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# Numerical analysis of hydrodynamic journal bearing under transient dynamic conditions

**M. Senthil Kumar*, P.R. Thyla**, E. Anbarasu*****
**PSG College of Technology, Coimbatore 641 004, India, E-mail: msenthil_kumar@hotmail.com*
***PSG College of Technology, Coimbatore 641 004, India, E-mail: thyla_pr@yahoo.co.in*
****PSG College of Technology, Coimbatore 641 00, India, E-mail: anbu_033@yahoo.co.in*

## 1. Introduction

A bearing is a system of machine elements whose function is to support an applied load by reducing friction between the relatively moving surfaces. The hydrodynamic bearing is to develop positive pressure by virtue of relative motion of two surfaces separated by a fluid film. If two mating surfaces during operating conditions are completely separated by lubricant film, such a type of lubrication is called fluid film lubrication. Elliptical bearings have been solved based on the numerical solution of Reynolds equation for finite bearings [1]. Reynolds differential equation has been analyzed for journal bearings having 100 and 75 deg arcs using digital computer [2].

The linear zed Reynolds equation of self-acting bearings, has been investigated the stability of the static equilibrium position of the shaft in gas-lubricated journals [3]. The nonlinear transient analysis of an oil-film journal bearing under different dynamic loads with Reynolds boundary conditions to predict the threshold of stability have been carried out by [4]. Numerical simulation of tooth mobility using nonlinear model of the periodontal ligament has been carried out [5]. The dynamic behavior of relatively short gas film rotor-bearing systems at various values of the rotor mass and bearing number have been characterized [6-9]. Finite difference Method is one of the most widely used technique for solving Reynolds differential equations [10-13]. Also, it has a rapid convergence rate and minimal calculation error. Characteristics of lubrication at nano scale on the performance of transversely rough slider bearing has been studied using Reynold's equation [14].

In Section 2, a mathematical model of steady state behavior of the center of a rigid rotor supported by hydrodynamic journal bearing has been developed. The static oil film pressure on this bearing is obtained by the steady state Reynolds equation. In Section 3, the mathematical model of the time-dependent motion of the rigid rotor supported by oil journal bearing has been developed. The nonlinearity of the oil film pressure significantly complicates the task of solving the time-dependent Reynolds equation. Section 4 presents the simulation results obtained using the proposed method for the pressure distribution, oil film thickness for static and dynamic conditions. Finally, section 5 draws some brief conclusions.

## 2. Mathematical model for steady state condition

In this section a numerical solution of two dimensional Reynolds equations for a finite journal bearing is given.

The governing differential equation for a finite bearing using incompressible lubricant of constant viscosity is given by

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)=6\eta\omega r\frac{dh}{dx} \tag{1}$$

where $p$ is the dimensionless pressure corresponding to the atmospheric pressure; $h$ is the dimensionless gap between the rotating shaft and the bushing, $r$ is radius of the bearing; $\mu$ is oil viscosity; $\omega$ rotational speed.

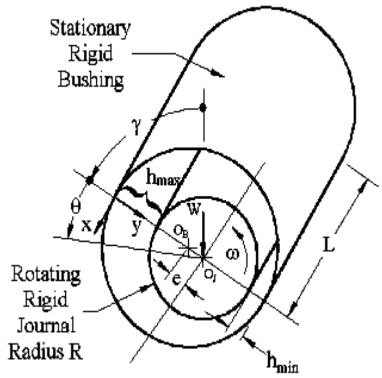


Fig. 1 Journal bearing geometry and coordinate system

Using the nondimensionalization scheme as

$$\theta=\frac{x}{r},\ \bar{z}=\frac{z}{(L/2)},\ \bar{h}=\frac{h}{c},\ \bar{p}=\frac{pc^2}{6\mu\omega r^2}$$

The Eq. (1) results in

$$\frac{\partial}{\partial\theta}\left(\bar{h}^3\frac{\partial\bar{p}}{\partial x}\right)+\left(\frac{D}{L}\right)^2\bar{h}^3\frac{\partial^2\bar{p}}{\partial\bar{z}^2}=\frac{d\bar{h}}{d\theta} \tag{2}$$

here $D$ is the diameter of the journal (=2$r$) and $\bar{h}$ is assumed to be only a function of $\theta$, i.e., $\bar{h}=1+\varepsilon\cos\theta$.

Equation (2) can be expressed as

$$\frac{\partial^2\bar{p}}{\partial\theta^2}+\left(\frac{D}{L}\right)^2\bar{h}^{-3}\frac{\partial^2\bar{p}}{\partial\bar{z}^2}+\frac{3}{\bar{h}}\frac{\partial\bar{p}}{\partial\theta}\frac{d\bar{h}}{d\theta}=\frac{d\bar{h}}{d\theta}\Big/\bar{h}^{3} \tag{3}$$

A developed view of the bearing is shown in Fig. 2. The area is divided into a number of mesh sizes ($\Delta\theta\ x\ \Delta z$) and using central difference quotients

$$\frac{p_{i+1,j}-2p_{ij}+p_{i-1,j}}{(\Delta\theta)^2}+\left(\frac{D}{L}\right)^2\left[\frac{p_{i,j+1}-2p_{i,j-1}+p_{i,j-1}}{\left(\Delta\bar{z}\right)^2}\right]-3\frac{\xi\, sin\theta_i}{\bar{h}_i}\left(\frac{p_{i+1,-j}-p_{i-1,j}}{2\Delta\theta}\right)=-\frac{\xi\, sin\theta_i}{\bar{h}_i^{3}} \tag{4}$$

where $P_{i,j}$ is the pressure at any mesh point $(i,j)$; $h_i$ is the film thickness at any point $(i,j)$; $P_{i+1,j}$, $P_{i-1,j}$, $P_{i,j+1,}$ and $P_{i,j-1}$ are pressures at the four adjacent points; $\theta_i=2(\Delta\theta)i/D$, $(i,j)$ is the numerical coordinate system.

Simplifying Eq. (4) for $P_{i,j}$ gives

$$P_{i,j}=\left[\begin{array}{l}\left(P_{i+1,j}+P_{i-1,j}\right)+\left(\frac{D}{L}\right)^2\left(\frac{\Delta\theta}{\Delta\bar{z}}\right)^2\left(P_{i,j+1}+P_{i,j-1}\right)-\\ -\frac{3}{2}\frac{\varepsilon\left(P_{i+1,j}+P_{i-1,j}\right)}{\bar{h}_i}\left(\Delta\theta\right)sin\theta_i+\varepsilon\frac{sin\theta_i}{\bar{h}_i^{3}}\left(\Delta\theta\right)^2\end{array}\right]\Bigg/\left[2\left\{1+\left(D/L\right)^2\left(\frac{\Delta\theta}{\Delta\bar{z}}\right)^2\right\}\right] \tag{5}$$

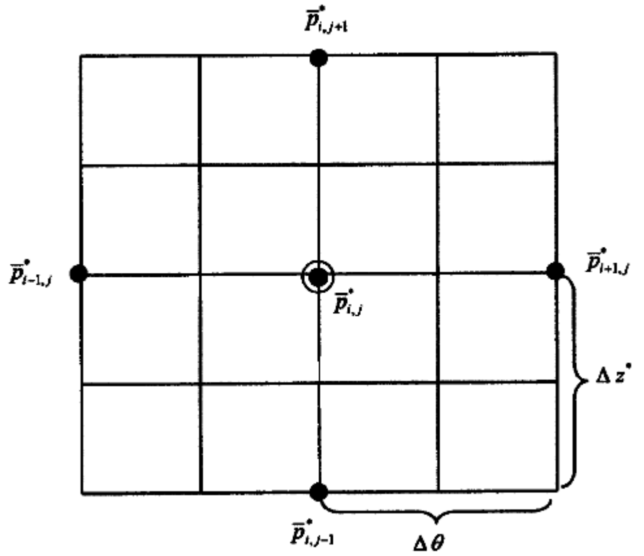


Fig. 2 Grid point notation

For this problem, a grid of about 60 points has been picked and the equation has been solved by using Matlab program. Image points were assumed to insure zero boundary conditions.

### 3. Mathematical model for dynamic conditions

Pressure distribution in the oil film between the shaft and the bushing is described by the Reynolds equation

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)=6\eta\omega r\frac{dh}{dx}+\sigma\frac{\partial h}{\partial\tau} \tag{6}$$

and

$$\sigma=\frac{12\mu\upsilon}{Pa}\left(\frac{R}{C}\right)^2 \tag{7}$$

where $C$ is radial clearance.

$$\left[3h^2\left(\frac{h_{i+1,j}-h_{i-1,j}}{2\Delta\theta}\right)\times\left(\frac{D}{L}\right)^2\left(\frac{p_{i+1,j}-p_{i-1,j}}{2\Delta\theta}\right)+\left(\frac{p_{i+1,j}-2p_{i,j}+p_{i-1,j}}{\Delta\theta^2}\right)\right]+h^3\left(\frac{p_{i+1,j}-2p_{i,j}+p_{i-1,j}}{\Delta z^2}\right)=$$
$$=6\eta\omega r\left(\frac{h_{i+1,j}-h_{i-1,j}}{2\Delta\theta}\right)+\sigma\left(\frac{h^{n+1}_{i,j}-h^{n}_{i,j}}{\Delta t}\right) \tag{8}$$

Simplifying Eq. (4) for $h_{i,j}$ gives

$$h^{n+1}_{i,j}=\frac{\Delta t}{\sigma}\left[\begin{array}{l}\left[\left(h^3\left(\frac{p_{i+1,j}-2p_{i,j}+p_{i-1,j}}{\Delta\theta^2}\right)+\left(\frac{D}{L}\right)^2\left(\frac{p_{i+1,j}-2p_{i,j}+p_{i-1,j}}{\Delta z^2}\right)\right)+\right.\\ \left.+\left[\left(\frac{h_{i+1,j}-h_{i-1,j}}{2\Delta\theta}\right)\times\left(3h^2\left(\frac{p_{i+1,j}-p_{i-1,j}}{2\Delta\theta}\right)-6\eta\omega r\right)\right]\right.\end{array}\right]+h_{i,j} \tag{9}$$

For this problem, a grid of about 60 points has been picked and the equation has been solved by using Matlab program. For the first time step, the boundary conditions for film thickness $h$ assumed to be C/2 and pressure ($p$) values are initialized to get the film thickness. The first time step film thickness $h$ values are substituted in Eq. (5). The new pressure distribution has been obtained for first time step. The process has been repeated to different time steps to get dynamic pressure distribution, and film thickness with respect to time.

### 3. Results and discussions

Investigations on the transient dynamic behavior of an oil lubricated journal bearing have been carried out by employing the aforesaid methodology. The results obtained for a bearing with the following parameters are presented here: journal diameter $D = 100$ mm; journal length $L$ = 100 mm; length and diameter ratio $L/D$ = 1.0; radial clearance $C$ = 0.025 mm; journal speed $n$ = = 3000 rpm; eccentricity $\varepsilon$ =0.6 mm; viscosity of lubricant $\mu$ = 0.02 Pa.s.

The transient variation of oil film thickness and oil pressure are studied. Figs. 4-9 show the circumferential variation of pressure at different instants of time viz., at the 1st, 2nd, 3rd, 4th, 20th, 25th revolution from the start. Figs. 10-15, show the corresponding film thickness variation.

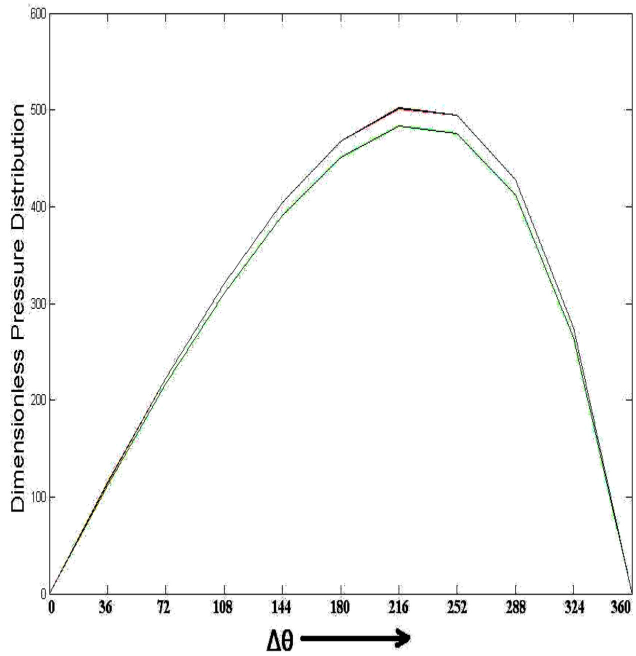


Fig. 4 Dimensionless pressure distribution with 1 revolution

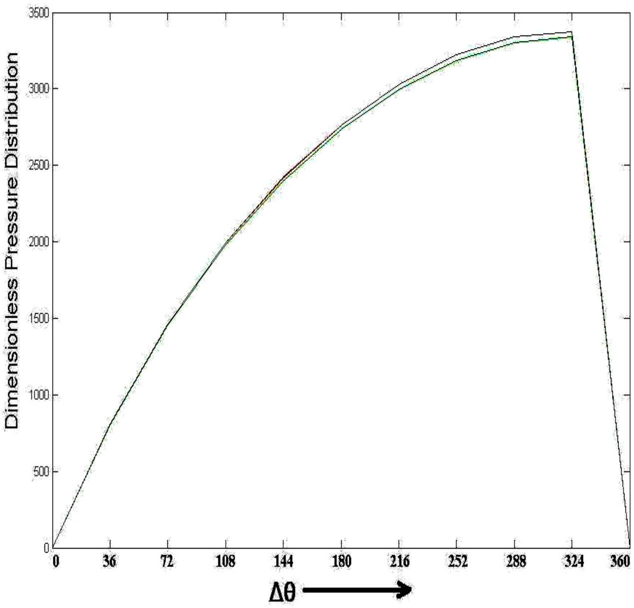


Fig. 5 Dimensionless pressure distribution with 2 revolutions

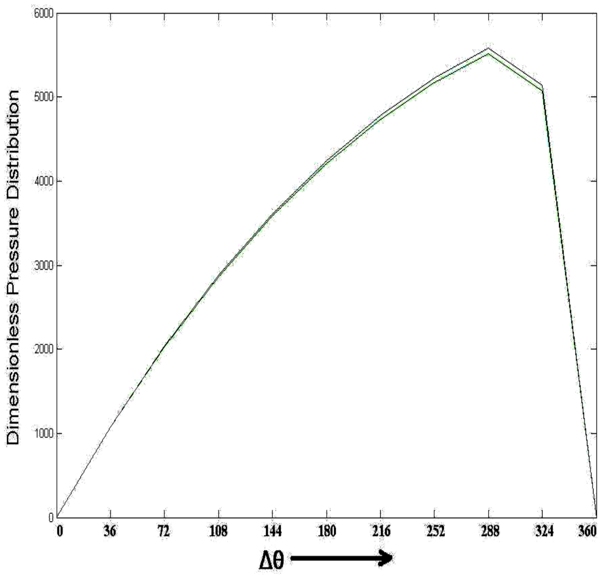


Fig. 6 Dimensionless pressure distribution with 3 revolutions

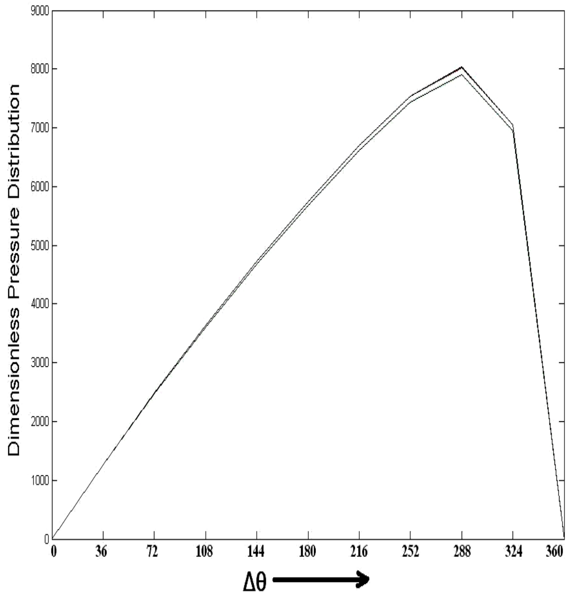


Fig. 7 Dimensionless pressure distribution with 4 revolutions

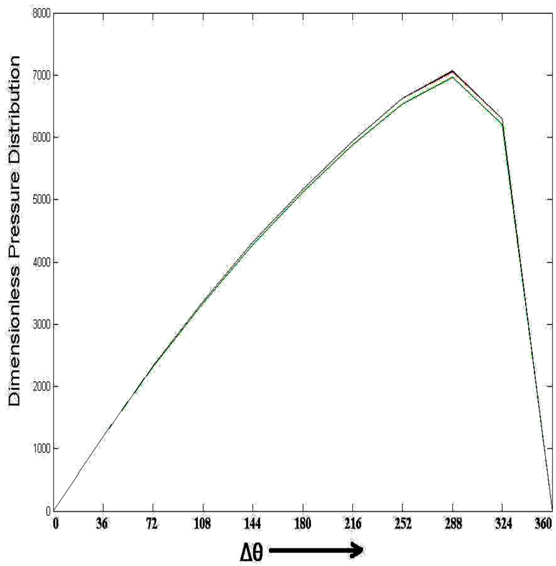


Fig. 8 Dimensionless pressure distribution with 20 revolutions

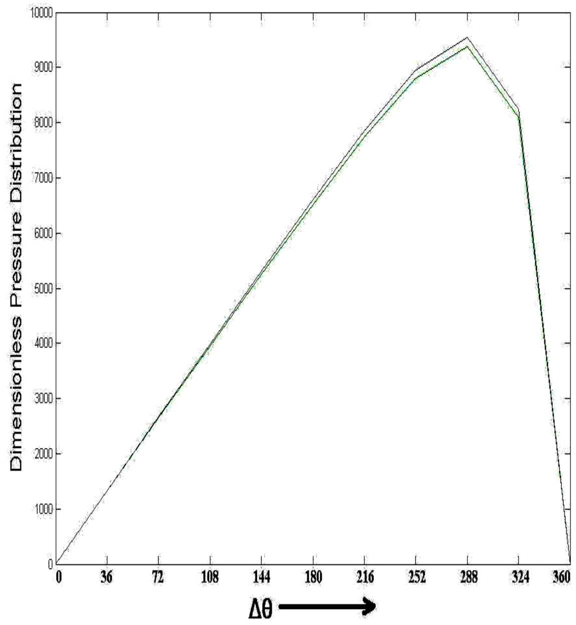


Fig. 9 Dimensionless pressure distribution with 25 revolutions

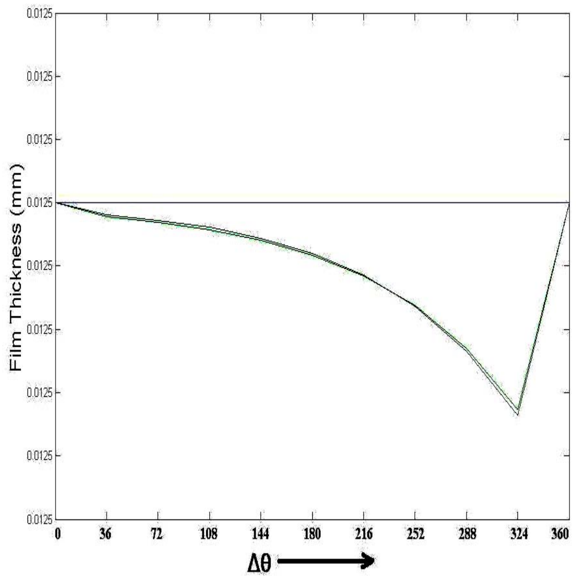


Fig. 10 Film thickness with respect to 1 rpm

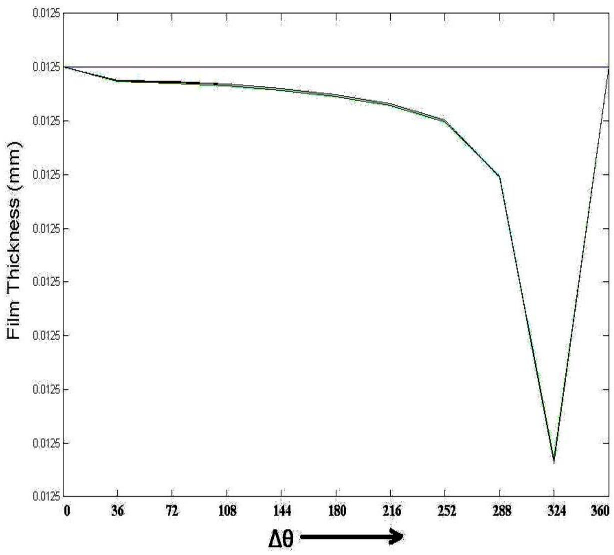


Fig. 11 Film thickness with respect to 2 rpm

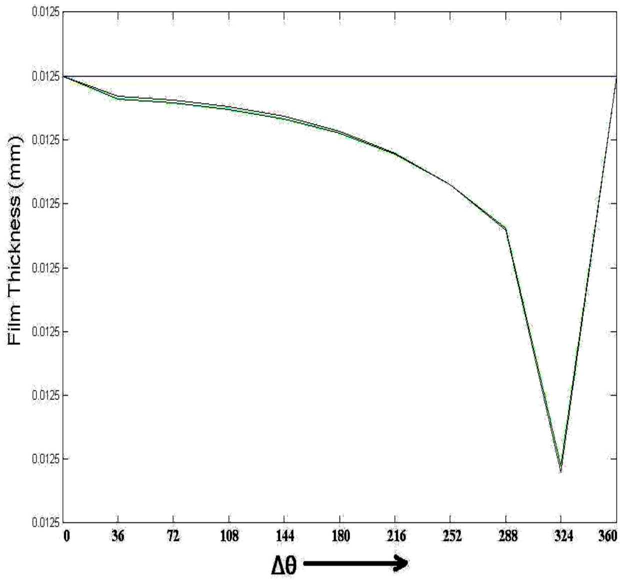


Fig. 12 Film thickness with respect to 3 rpm

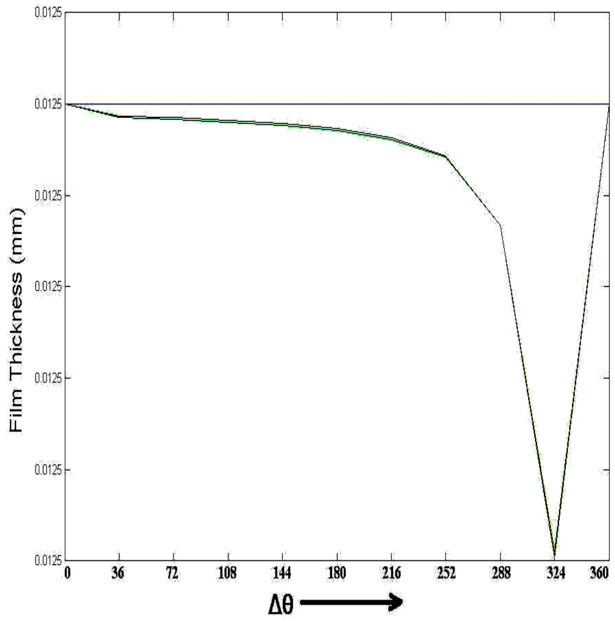


Fig. 13 Film thickness with respect to 4 rpm

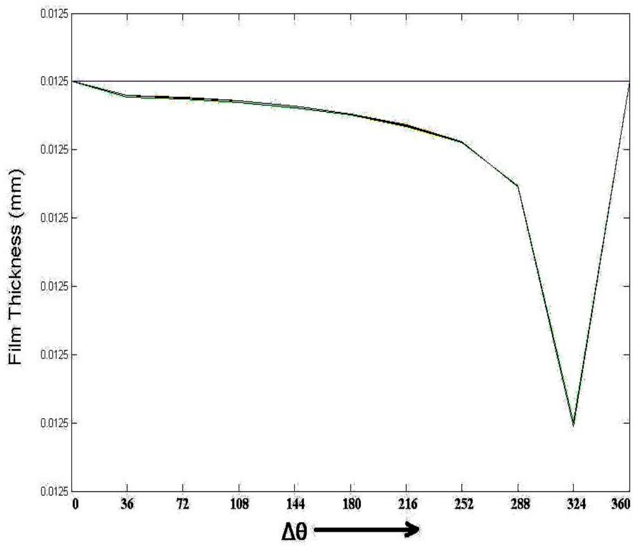


Fig. 14 Film thickness with respect to 20 rpm

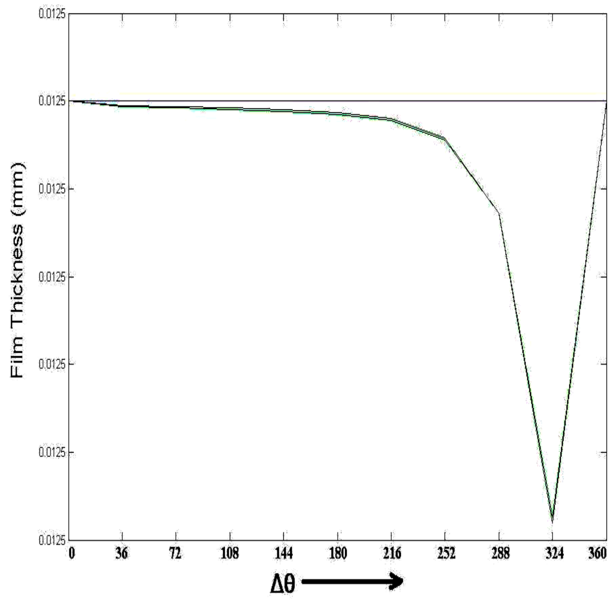


Fig. 15 Film thickness with respect to 25 rpm

It is observed that the maximum dimensionless pressure increases from 502.477 to 9708.9 during the first 20 revolutions from the start. The pressure distribution is found to become steady in about 20 revolutions from the start. Fig 16 shows that the pressure reaches the steady state in about 20 revolutions. The steady state pressure distribution in the circumferential direction is shown in Fig. 17. The maximum dimensionless pressure is found to be 502.447.

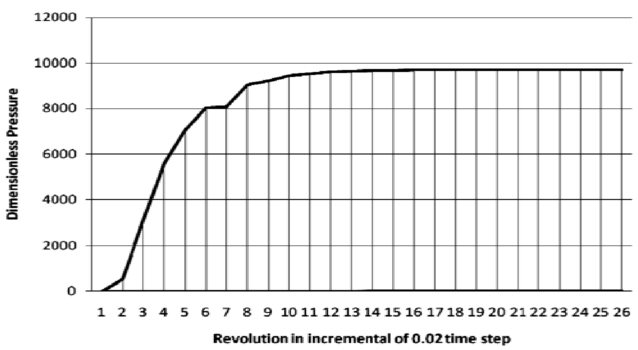


Fig. 16 Transient dynamic variations of dimensionless pressure

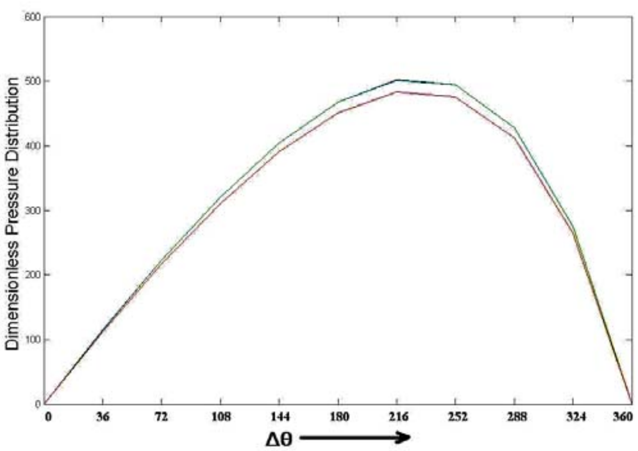


Fig. 17 Steady state pressure distribution

## 5. Conclusions

The steady state and transient dynamic behavior of hydrodynamic journal bearing system have been studied and presented in this paper. The steady state and transient dynamic behavior have been analyzed for different time steps at a particular speed. The result reveals that steady state is achieved within 20 revolutions corresponding to a time 0.4 sec from the start. It can be further extended to predict the dynamic behavior of the journal bearing under varying load and speed conditions.

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M. Senthil Kumar, P.R. Thyla, E. Anbarasu

## HIDRODINAMINIŲ GUOLIŲ SKAITINIS TYRIMAS PEREINAMAJAME DINAMINIAME REŽIME

R e z i u m ė

Straipsnyje pateiktas standaus rotoriaus hidrodinaminiuose guoliuose stabilumo matematinis modeliavimas. Tepalo plėvelės slėgis guolyje nustatomas sprendžiant Reynoldso lygtį. Sudarytas rotoriaus slydimo guoliuose dinamikos matematinis modelis. Tepalo plėvelės slėgio netiesiškumas gerokai apsunkina spręsti Reynoldso lygtį. Modeliavimo rezultatai leido nustatyti tepalo plėvelės storį ir slėgio pasiskirstymą, esant statinei ir dinaminei apkrovai.

M. Senthil Kumar, P.R. Thyla, E. Anbarasu

## NUMERICAL ANALYSIS OF HYDRODYNAMIC JOURNAL BEARING UNDER TRANSIENT DYNAMIC CONDITIONS

S u m m a r y

This paper deals with mathematical modeling of steady state behavior of the center of a rigid rotor supported by hydrodynamic journal bearing. The static oil film pressure on the bearing is obtained by the steady state Reynolds equation. The mathematical model of the time-dependent motion of the rigid rotor supported by oil journal bearing has been developed. The nonlinearity of the oil film pressure significantly complicates the task of solving the time-dependent Reynolds equation. Also, the simulation results obtained using the proposed method for the pressure distribution, oil film thickness for static and dynamic conditions have been presented. Some brief conclusions drawn are also given.

М.Сентгил Кумар, П.Р. Тгила, Е. Анбарасу

## ЧИСЛЕННОЕ ИССЛЕДОВАНИЕ ПЕРЕХОДНОГО ДИНАМИЧЕСКОГО РЕЖИМА В ГИДРОДИНАМИЧЕСКИХ ПОДШИПНИКАХ

Р е з ю м е

В статье представлено математическое моделирование стабильности жесткого ротора на гидродинамических подшипниках. Давление смазочной пленки в подшипнике определено при решении уравнения Рейнольдса. Создана математическая модель динамики ротора на подшипниках скольжения. Нелинейность давления смазочной пленки существенно осложняет решение уравнения Рейнольдса. Представленные результаты моделирования позволяют определить толщину смазочной пленки и распределение давления при статической и динамической нагрузке.

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