 = e_{e1}$ the value of $e_{i1}(\xi)$ may be calculated at elastic plastic loading

$$e_{i1}(\xi) = e_{i1}(\xi_{p1}) \frac{e_{i1}^{e}(\xi)}{e_{i1}^{e}(\xi_{p1})} = e_{e1} \frac{\sigma_{i1}^{L}(\xi)}{\sigma_{i1}^{L}(\xi_{p1})} \;^{***} \tag{8}$$

where ξ_{p1} is the maximum value elasto plastically deformed zone of layer 1.

The strain intensity value in layer 1

$$e_{i1}(\xi) = \frac{\sigma_{e1}}{E_1} \sqrt{\frac{\left[1 - C_p(1+s_1)^2\right]^2 + 3\left(1 - C_p\right)^2 \left(\frac{1+s_1}{1+\xi}\right)^4}{\left[1 - C_p(1+s_1)^2\right]^2 + 3\left(1 - C_p\right)^2 \left(\frac{1+s_1}{1+\xi_{p1}}\right)^4}} \tag{9}$$

is calculated from Eq. (8) by estimating dependence (5).

Stress intensity which at elastic plastic loading corresponds $e_{i1}(\xi)$ is

$$\sigma_{i1}(\xi) = \sigma_{e1} \left(\frac{e_{i1}(\xi)}{e_{e1}}\right)^{m_{01}} \tag{10}$$

When $\sigma_{i\,1}(\xi) > \sigma_{e\,1}$ secant modulus of stress strain curve

$$E_1'(\xi) = \sigma_{i\,1}(\xi) / e_{i\,1}(\xi) \tag{11}$$

Stiffness of internal layer 1 and increasing rate of internal pressure p at elastic plastic loading decrease with increasing ξ_{p1}. The value of internal pressure $p_{\xi_{p1}}$, which corresponds elasto-plastically deformed zone ξ_{p1} is determined by approaching method from the condition $\sigma_{i1}(\xi_{p1}) = \sigma_{e1}$, i.e.

$$\frac{\sigma_{e1}}{p_{\xi_{p1}}} = \frac{\sqrt{\left[1 - C_p(1+s_1)^2\right]^2 + 3\left(1 - C_p\right)^2 \left(\frac{1+s_1}{1+\xi_{p1}}\right)^4}}{s_1(2+s_1)} \tag{12}$$

** upper index e denotes elastic loading

*** (0), (ξ_{p1}), (s_1) denotes the values when $\xi = 0$, $\xi = \xi_{p1}$, $\xi = s_1$

In the first approaching it may be accepted

$$p_{\xi_{p1}} \approx \frac{\sigma_{e1}\, s_1 (2+s_1) \dfrac{E_{1c}}{E_1}}{\sqrt{\left[1 - C_p(1+s_1)^2\right]^2 + 3\left(1 - C_p\right)^2 \left(\frac{1+s_1}{1+\xi_{p1}}\right)^4}} \tag{13}$$

where $E_{1c} \approx \{\, 0.5\, [\, E'_1(0) + E_1\,]\, \xi_{p1} + E_1\, (s_1 - \xi_{p1})\,\} \,/\, s_1$ is the conditional elasticity modulus in relative thickness s_1 of layer 1.

Maximum value of $p_{\xi_{p1}}$ is recommended to determine from condition (12) when $\xi_{p1} = \xi_{p1\,max} = 0.5\, s_1$. With the further increasing $p_{\xi_{p1}}$ the elasto-plastically deformed zone rapidly increases. When $p = 1.055\, p_{\xi_{p1\,max}}$ elasto-plastically deformed zone reaches the external surface of pipe wall [7]. Therefore, stability of the structure may be loosed. For increasing stability of the structure and satisfaction the dependence $e_{\theta 1}(s_1) = e_{\theta 2}(s_1)$ is recommended choose the relative thickness s_2 from the condition $\sigma_{i2\,max} = \sigma_{i\,2}\big|_{\xi = s_1} \le \sigma_{e\,2}$. Determination of the recommended value s_{2r} is described in the third chapter. Radial stresses at elastic plastic loading

$$\sigma_{r1}(\xi) = p_{\xi_{p1}}\; \sigma_{r1}^{L}(\xi) / p \tag{14}$$

and

$$\sigma_{r2}(\xi) = p_{\xi_{p1}}\; \sigma_{r2}^{L}(\xi) / p \tag{15}$$

Stress $\sigma_{\theta 1}$ when $\xi \le \xi_{p1}$ may be determined from Eqs. (4), (10) and (14):

$$\sigma_{\theta 1}(\xi) = \frac{\sigma_{r1}(\xi)}{2} + \sqrt{\frac{\sigma_{r1}^{2}(\xi)}{4} + \sigma_{i1}^{2}(\xi) - \sigma_{r1}^{2}(\xi)} \tag{16}$$

In elastically deformed zone of interlayer 1 when $\xi > \xi_{p1}$ circumference stress may be determined by the dependence

$$\sigma_{\theta 1}(\xi) = \sigma_{\theta 1}(\xi_{p1})\; \sigma_{\theta 1}^{L}(\xi) / \sigma_{\theta 1}^{L}(\xi_{p1}) \tag{17}$$

which is obtained analogically as Eq. (8) (Fig. 2).

Stress intensity $\sigma_{i1}(\xi)$ when $\xi > \xi_{p1}$ is calculated from Eq. (4) by estimating Eqs. (13) and (17).

Strain intensity of layer 1 is calculated by the dependences

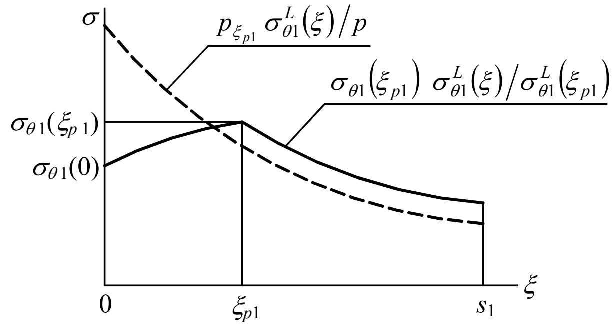

Fig. 2 Scheme for $\sigma_{\theta 1}(\xi)$ determination at elastic plastic deforming of layer 1

$$\left.\begin{aligned} e_{i1}(\xi) &= \frac{\sigma_{e1}}{E_1}\left[\frac{\sigma_{i1}(\xi)}{\sigma_{e1}}\right]^{1/m_{01}} && \text{when} \quad \sigma_{i1}(\xi) > \sigma_{e1} \\ e_{i1}(\xi) &= \sigma_{i1}(\xi)/E_1 && \text{when} \quad \sigma_{i1}(\xi) \le \sigma_{e1} \end{aligned}\right\} \qquad (18)$$

Material of layer 1 is deformed elastically when $\xi \ge \xi_{p1}$. In this case strains e_θ, e_r and e_z are calculated by Hooke's law when $\sigma_{z1} = 0$. In elasto-plastically deformed zone of layer 1 ($\xi < \xi_{p1}$) strains are equal: $e_{\theta 1} = (\sigma_{\theta 1} - \nu_{N1}\sigma_{r1})/E_1'$, $e_{r1} = (\sigma_{r1} - \nu_{N1}\sigma_{\theta 1})/E_1'$ and $e_{z1} = -\nu_{N1}(\sigma_{\theta 1} + \sigma_{r1})/E_1'$. Poisson's ratio at elastic plastic loading may be calculated by Nadai's dependence $\nu_{N1} = 0.5 - (0.5 - \nu_1)\,E_1'/E_1$. At elastic plastic loading contact pressure negligibly depends on $\nu_2 - \nu_1$ [5]. Analysis ν_1^* on ξ_{p1} showed that $\nu_{N1} = 1.03\,\nu_1$ when $\xi_{p1} = 0.5\,s_1$. For simplifying calculations at elasto plastic loading instead of ν_{N1} Poisson's ratio at elastic loading ν_1 may be used. Strains $e_{\theta 2}$, e_{r2} and e_{z2} are also calculated from Hooke's law by estimating that $\sigma_{r2} = p_{\xi_{p1}}\,\sigma_{r2}^L/p$, $\sigma_{\theta 2} = p_{\xi_{p1}}\,\sigma_{\theta 2}^L/p$ and $\sigma_{z2} = 0$.

3. Determination of the rational geometrical parameters of two-layer pipe subjected to internal pressure

Serviceability of two-layer pipe depends on location of the layers (structure H-M or M-H), admissible internal pressure p_{adm}, $\sigma_{i\,2\,max}$ and the relative thickness of layers s_1 and s_2. Distribution of stresses and strains in mechanically inhomogeneous two-layer pipe, subjected to internal pressure p, at elastic plastic loading of H-M and M-H structures when $\xi_{p1\,max}$, $E_H = 21\cdot 10^4$ MPa, $E_M = 7\cdot 10^4$ MPa, $\sigma_{eH} = 350$ MPa, $\sigma_{eM} = 100$ MPa, $\nu_H = \nu_M = 0.3$, $r_1 = 50$ mm, $s_1 = s_2 = 0.25$, $m_{0H} = 0.15$, $m_{0M} = 0.2$ are analyzed in this work. Distribution of stress intensity σ_i and stresses σ_θ, σ_r at elastic loading in this pipe is presented in work [5]. Strength of homogeneous pipe may be expressed by the admissible value of internal pressure p_{adm} [7]. Strength of internal pipe 1, as follows from Eq. (7), increases with increasing contact pressure p_c.

Increasing p_{max}^e of internal pipe at elastic loading may be expressed by the ratio

$$K = \frac{p_{max}^e}{p_{max\,|\,C_p=0}^e} \qquad (19)$$

where $p_{max}^e = \dfrac{\sigma_{e1}\,s_1\,(2+s_1)}{\sqrt{\left[1 - C_p(1+s_1)^2\right]^2 + 3\,(1-C_p)^2\,(1+s_1)^4}}$ is maximum value of internal pressure, when $\sigma_{i1\,max} = \sigma_{e1}$ and $p_{max\,|\,C_p=0}^e = \dfrac{\sigma_{e1}\,s_1\,(2+s_1)}{\sqrt{1+3\,(1+s_1)^4}}$. Lower index "$C_p = 0$" denote value of homogeneous pipe without reinforcement layer ($C_p = 0$ and $s_2 = 0$).

At elastic loading of two-layer pipe, which is shown in work [5], the ratio K for structure H-M is 1.13 and for structure M-H – $K = 2.397$. Therefore, structure M-H, when layer 2 is reinforcing cover of internal pipe 1, is frequently used. When s_1 is known the recommended relative thickness s_{2r} at elastic loading of layer 2 in structure M-H may be determined from the condition

$$\sigma_{i2\,max} = \frac{p_{\xi_{p1}}\,C_p\,(1+s_1)^2\,\sqrt{1+3\,(1+s_1+s_{2r})^4}}{s_{2r}\,(2+2\,s_1+s_{2r})} = \sigma_{e2} \qquad (20)$$

by approaching method.

Ratio $\sigma_{i2\,max}/\sigma_{e2}$ depends on σ_{e2}/σ_{e1}, E_2/E_1 and s. When

$$\sigma_{i1}(s_1)_{|\,C_p=0}\,\frac{E_2}{E_1} < \sigma_{e2} \qquad (21)$$

the s_{2r} can not be obtained by decreasing of s_2 (Fig. 3).

In structure M-H for plastic pipes with the reinforcing metal layer 2 $E_2 \ge 3\,E_1$ and $\sigma_{e2} = (2.5\ ...\ 3)\,\sigma_{e1}$ are recommended.

Polypropylene tubes with the aluminum alloy reinforcement layer are frequently used [4]. Also reinforced high density polyethylene tubes which mechanical properties are presented in work [8] may be used.

Structure H-M is often used when layer 2 is the rust preventive cover of metal internal pipe 1. In this case $E_2 < E_1/3$, $\sigma_{e2} < 0.3\,\sigma_{e1}$ and $s_2 < s_1$ are recommended.

Relative thickness s_2 may be determined by the condition: $\sigma_{i2\,max} \le \sigma_{e2}$. Pressure p_{max}^e in the both cases is determined by Eqs. (5) and (7) when $\xi = 0$.

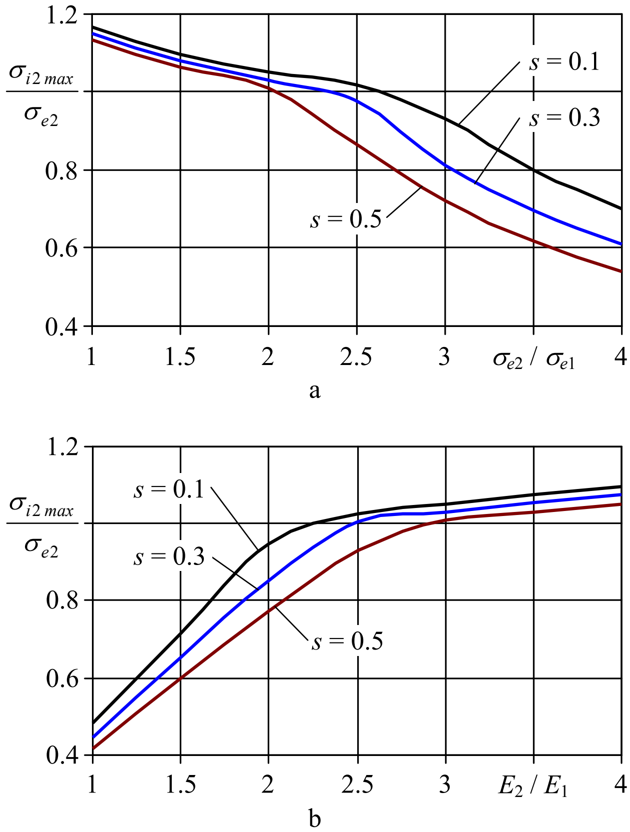

Fig. 3 Dependence of $\sigma_{i2\,max}/\sigma_{e2}$ on σ_{e2}/σ_{e1} (a), E_2/E_1 (b) and s for M-H structure, when $s_1 = s_2$: a – $E_2/E_1 = 3.0$; b – $\sigma_{e2}/\sigma_{e1} = 2.0$

When admissible elasto-plastically deformed zone of layer 1 is $0 < \xi_{p1\,adm} \leq \xi_{p1\,max}$ the recommended value s_{2r} is determined analogically as at elastic loading [5]. In this case instead of p^{e}_{max} the $p_{\xi_{p1\,adm}}$ determined from condition (12) is used.

Elastic plastic strains appear in layer 2 when $s_2 < s_{2r}$. Values of $\sigma_{i\,2}(s_1)$, $\sigma_{\theta 2}(s_1)$, $E'_2(s_1)$, $\sigma_{\theta 2}(\xi_{p\,2})$, $\sigma_{\theta 2}(\xi)$ are determined analogically as $\sigma_{i\,1}(0)$, $\sigma_{\theta 1}(0)$, $E'_1(0)$, $\sigma_{\theta 1}(\xi_{p\,1})$, $\sigma_{\theta 1}(\xi)$ by using $p_{\xi_{p1}}$.

The elasto-plastically deformed zone of layer 2 ξ_{p2} is determined by approaching method from the condition $\sigma_{i2}(\xi_{p2}) = \sigma_{e2}$, i.e.

$$\frac{\sigma_{e2}}{p_{\xi_{p1}}} = \frac{C_p\left(1+s_1\right)^2 \sqrt{\left(1+\xi_{p2}\right)^4 + 3\left(1+s_1+s_2\right)^4}}{s_2\left(2+2\,s_1+s_2\right)\left(1+\xi_{p2}\right)^2} \qquad (22)$$

In the first approaching may be accepted

$$\xi_{p2} \approx \sqrt[4]{\frac{3\,p_c^{\,2}\left(1+s_1\right)^4\left(1+s_1+s_2\right)^4}{\left[\sigma_{e2}\,s_2\left(2+2\,s_1+s_2\right)\right]^2 - p_c^{\,2}\left(1+s_1\right)^4}} - 1 \qquad (23)$$

The theory of small elastic plastic strains for pipes, subjected to internal pressure, may be used when $\xi_p \leq 0.3\ ...\ 0.35$ [7]. Therefore is recommended $\xi_{p1} \leq 0.35\, s_1$ and $\xi_{p2} \leq s_1 + 0.35\, s_2$.

4. Results of stresses and strains investigations in two-layer pipe at elastic plastic loading

Distribution of stress state components in the investigated two-layer pipe at elastic plastic loading strain intensity e_i is shown in Fig. 4 and circumference strain e_θ – in Fig. 5.

Because in structure H-M $\sigma_{i2\,max} = \sigma_{e2}$, when $p_{\xi_{p1\,max}}$ and $s_2 = s_1 = 0.25$, the recommended value of relative thickness of layer 2 $s_{2r} = 0.25$ (Fig. 4, a). In structure M-H, when $s_2 = s_1 = 0.25$, $\sigma_{i2\,max} = 0.61\ \sigma_{e2}$. Therefore, in this case elastic properties of layer 2 are not exploited to the maximum (Fig. 4, b).

In elastic plastic deformed zone ξ_{p1} stress $\sigma_{\theta 1}$ increases and $\sigma_{i\,1}$ – decreases with increasing ξ. Circumference strain $e_{\theta 1}$ and strain intensity $e_{i\,1}$ in elasto plastically deformed zone ξ_{p1} approximately linearly decrease with increasing ξ.

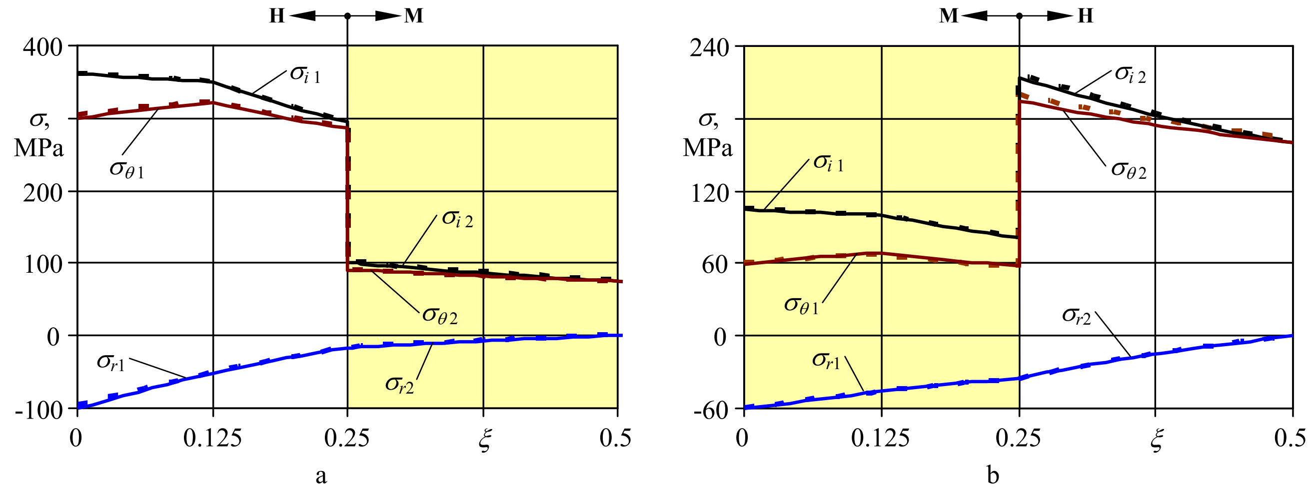

Fig. 4 Distribution of σ_i, σ_θ and σ_r in the thickness of pipe wall (——) determined analytically and (– – –) by FEM: a – structure H-M when $p_{\xi_{p1\,max}} = 101.6$ MPa; b – structure M-H when $p_{\xi_{p1\,max}} = 61.2$ MPa

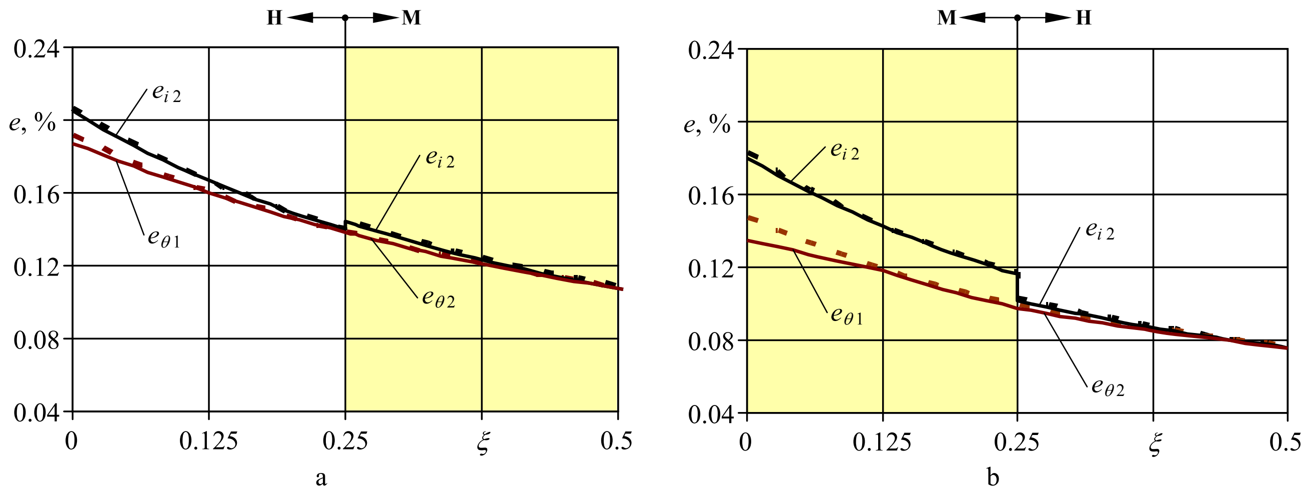

Fig. 5 Distribution of e_i and e_θ in the thickness of pipe wall (——) determined analytically and (– – –) by FEM: a – structure H-M when $p_{\xi_{p1\,max}} = 101.6$ MPa; b – structure M-H when $p_{\xi_{p1\,max}} = 61.2$ MPa

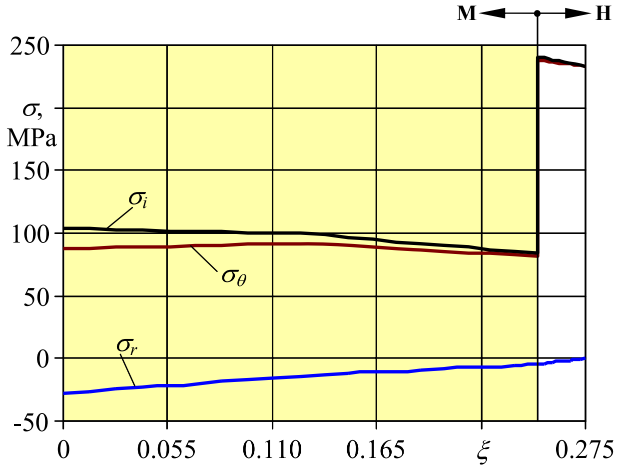

Fig. 6 Distribution of stresses determined analytically in the thickness of pipe wall of structure M-H when $\xi_{p1}=\xi_{p1\,max}=0.5\,s_1$, $p=28.1$ MPa, $s_1=0.25$, $s_2=s_{2r}=0.025$ and $\sigma_{i2\,max}=0.687\,\sigma_{e2}$

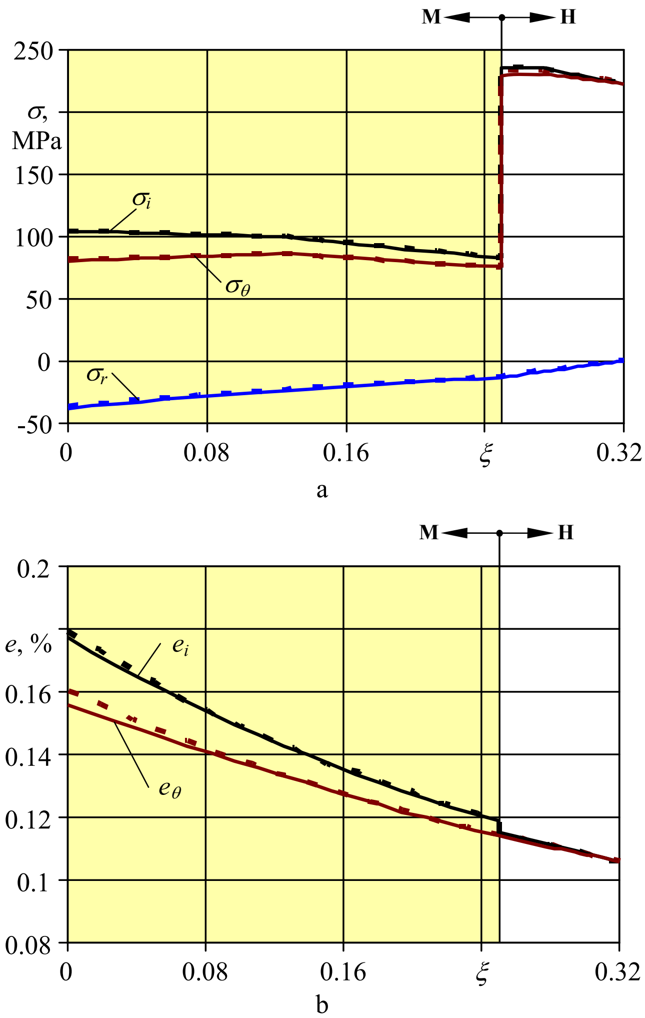

Fig. 7 Distribution of stress strain components in the thickness of pipe wall of structure M-H when both layers are deformed elasto plastically (——) determined analytically and (– – –) by FEM, when $\sigma_{e2}=235$ MPa, $s_2=0.07$, $\xi_{p2}\approx s_1+0.35\,s_2$, $p=37.8$ MPa (other parameters of the pipe are the same as in Fig. 4, b and Fig. 5, b): a – distribution of stresses; b – distribution of strains

Elastic properties of layer 2 in structure M-H can not be exploited to the maximum because Eq. (21) can not be satisfying: $\sigma_{i\,1}(s_1)_{|\,C_p=0;\ \xi_{p1\,max}}\,E_2\,/\,E_1=255$ MPa, while $\sigma_{e\,2}=350$ MPa. Therefore taking into account workability was accepted $s_2=0.025$ (Fig. 6).

Distribution of stress strain state components in the thickness of pipe wall of structure M-H when both layers are deformed elasto plastically is shown in Fig. 7.

The obtained analytical dependencies for stress strain state determination at elastic plastic loading were confirmed by FEM. Difference parameters $\Delta_{\,\xi_{p1\,max}}$ of analyzed two-layer pipe calculated analytically and determined by FEM of structure H-M does not exceed: σ_θ, σ_i – 1.5 %; σ_r, $p_{\xi_{p1}}$ – 3.5 %; e_θ – 2 %; e_i – 1.5 % and in structure M-H – σ_θ – 3 %; σ_r, $p_{\xi_{p1}}$ – 4 %; σ_i – 1 %; e_θ – 7 %; e_i – 1.5 %. When $\xi_{p1\,adm}<0.5\,s_1$, difference of parameters $\Delta_{\xi_{p1}}\approx\Delta_{\xi_{p1\,max}}\,\xi_{p1\,adm}/\xi_{p1\,max}$.

The assumption $\sigma_z=0$ in model analytical solution is made. In FEM model layers of the pipe are connected tight. Stress state, determined by FEM when $\xi_{p1}=0.5\,s_1$, $\xi_{p2}\le s_1$ and $e_{z1}(s_1)=e_{z2}(s_1)$, showed that the mean value $\sigma_{z\,m\,FEM}\approx 0.11\,p_c\left(1+s_1\right)/\left[s_2\left(2+2\,s_1+s_2\right)\right]$ for hard layer of two-layer pipe and $\sigma_{z\,m\,FEM}\approx$ $\approx -0.11\,p_c\left(1+s_1\right)/\left[s_1\left(2+s_1\right)\right]$ for mild layer. Therefore, difference of $e_\theta(0)$ calculated analytically and determined by FEM, caused by stress σ_z, may be up to 8 % (Fig. 5, b). Deforming conditions at the contact surface of pipe layers and value of σ_z are intermediate between analytical and FEM models [5]. Therefore, mean value $\sigma_{z\,m}$, which corresponds the real deforming conditions of two-layer pipe, $\sigma_{z\,m\,tr}\approx 0.5\,\sigma_{z\,m\,FEM}$. Strains of layers at elastic plastic loading, determined by estimating real deforming condition at contact surface of the pipe, approximately may be calculated by these dependences: $e_{\theta\,tr}\approx[\,\sigma_\theta-\nu\,(\sigma_r+\sigma_{z\,m})\,]/E'$, $e_{r\,tr}\approx[\,\sigma_r-\nu\,(\sigma_\theta+\sigma_{z\,m})\,]/E'$, $e_{z\,tr}\approx[\,\sigma_{z\,m}-\nu\,(\sigma_\theta+\sigma_r)\,]/E'$. When $\sigma_i\le\sigma_e$ modulus of elasticity E instead of E' must by used. Difference of e_θ determined analytically by dependencies of this work and true value $e_{\theta\,tr}$ in structure M-H, when $\xi_{p1\,adm}=0.5\,s_1$, does not exceed 4 %. Therefore, accuracy of this solution is fully acceptable.

The more accurate solution of real structure may be obtained by estimating $\sigma_{z\,m}$, when real conditions of deforming on the contact plane surface of materials M and H are evaluated, is very complicated.

5. Conclusions

1. Dependencies for stresses and strains determination in two-layer inhomogeneous pipe subjected to internal pressure at elastic plastic loading are presented in this paper. It is proved that accuracy of these dependencies is quite acceptable.

2. The internal pressure $p_{\xi_{p1}}$ increases with increasing elasto plastically deformed zone ξ_{p1}, relative thickness s_1 and s_2, $E_H\,/\,E_M$, $\sigma_{eH}\,/\,\sigma_{eM}$ and power index m_{01}.

3. The maximum value of elasto plastically deformed zone ξ_{p1} of layer 1 can not exceed $0.5\,s_1$.

4. A method for determination of recommended thickness s_{2r}, when s_1 is known, is proposed. At elastic plastic loading of structure M-H relative thickness s_{2r} may

be determined by approaching method from the condition that $\sigma_{i\,2\,max} = \sigma_{e2}$, when $p_{\xi_{p1}} = p_{\xi_{p1}\,adm}$. In this case elastic properties of layer 2 can be exploited to the maximum when $\sigma_{i1}(s_1)_{|\,C_p=0;\,\xi_{p1\,max}} E_2/E_1 \geq \sigma_{e2}$.

5. Stress $\sigma_{\theta 1}$ in elasto plastically deformed zone ξ_{p1} approximately linearly increases with increasing ξ.

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DVISLUOKSNIO MECHANIŠKAI NEVIENALYČIO TAMPRIAI PLASTIŠKAI DEFORMUOJAMO VAMZDŽIO, VEIKIAMO VIDINIO SLĖGIO, ĮTEMPIŲ IR DEFORMACIJŲ NUSTATYMAS

R e z i u m ė

Darbe nagrinėjamas dvisluoksnio mechaniškai nevienalyčio vamzdžio, veikiamo vidinio slėgio, įtempių deformacijų būvis. Pateiktas matematinis modelis vamzdžio žiediniams σ_θ bei radialiniams σ_r įtempiams ir deformacijoms e_θ, e_r ir e_z apskaičiuoti, kai vidinis vamzdis yra tampriai plastiškai deformuotas. Pasiūlyta rekomenduotino santykinio išorinio sluoksnio storio s_{2r}, leidžiančio geriausiai panaudoti jo tampriąsias savybes, nustatymo metodika. Baigtinių elementų metodu įrodyta, kad darbe gautų priklausomybių dvisluoksnio mechaniškai nevienalyčio, tampriai plastiškai deformuojamo vamzdžio, analitiškai apskaičiuotų įtempių ir deformacijų tikslumas yra visai priimtinas.

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DETERMINATION OF STRESSES AND STRAINS TWO-LAYER MECHANICALLY INHOMOGENEOUS PIPE SUBJECTED TO INTERNAL PRESSURE AT ELASTIC PLASTIC LOADING

S u m m a r y

The stress strain state of two-layer mechanically inhomogeneous pipe subjected to internal pressure at elastic plastic loading is analyzed. A mathematical model for the calculation of circumference stress σ_θ, radial stress σ_r and strains e_θ, e_r and e_z when internal pipe is deformed elasto plastically is presented by the finite element method. A method for determination of the recommended relative thickness s_{2r} of external layer, which insures that elastic properties of external layer exploited to the maximum, is proposed. It is proved that the accuracy of dependencies for stress and strain determination at elastic plastic loading of mechanically inhomogeneous pipe is quite acceptable.

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ОПРЕДЕЛЕНИЕ НАПРЯЖЕНИЙ И ДЕФОРМАЦИЙ МЕХАНИЧЕСКИ НЕОДНОРОДНОЙ ДВУХСЛОЙНОЙ ТРУБЫ, НАГРУЖЕННОЙ ВНУТРЕННИМ ДАВЛЕНИЕМ, ПРИ УПРУГОПЛАСТИЧЕСКОМ ДЕФОРМИРОВАНИИ

Р е з ю м е

В настоящей работе рассмотрено напряженно деформированное состояние двухслойной трубы, нагруженной внутренним давлением, при упруго-пластическом деформировании. Приведена математическая модель определения окружного напряжения σ_θ, радиального напряжения σ_r и деформаций e_θ, e_r и e_z при упругопластическом деформировании внутренней трубы. Предложен метод для определения рекомендуемой относительной толщины наружного слоя s_{2r}, обеспечивающей максимальное использование его упругих свойств. Используя метод конечных элементов доказано, что точность в работе полученных зависимостей для определения напряжений и деформаций механически неоднородной двухслойной трубы при упругопластическом деформировании вполне приемлема.

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