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Control of physical surface by detecting coordinates

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1. Introduction

There are various methods for determination and control of physical surfaces (products) parameters [1-8]. The method of surface scanning by using laser technologies is easy to realize from organizational point of view but more complex from technical side. Such method of surface scanning is accurate. Standard deviation estimate reaches of point position few μm [2, 8]. This method can't be used for determining points on complex surfaces, which are not seen for scanning ray. Therefore other methods should be used when due to the complex physical surfaces scanning ray can't "see" some surface points. A method for determination of physical surface points 3D coordinates by charge coupled device (CCD) camera with focal lens is presented here. Proper scale is provided for the elements of linear converter of photodiodes during CCD camera calibration. Point coordinates are determined by the method of linear intersection in a free coordinate system by using two CCD cameras. Necessary distances between CCD detector and the point in question are determined from adequate elements measured in linear detector by using derived formulas. The size of examined physical surfaces (products) could be up to some tens of meters. Accuracy of suggested method is analysed.

2. Theoretical background

Linear intersection method could be used for the determination of physical surface coordinates. Distances from starting base points (Figs. 1 and 2) with known coordinates to the points in question should be measured. The number of minimal starting points is two. Coordinates can be measured in any system, for example free system. Plane coordinates of a single point i (x_i, y_i) are determined by linear intersection method, which is based here on measuring slope distances S_{1i} and S_{2i} (Fig. 1).

Projections of distances to the horizontal plane (or

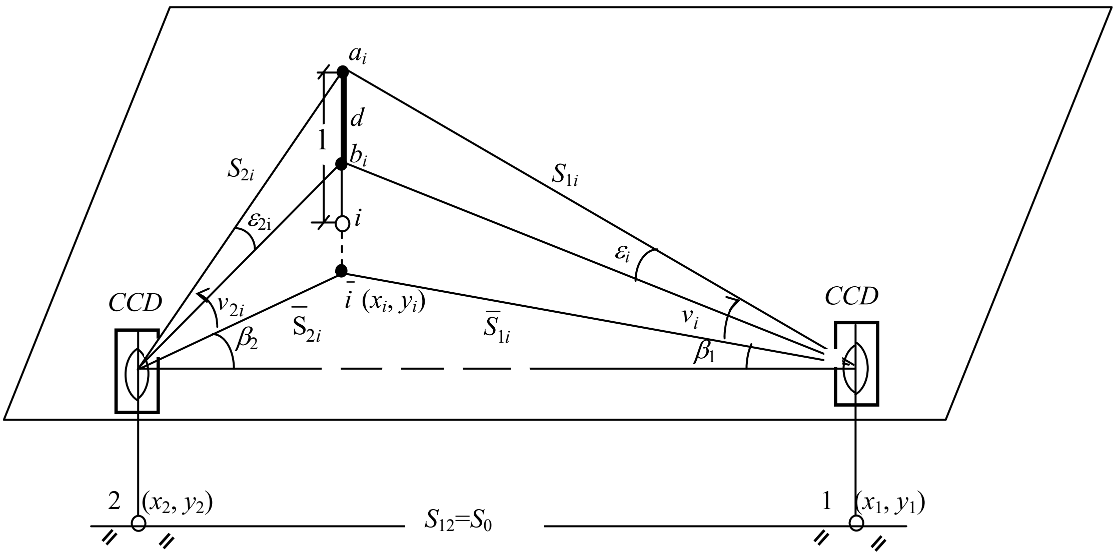

Fig. 1 Scheme of coordinate determination by linear intersection method: $\bar{i}$ - projection at point i into the horizontal plane

any other) $\bar{S}_{1i}$ and $\bar{S}_{2i}$ are determined form observed angles ν_i. CCD cameras are set by the help of electronic level at the starting points 1 and 2. Their coordinate system is fixed by calibrated distance $S_0 = \bar{S}_{12}$ from selected freely whole circle bearing α_{12}, e.g. $\alpha_{12} = 0°$. Deviation $\bar{h}_i$ of the point i is obtained by height from the horizontal plane by using computed angle ν_i. The value of this angle is determined from CCD detector reading. This is shown below. Measuring principle of distances S_{1i} and S_{2i} as well as computation of points coordinates are shown in Fig. 2 and presented ин formulas.

CCD cameras are set at starting points 1 and 2 while length of the cylindrical ruler (with electronic level for verticality control) with two stroke calibrated distance between them d (e.g. 100 mm) at the point i is fixed. Starting reading of CCD detector a_0 is accepted as equal to zero when collimation line is horizontal or parallel to the set plane. Based on Fig. 2 one can write an equation for measured distance S_{12} between the points 1 and 2

$$S_{12} = S'_{12}\frac{d}{d'} = \frac{dS'_0}{d'cos\,\nu_2} = \frac{k}{d'cos\,\nu_2} \tag{1}$$

where $k = dS'_0$ is CCD constant, S'_0 is distance between lens centre and linear detector, d' is projection of the calibrated segment d into CCD from the point 2, ε_2 is viewing angle of the segment d from CCD at point 2, ν_2 is viewing angle of point a_2 of deflection of the segmnent d from horizontal plane, S'_{12} is projection of the slope line S_{12} in the CCD camera.

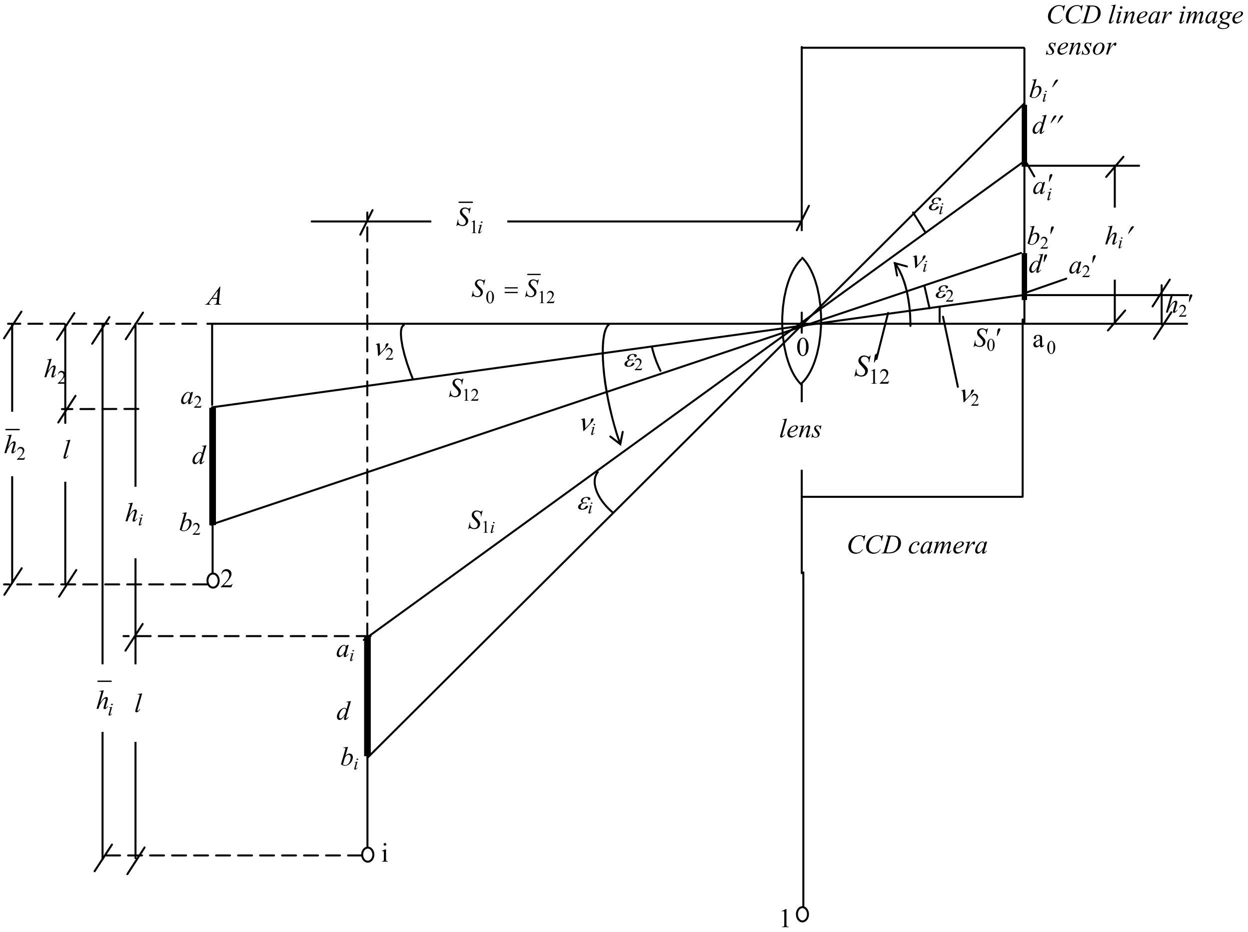

Fig. 2 Diagram of distance measurement applying CCD camera

The projection $\overline{S}_{12}$ of distance S_{12} to horizontal plane, when Eq. (1)

$$\overline{S}_{12} = S_{12}\cos\nu_2 = k\frac{1}{d'} \tag{2}$$

Analogically we get the inclined distance S_{1i}

$$S_{1i} = S'_{1i}\frac{d}{d'} = \frac{dS'_0}{d''\cos\nu_i} = \frac{k}{d''\cos\nu_i} \tag{3}$$

where d'' is projection of the segment d from point i into the CCD detector, ν_i is viewing angle of point a_i of the segment d deflection from horizontal plane; ε_i is viewing angle of the segment d from CCD at the point i.

Projection of the distance S_{1i} to horizontal plane $\overline{S}_{1i}$, when Eq. (3) is taken into account

$$\overline{S}_{1i} = S_{1i}\cos\nu_i = k\frac{1}{d''} \tag{4}$$

Values of angles ν_2 and ν_i are computed from the formula

$$\nu_2 = arctan\frac{h'_2}{S'_0} \tag{5}$$

$$\nu_i = arctan\frac{h'_i}{S'_0} \tag{6}$$

where $h'_2 = a'_2 - a_0$, $h'_i = a'_i - a_0$ is segments of CCD detector.

Single deflection of point i from the horizontal plane $\overline{h}_i$ is (Fig. 2)

$$\overline{h}_i = h_i + l \tag{7}$$

where h_i is deflection of the cylindrical ruler point a_i from horizontal plane, l is known ruler length.

Value h_i is determined from known angle ν_i by applying the formula

$$h_i = S_{1i}\sin\nu_i = \overline{S}_{1i}tg\nu_i \tag{8}$$

By using this measurement method with known CCD detector constant k value and projection of the calibrated segment d to CCD d' and d'' values, we can compute distances between CCD and projections of single points to the horizontal plane $\bar{S}_{1i}$, $\bar{S}_{2i}$ and further coordinates of these points.

Coordinates of point i are computed from the formula

$$x_i = x_1 + \bar{S}_{1i} \cos\alpha_{1i} = x_2 + \bar{S}_{2i} \cos\alpha_{2i} \quad (9)$$

$$y_i = y_i + \bar{S}_{1i} \sin\alpha_{1i} = y_2 + \bar{S}_{2i} \sin\alpha_{2i} \quad (10)$$

Whole circle bearing angles of lines S_{1i} and S_{2i} are computed

$$\alpha_{1i} = \alpha_{12} + \beta_1 \quad (11)$$

$$\alpha_{2i} = \alpha_{12} \pm 180° - \beta_2 \quad (12)$$

Angle α_{12} of baseline S_{12} whole circle and the values of angles β_1 and β_2 are computed from the formulas

$$\alpha_{12} = \arctan\frac{y_2 - y_1}{x_2 - x_1} \quad (13)$$

$$\beta_1 = \arccos\left\{\frac{1}{2\bar{S}_{12}\bar{S}_{1i}}\left(\bar{S}_{12}^2 + \bar{S}_{1i}^2 - \bar{S}_{2i}^2\right)\right\} = \arccos A \quad (14)$$

$$\beta_2 = \arccos\left\{\frac{1}{2\bar{S}_{12}\bar{S}_{2i}}\left(\bar{S}_{12}^2 + \bar{S}_{2i}^2 - \bar{S}_{1i}^2\right)\right\} = \arccos B \quad (15)$$

where A and B are functional values in the braces, $\bar{S}_{12}$ is projection of line S_{12} in the horizontal plane.

Computations can be performed in a free coordinate system by introducing, e.g. $\alpha_{12} = 0°$ and $x_1 = y_1 = 0$. Then $x_2 = \bar{S}_{12} \cos 0° = \bar{S}_{12}$, $y_2 = \bar{S}_{12} \sin 0° = 0$.

Further we can write: $\alpha_{1i} = 360° + \beta_1$; $\alpha_{2i} = 180° - \beta_2$; $x_3 = \bar{S}_{1i} \cos\alpha_{1i} = \bar{S}_{12} + \bar{S}_{2i} \cos\alpha_{2i}$; $y_i = \bar{S}_{1i} \sin\alpha_{1i} = \bar{S}_{2i} \sin\alpha_{2i}$.

3. Coordinate accuracy estimation

Distance between CCD camera 1 and projection $\bar{S}_{1i}$ of point i is computed from the Eq. (4)

$$\bar{S}_{1i} = k\frac{1}{d''}$$

Standard deviation of this distance by applying mathematical statistic laws is computed from the formula

$$\sigma_{\bar{S}}^2 = k^2\frac{1}{d''^4}\sigma_{d''}^2 + \frac{1}{d''^2}\sigma_k^2 \quad (16)$$

where $\sigma_{\bar{S}}, \sigma_{d''}, \sigma_k$ are standard deviations of adequate quantity (variable).

Accuracy of CCD camera constant $k = dS'_0$ is estimated by standard deviation σ_k from the formula [6]

$$\sigma_k^2 = d^2\sigma_{S'_0}^2 + S_0'^2\sigma_d^2 \quad (17)$$

We will analyse estimate of standard deviations for measured distance, when parameters of CCD camera and cylindrical ruler have the following values: $S'_0 = 100$ mm, $d'' = 10$ mm, $d = 100$ mm, $\sigma_{S'_0} = \sigma_d = \sigma_{d''} = 1\,\mu$m, $\bar{S}_{1i} = 1000$ mm, $k = 10^4$ mm^2.

From Eq. (17) we get: $\sigma_k^2 = 2 \cdot 10^{-2}$ and $\sigma_k = 0.14$ mm^2.

Further on from Eq. (16) we compute: $\sigma_{\bar{S}_{1i}}^2 \approx 10^{-2}\,\text{mm}^2$ and $\sigma_{\bar{S}_{1i}} \approx 0.1$ mm.

Standard deviations the coordinates of single point i are determined by taking into account Eqs. (9) and (10)

$$\sigma_{x_i}^2 = \sigma_{x_1}^2 + \bar{S}_{1i}^2 \sin^2\alpha_{1i}\sigma_{\alpha_{1i}}^2 + \cos^2\alpha_{1i}\sigma_{\bar{S}_{1i}}^2 \quad (18)$$

$$\sigma_{y_i}^2 = \sigma_{y_1}^2 + \bar{S}_{1i}^2 \cos^2\alpha_{1i}\sigma_{\alpha_{1i}}^2 + \sin^2\alpha_{1i}\sigma_{\bar{S}_{1i}}^2 \quad (19)$$

Standard deviation $\sigma_{\alpha_{1i}}$ of line S_{1i} whole circle bearing α_{1i} is computed from the formula

$$\sigma_{\alpha_{1i}}^2 = \sigma_{\alpha_{12}}^2 + \sigma_{\beta_1}^2 \quad (20)$$

Standard deviation $\sigma_{\alpha_{12}}$ of starting line S_{12} whole circle bearing α_{12} expression, from Eq. (13) is equal (when $x_1 = y_1 = 0$)

$$\sigma_{\alpha_{12}}^2 = \left(1 + \frac{y_2^2}{x_2^2}\right)^{-2}\sigma^2\left(\frac{y_2}{x_2}\right) = \left(1 + \frac{y_2^2}{x_2^2}\right)^{-2}\left(y_2^2 x_2^{-4}\sigma_{x_2}^2 + x_2^{-2}\sigma_{y_2}^2\right) \quad (21)$$

By accepting whole circle bearing $\alpha_{12} = 0°$, for line S_{12} we get $\sigma_{\alpha_{12}} = 0$.

Expressions of standard deviations of σ_{β_1} and σ_{β_2} of angles β_1 and β_2 are analogical

$$\sigma_{\beta_1}^2 = \left(1 - A^2\right)^{-1}\sigma_A^2 = \frac{1}{4}\left(1 - A^2\right)^{-1} \times \left\{\left(\bar{S}_{1i}^{-1} - \bar{S}_{1i}\,\bar{S}_{12}^{-2} + \bar{S}_{12}^{-2}\,\bar{S}_{1i}^{-1}\,\bar{S}_{2i}^2\right)^2 \times \sigma_{\bar{S}_{12}}^2 + \left(-\bar{S}_{12}\,\bar{S}_{1i}^{-2} + \bar{S}_{12}^{-1} + \bar{S}_{12}^{-1}\,\bar{S}_{1i}^{-2}\,\bar{S}_{2i}^2\right)^2\sigma_{\bar{S}_{1i}}^2 + \left(2\bar{S}_{12}^{-1}\,\bar{S}_{1i}^{-1}\,\bar{S}_{2i}\right)^2\sigma_{\bar{S}_{2i}}^2\right\} \quad (22)$$

where $A=\frac{1}{2\bar{S}_{12}\bar{S}_{1i}}\left(\bar{S}_{12}^2+\bar{S}_{1i}^2-\bar{S}_{2i}^2\right)$.

If $\bar{S}_{12}\approx\bar{S}_{1i}\approx\bar{S}_{2i}\approx S$ and $\sigma_{S_{12}}\approx\sigma_{S_{1i}}\approx\sigma_{S_{2i}}=\sigma_S$ we get $\sigma_{\beta_1}^2=2S^{-2}\sigma_S^2$ and

$$\sigma_{\beta_1}=\frac{\sqrt{2}\sigma_S}{S} \tag{23}$$

Analogically we get

$$\sigma_{\beta_2}=\frac{\sqrt{2}\sigma_S}{S}$$

By using Eqs. (14) - (17), (19) and mean values: $S_{1i}=1000\,\text{mm}$, $\alpha_{1i}=45°$, $\sigma_S=0.1$ mm we get $\sigma_{x_i}=0.12$ mm, $\sigma_{y_i}=0.12$ mm.

Accuracy of deviation of the determined point i from horizontal plane $\bar{h}_i$ Eq. (7) can be assessed as

$$\sigma_{\bar{h}_i}^2=\sigma_{h_i}^2+\sigma_l^2 \tag{24}$$

Standard deviation of value h_i Eq. (8) is equal

$$\sigma_{h_i}^2=\bar{S}_{1i}^2\frac{1}{cos^4 v_i}\sigma_{v_i}^2+\sigma_{\bar{S}_{1i}}^2 tan^2 v_i \tag{25}$$

As accuracy of v_i is evaluated using Eq. (6)

$$\sigma_{v_i}^2=\frac{1}{\left(1+z^2\right)^2}\left\{\left(\frac{\partial z}{\partial h_i'}\right)^2\sigma_{h_i'}^2+\left(\frac{\partial z}{\partial S_0'}\right)^2\sigma_{S_0'}^2\right\}=$$

$$=\frac{1}{\left(1+z^2\right)^2}\left\{\frac{1}{S_0'^2}\sigma_{h_i'}^2+\left(\frac{h_i'}{S_0'^2}\right)^2\sigma_{S_0'}^2\right\} \tag{26}$$

here $z=h_i'/S_0'$.

Using the values of parameters of the CCD camera and cylindrical ruler from the earlier analysed example, we receive such evaluates of standard deviation, when $v_i=45°$, $\sigma_l=0.01\,\text{mm}$: $\sigma_{v_i}\approx 0.7\cdot 10^{-5}$ rad., $\sigma_{h_i}\approx$ ≈ 0.1 mm, $\sigma_{\bar{h}_i}\approx 0.1$ mm.

These results of computations point out, that accuracy of the computed coordinates is almost equal to the accuracy of measured line lengths accuracy.

4. Conclusions

1. A method of linear intersections for determinations of significance of parameters of physical surfaces in remote sensing, by employment spatial coordinates of surface points in a free coordinate system is presented. Spatial coordinates of points are established by baseline of two CCD cameras (matrix of photodiodes) with focal lens by the application of adequate elements measured in CCD linear detector.

2. For estimate of accuracy of the suggested method formulas to compute standard deviation of spatial coordinate of physical surfaces are derived. The accuracy of computed coordinates of points is almost equal to the accuracy of measured line lengths accuracy by the suggested method.

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KOORDINATINIS FIZINIŲ PAVIRŠIŲ PARAMETRŲ KONTROLĖS METODAS

R e z i u m ė

Pateikiamas teorinis ir praktinis fizinių paviršių (gaminių) parametrų reikšmių nustatymo ir kontrolės principas. Šis metodas įgyvendinamas naudojant CCD (charge coupled device) kamerą (fotodiodų matricą) su fokusavimo lęšiu paviršiaus taškų erdvinėms koordinatėms nustatyti. Taškų koordinatės nustatomos linijinės sankirtos laisvoje koordinačių sistemoje metodu, panaudojant dviejų CCD kamerų bazinę liniją. Atstumas tarp bazinės CCD kameros ir nustatomo taško gaunamas pagal CCD detektoriuje išmatuotus atitinkamus elementus. Tikrinamųjų fizinių paviršių (gaminių) matmenys gali siekti kelias dešimtis metrų. Pateikiamos formulės nustatomų taškų koordinačių tikslumui įvertinti.

J. Skeivalas, D. Čygas

CONTROL OF PHYSICAL SURFACE BY DETECTING COORDINATES

S u m m a r y

Theoretical and practical principles of the determination of physical surfaces (of products) parameters and their control are presented. This method is realizable in the determination of spatial coordinates of surface points when CCD (coupled charge device) camera (matrix of photodiodes) with focal lens is used. The method of linear intersections is used for recording point's coordinates in a free coordinate system, when a baseline of two CCD cameras is established. Distance between the basic CCD camera and the point in question is obtained by adequate elements measured in CCD detector. Size of the examined physical surfaces (of products) could be up to some tens of meters. Formulas presented could be used for accuracy determination of coordinates in question points.

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КООРДИНАТНЫЙ МЕТОД КОНТРОЛЯ ПАРАМЕТРОВ ФИЗИЧЕСКИХ ПОВЕРХНОСТЕЙ

Р е з ю м е

Предложен теоретический и практический принцип для определения и контроля значений параметров физических поверхностей. Настоящий метод реализуется путем определения пространственных координат точек физических поверхностей при использовании камеры CCD (матрица фотодиодов) с линзой фокусировки. Для определения координат точек использован метод линейной засечки в свободной системе координат при применении базовой линии между двумя камерами CCD. Расстояние между базовой камерой CCD и фиксируемой точкой определяется по измеренным элементам в линейном детекторе CCD. Проверяемые физические поверхности могут иметь размеры в несколько десятков метров. Предложены формулы для оценки точности определяемых координат точек.

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