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Strength and fracture criteria application in stress concentrators areas

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1. Introduction

Strength and fracture parameters are very important in fracture mechanics. Hereupon when at crack tip exists stress concentration complex stress state is obtained. Here is not enough to determine highest stresses while strength and fracture are governed by equivalent effect. This effect also depends on material properties: elasticity and plasticity. In various studies [1-2] mostly brittle fracture is researched and plastic fracture studies are much more rare [3]. Fracture depend on crack size as well [4, 5]. Considering defect like round hole strength can be computed according theory of elasticity [6] and especially for fracture of brittle materials Griffiths criteria shall be applied [4]. But using this criteria it is complicated to determine crack growth and its area size. This task requires additional studies.

2. Brittle fracture criteria

Griffith's energy growth criteria

$$G = -\frac{\Delta\Pi}{\Delta S} > G_C \tag{1}$$

where $\Delta\Pi$ is system potential energy variation on increased crack surface ΔS; G_C is critical energy value.

However applying this criteria becomes hard to determine increase value of ΔS. Griffiths energy growth criteria is easier to calculate applying stress intensity coefficient K_I. Then

$$G = \frac{K_I^2}{E'} \tag{2}$$

and

$$E' = \begin{cases} E & - \textit{plane stress} \\ \dfrac{E}{1-\nu^2} & - \textit{plane strain} \end{cases}$$

where E is modulus of elasticity; υ is Poisons' ratio, K_I is stress intensity factor calculated on opening case (I moda).

Critical energy G_C value calculated by the equation

$$G_C = \frac{K_{IC}^2}{E'} \tag{3}$$

where K_{IC} is critical stress intensity factor.

Critical stress intensity factor K_{IC} is obtained by the equation

$$K_{IC} = \sigma_\infty \sqrt{2\pi l_c} F_0 \tag{4}$$

where σ_∞ is distant stresses, $2l_c$ is critical crack length, F is corrective function, taking into account crack and element geometry shown in Fig. 1.

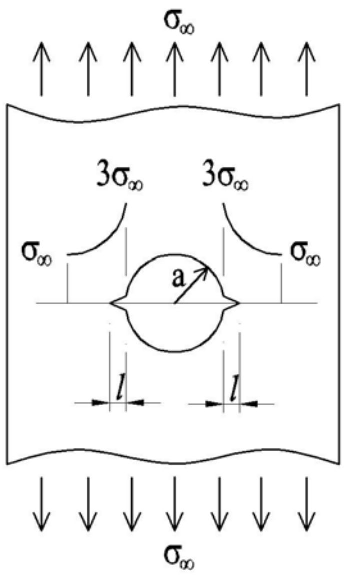

Fig. 1 Two opposite short cracks radiating from a circular hole in an infinite plate under tension

For a plate with a far field uniform stress σ_∞ we know that there is a stress concentration factor of 3 [6]. For a crack radiating this hole we consider two cases. In one the crack is short $l/2a \rightarrow 0$ and thus we have an approximate for field stress of 3σ and for an edge crack F_1 = 1.12. Thus

$$K_{IC} = 1.12(3\sigma_\infty)\sqrt{2\pi\ l_c} = 3.36\sigma_\infty\sqrt{2\pi\ l_c} \tag{5}$$

In second case the crack is long $2a \leq 2l+2a$ and we can for all practical purposes ignore the presence of hole and assume that we have central crack with an effective length. Then

$$l_{eff} = \frac{2l+2a}{2} = l + a \tag{6}$$

thus

$$K_{IC} = \sigma_\infty \sqrt{2\pi(l_c + a)} \quad (7)$$

In studies [7] the plate with a hole and at its edge appeared crack K_{IC} is obtained by the equation

$$K_{IC} = \sigma_\infty \sqrt{2\pi(a + l_c)F} \quad (8)$$

where a is radius of the hole.

Then

$$\sigma_\infty = \frac{K_{IC}}{\sqrt{2\pi(a + l_c)F}}. \quad (9)$$

3. Multiparametric fracture criteria

McClintock and Leguilon [3, 4] offered strain fracture criteria when plastic zone at crack tip is taken into account and stresses σ_∞ are calculated:

$$\sigma_\infty = \frac{2\sigma_C}{\left[2 + \frac{a^2}{[2(a + l_c)]^2} + 3\frac{a^4}{[2(a + l_c)]^4}\right]} \quad (10)$$

where σ_c is critical stresses at crack tip.

If no hole exists ($a = 0$), $\sigma_\infty = \sigma_c$.

When investigating maximum stresses at the crack tip σ_{max} and crack tip area length Δl_{max} in studies [5, 8] fracture resistance stresses σ_{coh} were obtained

$$\sigma_{coh} = \sigma_{max}\left(1 - \frac{\Delta l}{\Delta l_{max}}\right) \quad (11)$$

As a reference of studies [9]

$$G_c = \frac{1}{2}\Delta l_{max}\sigma_{max} \quad (12)$$

In paper [10] an obtained criterion is described

$$G_c^s = h_c \gamma \quad (13)$$

where γ is fracture energy density (quantity of energy for unit volume), h_c is critical crack area width dependable on stress concentration, G_c^s is specific fracture energy.

With these principles three parameters criteria are formed [10]

$$G_c^s = \alpha G_c + (1 - \alpha)G_c^u \quad (14)$$

where α is the parameter indicating level of stress concentration ($0 < \alpha < 1$), G_c is material fracture energy with existing crack, G_c^u is fracture energy under tensile ultimate strength. Fracture energy is determined according strength and displacement tension diagram.

When fracture is specified by specific fracture energy then $\alpha = 1$, and when fracture is specified by stresses, then $\alpha = 0$.

4. Crack area studies

There are no analytic references in study [10] on how to specify crack area width h_c and for parameter α assessment only the volume at crack tip analysis is proposed. But introduced analysis is approximate because complex stress and strain state exists. Therefore to determine crack area width would be the most proper by known stresses and fracture parameters equations [6]. Those equations for plane stress are

$$\sigma_1 = \frac{K_I}{\sqrt{2\pi r}} cos\frac{\theta}{2}\left(1 + sin\frac{\theta}{2}\right) \quad (15)$$

here

$$r_{cr} = \left[\frac{K_{IC}}{\sqrt{2\pi\sigma_1}} cos\frac{\theta}{2}\left(1 + sin\frac{\theta}{2}\right)\right]^2 \quad (16)$$

This radius shows area width before crack growth and it is calculated according fracture parameters. Radius r_{cr} describes area width with critical stress σ_c.

Radial normal stresses around the hole σ_{rr} are calculated

$$\sigma_{rr} = \frac{\sigma_\infty}{2}\left(1 - \frac{a^2}{r^2}\right) - \frac{\sigma_\infty}{2} cos\,2\theta\left(1 - \frac{a^2}{r^2}\right) \times \left(1 - \frac{3a^2}{r^2}\right) \quad (17)$$

Circular normal stresses are calculated

$$\sigma_{\theta\theta} = \frac{\sigma_\infty}{2}\left(1 + \frac{a^2}{r^2}\right) + \frac{\sigma_\infty}{2} cos\,2\theta\left(1 + 3\frac{a^4}{r^4}\right) \quad (18)$$

Tangential stresses are obtained

$$\sigma_{r\theta} = -\frac{\sigma_\infty}{2} sin\,2\theta\left(1 - \frac{a^2}{r^2}\right) \times \left(1 + 3\frac{a^2}{r^2}\right) \quad (19)$$

With angle $\theta = 0$, $\sigma_{r\theta} = 0$, circular stresses $\sigma_{\theta\theta}$ are the main stresses σ_1 and radial stresses are the main stresses σ_2.

Adopting $\sigma_c = \sigma_{11} = \sigma_{\theta\theta}$ radius r_α is calculated from Eq. (18) taking into account $\theta = 0$. Then

$$\sigma_{\theta\theta} = 3\sigma_\infty = \frac{\sigma_\infty}{2}\left(1 + \frac{a^2}{r^2}\right) + \frac{\sigma_\infty}{2}\left(1 + 3\frac{a^4}{r^4}\right) \quad (20)$$

Parameter α will be obtained

$$\alpha = \frac{r_{cr}}{r_\alpha} \quad (21)$$

when $r_{cr} = 0$, $\alpha = 0$ fracture with no crack exists.

Taking into account that $\theta = 0$, radius r_{cr} for plane stress calculated by Eq. (16)

$$r_{cr} = \frac{K_{IC}{}^2}{\sigma_{1,c}^2 2\pi} \quad (22)$$

For plane strain

$$r_{cr} = \frac{K_{IC}{}^2}{\sigma_{1,c}^2 6\pi} \quad (23)$$

Then three parameters criteria (14) can be obtained taking into account:

1) G_C which is calculated from Eq. (3);
2) fracture energy G_c^u obtained by plate tension with no crack and hole in F-Δu coordinates till ultimate strength;
3) parameter α which is obtained from Eq. (21).

5. Experiment

For the experiment three different steel grade (Table 1) specimens were chosen: 1) Steel 45, 2) Steel 15ХСНД, 3) Steel 35ХГСА.

Table 1

Mechanical properties of chosen materials

Material	Yield strength σ_Y, MPa	Ultimate strength σ_U, MPa
Steel 45	320	680
Steel 15ХСНД	350	630
Steel 35ХГСА	404	730

Round hole was drilled through the plate and cracks were made inside the hole with the help of laser cuts.

Energy G_c^u was calculated and expressed by tension diagram area till ultimate strength. For the plane stress plate dimensions – 2x50 mm and diameter of holes – 2; 5 mm. When 2 mm plate is broken by tension force no significant plate cross-section shrinkage was noticed. That‘s why this deformation is considered as a plain stress.

Fracture characteristics σ_∞, $\sigma_{1,c}$, l_c and K_{IC} were obtained during the experiment and calculative area r_α are shown in Table 2. Calculative fracture parameters values r_{cr}, α, G_C, G_c^u and G_c^S are shown in Table 3.

Table 2

Strength, fracture and crack tip area parameters under plane stress

No.	a, mm	$2l_c$, mm	σ_∞, MPa	$\sigma_{1,c}$, MPa	r_α, mm	r_{cr}, mm	α
Steel 45							
1	2	1	424	1427	2	0.5	0.250
2	5	1.3	372	1251	3.8	0.65	0.17
Steel 15ХСНД							
3	2	1.6	440	1480	2	0.8	0.4
4	5	2	394	1324	3.8	1	0.26
Steel 35ХГСА							
5	2	3	490	1648	2	1.5	0.75
6	5	3.7	441	1484	3.8	1.85	0.49

For the plain strain, plate dimensions – 4x50 mm and diameter of holes – 2; 5 mm. When 4 mm plate is broken by tension force, plate cross-section shrinkage is noticed. That‘s why this deformation is considered as a plain strain.

Fracture characteristics σ_∞, $\sigma_{1,c}$, l_c and K_{IC} were obtained during the experiment and calculative area r_x is shown in Table 4. Calculative fracture parameters values r_{cr}, α, G_C, G_c^u and G_c^S are shown in Table 5.

Table 3

Values of energy and fracture parameters under plane stress

No.	K_{IC}, MPa·m$^{3/2}$	G_C, kJ/m^2	G_c^u, kJ/m^2	G_c^S, kJ/m^2
Steel 45				
1	80	32.0	4.84	11.63
2				9.45
Steel 15ХСНД				
3	105	55.1	8.03	26.9
4				20.2
Steel 35ХГСА				
5	160	128.0	7.05	97.8
6				66.3

Table 4

Strength, fracture and crack tip area parameters under plane strain

No.	a, mm	$2l_c$, mm	σ_∞, MPa	$\sigma_{1,c}$, MPa	r_α, mm	r_{cr}, mm	α
Steel 45							
1	2	0.8	450	1512	2	0.37	0.18
2	5	1.1	420	1411	3.8	0.42	0.11
Steel 15ХСНД							
3	2	1.4	480	1612	2	0.56	0.28
4	5	1.8	440	11478	3.8	0.66	0.17
Steel 35ХГСА							
5	2	2.5	520	1747	2	0.115	0.057
6	5	3.1	460	1545	3.8	0.146	0.038

Table 5

Values of energy and fracture parameters under plane strain

No.	K_{IC}, MPa·m$^{3/2}$	G_C, kJ/m^2	G_c^u, kJ/m^2	G_c^S, kJ/m^2
Steel 45				
1	72.8	29.1	4.4	8.94
2				7.14
Steel 15ХСНД				
3	95.5	50.1	7.3	19.28
4				14.79
Steel 35ХГСА				
5	145.6	116.5	6.4	12.67
6				10.63

Obtained fracture energy results show reliance on size of hole and crack relation and deformation state.

6. Conclusions

1. Fracture analysis indicates that structural elements made of plastic and semiplastic materials with stress concentration around holes edges generate plastic deformations and fracture with crack growth. For strength and fracture assessment is necessity to apply multiparametric fracture criteria.

2. As yet two parameters fracture criteria were used, therefore three parameters criteria describes fracture process in more details. Those parameters are: material fracture energy with existing crack, fracture energy obtained when ultimate strength is reached and parameter indicating stress concentration level.

3. Evaluating crack area width and deformation width of stress concentrator edges, similarly calculated fracture energy with and without crack – certain specific fracture energy calculations are provided. Strength and fracture characteristics determined from experimental data by testing plates with holes specimens.

References

1. **Leonavičius, M., Bobyliov, K., Krenevičius, A., Stupak, S.** Superoid graphyte cast iron specimiens with defects cracking investigation. -Mechanika. -Kaunas: Technologija, 2008, Nr.1(69), p.13-18.
2. **Bazant, Z.P., Pijaudier-Cabot, G.** Non- local continuum damage, localization, instability and convergence. -Journal of Applied Mechanics, 1988, v.55, p.287-293.
3. **McClintock, F.A.** Ductile fracture instability in shear. -Journal of Applied Mechanics, 1958, v.25, p.582-588.
4. **Leguilon, D.** Strength or toughness? A criterion for crack onset at a notch. -European Journal of Mechanics – A/Solids, 2002, v.21, No.1, p.61-72.
5. **Hutchinson, J.W., Evans, A.G.** Mechanics of materials: top – down approaches to fracture.-Acta Materialia, 2000, v.48, No.1, p.125-135.
6. **Žiliukas, A.** Strength and Fracture Criteria.-Kaunas: Technologija, 2006.-208p. (in Lithuanian).
7. **Newman, J.C**. NASA Report, TND-6367, 1971.
8. **Mohammed, I., Leichti, K.M.** Cohesive zone modelling of crack nucleation at bimaterial corners. -Journal of Mechanics Physycs and Solids, 2000, v.48, p.735-764.
9. **Camacho, G.T., Ortiz, M.** Computational modelling of impact damage in brittle materials.-International journal of Solids and Structures, 1996, v.33, No.20-22, p.2899-2938.
10. **Li, J., Zhang, X.B.** A criterion study for nonsingular stress concentrations in brittle or quasi– brittle materials. -Engineering Facture Mechanics, 2006, v.73, p.505-523.

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STIPRUMO IR IRIMO KRITERIJŲ TAIKYMAS ĮTEMPIŲ KONCENTRATORIŲ ZONOSE

R e z i u m ė

Straipsnyje analizuojamas pagamintų plokštelių su skylėmis, iš plastiškų plienų, plastinis deformavimas skylės kraštuose ir irimas atsiradus plyšiui. Stiprumui ir irimui įvertinti apskaičiuojama specifinė irimo energija, gaunama kaip plyšio irimo energijos ir deformacijos energijos dydžio, esant stiprumo ribai, suma, susieta parametru, parodančiu įtempių koncentracijos lygį. Skaičiavimams naudojamos bandymų metu nustatytos stiprumo ir irimo charakteristikos.

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STRENGTH AND FRACTURE CRITERIA APPLICATION IN STRESS CONCENTRATORS AREAS

S u m m a r y

This paper presents an investigation of plastic deformation of plates with holes made of plastic steels. Plastic deformation in plate hole edge area and fracture with appeared crack are studied. Specific fracture energy is calculated for strength and fracture evaluation which is obtained as a sum of fracture energy crack and deformation energy value with ultimate strength limit coherent with the parameter, describing stress concentration level. For the calculations strength and fracture parameters are used which were obtained during the experiment.

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ИСПОЛЬЗОВАНИЕ ПАРАМЕТРОВ ПРОЧНОСТИ И РАЗРУШЕНИЯ В ЗОНАХ КОНЦЕНТРАЦИИ НАПРЯЖЕНИЙ

Р е з ю м е

В настоящей работе проведено исследование пластического деформирования пластин с отверстием из пластических сталей. Рассматривается пластическое деформирование зон около отверстий пластин и их разрушение. Рассчитана относительная суммарная энергия деформирования и разрушения при предельных состояниях в зависимости от уровня концентрации напряжений. Расчётные параметры упругости и разрушения получены экспериментально.

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