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The finite element models of thin-walled branched structures in heat transfer problems

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1. Introduction

At present engineers are often faced with the task of designing thin-walled branched structures working under conditions of intensive heat interchange with the environment. In the junction zone of the plates of such structures, great temperature oscillations take place, which has an influence on the analysis of the stress–strain state of the structure [1]. In solving this problem, first of all we have to evaluate accurately the values of the temperature in the plate junction zone.

Transient heat conduction in an anisotropic material is described by the differential equation

$$k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2} + k_z \frac{\partial^2 T}{\partial z^2} + Q = \rho c \frac{\partial T}{\partial t} \qquad (1)$$

here k_x, k_y, k_z are anisotropic thermal conductivity coefficients; ρ is the mass density of the material; c is the specific heat capacity; $Q(x,y,z)$ is the heat generated within the body; $T(x,y,z)$ is the temperature; t is the time.

To solve the Eq. (1), initial and boundary conditions are introduced

$$T(x,y,z) = T_0 \qquad (2)$$

$$k_x \frac{\partial T}{\partial x} n_x + k_y \frac{\partial T}{\partial y} n_y + k_z \frac{\partial T}{\partial z} n_z + q + \alpha (T - T_\infty) = 0 \qquad (3)$$

where n_x, n_y, n_z are the direction cosines, $q(x,y,z)$ is the heat flux, α is the convection coefficient over the surface, T_∞ is the fluid temperature.

Because the analysed structure is of a complex geometrical shape, it is convenient to solve the problem of the temperature field by the finite element method. All mathematical dependences of the method are easily formalised and generated by computer technologies.

An Eq. (1) with initial conditions (2) and with boundary conditions (3) in accordance with [2] is written as the system of differential equations

$$[C]\frac{\partial \{T\}}{\partial t} + [K]\{T\} = \{F\} \qquad (4)$$

where $[K]$, $[C]$ and $\{F\}$ are respectively thermal conductivity matrix, heat capacity matrix and vector of thermal load.

The finite element model of the examined structure is composed of finite elements of several types: plane (triangular or quadrangular) and connective quadrangular prismatic finite elements. That is why the problem of modelling plate junction zones with finite elements appears. It is analysed in research works [3-5]. Plane finite elements are joined with triangular or quadrangular prismatic connective finite elements.

Practical calculations have shown that in those cases when connective finite elements join other finite elements in the nodes that are in the middle-surface of the plates, it is necessary to use the middle nodes of the prism base instead of corner nodes of the prism. To calculate temperature values in such nodes special transformation matrices are necessary.

In this work, transformation formulas of coordinates that translate the middle side nodes of the bases of a quadrangular prism to the corner nodes of the prism, and formulas that translate the temperature values of the corner nodes into the temperature values of the middle side nodes of the bases of the finite element are found. The problem is solved for the connective triangular prismatic finite element [3].

The aim of the present article is to assess the transient temperature field of the plate junction zone of a thin-walled branched structure of an anisotropic material, when the boundary conditions on the lateral surfaces of the plate can be expressed in three different forms: convection, heat source, and heat flow. The junction zone of the plates is going to be modelled by a connective quadrangular prismatic finite element, the main mathematical expressions of which for the isotropic material are described in [4].

2. Matrices of the connective quadrangular prismatic finite element

To make a discrete model of the junction zone of the plates, a connective quadrangular finite element of an equal cross-section is going to be used; with a quadrangle as the base (Fig. 1). The element has 8 degrees of freedom.

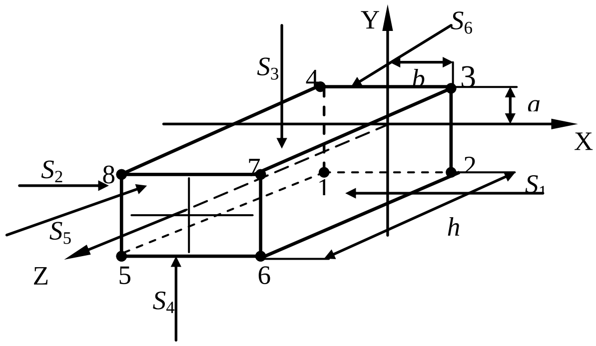

Fig. 1 The geometry of the connective finite element

The temperature inside the finite element e changes linearly

$$T^e = \sum_{i=1}^{8} N_i T_i = [N]\{T^e\} \tag{5}$$

where $[N]$ is the matrix of the shape functions of the element, $\{T^e\}$ is temperature values in the nodes of the element.

The shape functions, satisfying properties in the nodes of the element

$$\left.\begin{aligned} &N_i^e(x_i, y_i, z_i) = 1 \\ &N_i^e(x_j, y_j, z_j) = 0, \text{ when } i = 1,2,3,\dots,8 \end{aligned}\right\} \tag{6}$$

are entered as follows

$$\left.\begin{aligned}
N_1 &= \frac{1}{Sh}(b-x)(a-y)(h-z) \\
N_2 &= \frac{1}{Sh}(b+x)(a-y)(h-z) \\
N_3 &= \frac{1}{Sh}(b+x)(a+y)(h-z) \\
N_4 &= \frac{1}{Sh}(b-x)(a+y)(h-z) \\
N_5 &= \frac{1}{Sh}(b-x)(a-y)z \\
N_6 &= \frac{1}{Sh}(b+x)(a-y)z \\
N_7 &= \frac{1}{Sh}(b+x)(a+y)z \\
N_8 &= \frac{1}{Sh}(b-x)(a+y)z
\end{aligned}\right\} \tag{7}$$

where S is the cross-sectional area of the connective finite element, h is the height of the element. The values of a and b are calculated by using the formulae to calculate the distance between two points.

The contribution of the connective finite element to the matrices $[K]$, $[C]$ and vector $\{F\}$ of the Eq. (4) is expressed by the following formulae

$$[K]^e = \int_V [B]^T [D][B] dV + \int_S \alpha [N]^T [N] dS \tag{8}$$

$$[C]^e = \int_V \rho c [N]^T [N] dV \tag{9}$$

$$\{F\}^e = -\int_V Q [N]^T dV + \int_S q [N]^T dS - \int_S \alpha T_\infty [N]^T dS \tag{10}$$

here V is the volume of the finite element, S is the lateral area of the element, $[B]$ is a matrix of the derivatives of the shape functions of the element in the directions x, y and z, $[D]$ is a matrix of the thermal conductivities in the directions x, y and z.

The matrix $[D]$ in the discussed case is entered as follows

$$[D] = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix} \tag{11}$$

By differentiating matrix $[N]$ (7) with respect to x, y and z, we form matrix $[B]$, the inverse matrix of which is entered as follows

$$[B]^T = \frac{1}{Sh}\begin{bmatrix}
-(a-y)(h-z) & -(b-x)(h-z) & -(b-x)(a-y) \\
(a-y)(h-z) & -(b+x)(h-z) & -(b+x)(a-y) \\
(a+y)(h-z) & (b+x)(h-z) & -(b+x)(a+y) \\
-(a+y)(h-z) & (b-x(h-z)) & -(b-x)(a+y) \\
-(a-y)z & -(b-x)z & (b-x)(a-y) \\
(a-y)z & -(b+x)z & (b+x(a-y)) \\
(a+y)z & (b+x)z & (b+x)(a+y) \\
-(a+y)z & (b-x)z & (b-x)(a+y)
\end{bmatrix} \tag{12}$$

While calculating matrices of the finite element, let us assume that

$$\int_V dV = \int_{-a}^{a}\int_{-b}^{b}\int_{0}^{h} dz\, dy\, dx \tag{13}$$

The finite element thermal conductivity matrix $[K]$ (8) has two parts:

- the first integral describes the thermal conductivity of the finite element, determined by the thermal conductivity coefficients of the material;
- the second integral describes the thermal conductivity of the finite element, determined by the convection heat exchange with environment via the lateral surfaces and the ends of the finite element.

The thermal conductivity matrix of the element always has the first component, while the second has to be evaluated only when the lateral surfaces and the ends of the finite element are open, i.e. when they have a contact with the environment.

While calculating the first integral of the thermal conductivity matrix of the element (8), the values of matrices $[B]$, $[B]^T$ and $[D]$ are inserted in its expression. It can be seen that post-integral expressions are rather complex. That is why it is difficult to calculate them manually; besides, it is easy to make mistakes. That is why mathematical transformations are made by using computer algebra systems MAPLE and MATHEMATICA.

Having integrated the first integral of the thermal conductivity matrix of the connective finite element, we get

$$\int_V [B]^T [D][B]dV = \frac{1}{18Sh}\begin{bmatrix} 2[M_1] & [M_2] \\ [M_2] & 2[M_1] \end{bmatrix} \tag{14}$$

where

$$[M_1] = \begin{bmatrix} m_1 & -m_2 & -m_3 & m_4 \\ -m_2 & m_1 & m_4 & -m_3 \\ -m_3 & m_4 & m_1 & -m_2 \\ m_4 & -m_3 & -m_2 & m_1 \end{bmatrix}$$

$$[M_2] = \begin{bmatrix} m_5 & -m_6 & m_7 & -m_8 \\ -m_6 & m_5 & -m_8 & m_7 \\ m_7 & -m_8 & m_5 & -m_6 \\ -m_8 & m_7 & -m_6 & m_5 \end{bmatrix}$$

$$m_1 = 3a^4k_z + a^2\left(h^2k_2 + 10b^2k_z\right) + b^2\left(h^2k_1 + 3b^2k_z\right)$$

$$m_2 = 3a^4k_z + a^2\left(h^2k_2 - 8b^2k_z\right) + b^2\left(h^2k_3 - 3b^2k_z\right)$$

$$m_3 = 3a^4k_z + a^2\left(h^2k_4 - 10b^2k_z\right) - b^2\left(h^2k_3 - 3b^2k_z\right)$$

$$m_4 = 3a^4k_z + a^2\left(h^2k_4 + 8b^2k_z\right) - b^2\left(h^2k_1 + 3b^2k_z\right)$$

$$m_5 = -6a^4k_z + a^2\left(h^2k_2 - 20b^2k_z\right) + b^2\left(h^2k_1 - 6b^2k_z\right)$$

$$m_6 = -6a^4k_z + a^2\left(h^2k_2 + 16b^2k_z\right) + b^2\left(h^2k_3 + 6b^2k_z\right)$$

$$m_7 = 6a^4k_z - a^2\left(h^2k_4 + 20b^2k_z\right) + b^2\left(h^2k_3 + 6b^2k_z\right)$$

$$m_8 = 6a^4k_z - a^2\left(h^2k_4 - 16b^2k_z\right) + b^2\left(h^2k_1 - 6b^2k_z\right)$$

$$k_1 = k_x + 3k_y,\ k_2 = 3k_x + k_y$$

$$k_3 = k_x - 3k_y,\ k_4 = 3k_x - k_y$$

The value of the second component

$$\int_S \alpha [N]^T [N] dS \tag{15}$$

of the thermal conductivity matrix of the connective finite element (8) depends on the surface of the element (lateral or end) where the heat transfer with convection takes place. The value of this integral does not depend on the thermal conductivity coefficients, that is why by employing the outcomes of [4], we can calculate the value of integral (15) for the surface, e.g. $S_1 : x = b,\ y \in [-a, a]$, $z \in [0, h]$

$$S_1 = \frac{\alpha ha}{18}\begin{bmatrix} 2[S_1'] & [S_1'] \\ [S_1'] & 2[S_1'] \end{bmatrix}, \quad \text{here} \quad [S_1'] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The values of integral (15) are written accordingly for surfaces S_2, S_3, S_4, S_5, S_6, S_7, S_8, where the values of the elements of matrix $[S_i']$ depend on the numbers of the nodes belonging to lateral or end surfaces.

The matrix of the thermal capacity of the connective finite element (9) according to [3] has the following form

$$[C] = \frac{\rho chS}{216}\begin{bmatrix} 2[C^*] & [C^*] \\ [C'] & 2[C^*] \end{bmatrix}, \text{ here } [C^*] = \begin{bmatrix} 4 & 2 & 1 & 2 \\ 2 & 4 & 2 & 1 \\ 1 & 2 & 4 & 2 \\ 2 & 1 & 2 & 4 \end{bmatrix}$$

Because the integrals of the thermal load vector (10) do not depend on the thermal conductivity coefficients, on the basis of [4], the value of the first integral is

$$-\int_V Q[N]^{[T]} dV = -\frac{QhS}{8}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

The values of the other two integrals of the thermal load vector (10) depend on the numbers of the nodes of the lateral or end surfaces of the finite element. For instance, for surface S_1

$$\int_{S_1} q[N]^T dS_1 = \frac{qha}{2}\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad \int_{S_1} \alpha T_\infty [N]^T dS_1 = \frac{\alpha T_\infty ha}{2}\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}.$$

3. The transformation formulae of the coordinate nodes of the connective finite element

We shall now analyse the base (quadrangle) of the connective finite element (Fig. 2) with nodal points at middle side points, numbered counter-clockwise from the first freely chosen node. We shall transfer the middle side nodes of the base

$$1(x_1, y_1, z_1),\ 2(x_2, y_2, z_2),\ 3(x_3, y_3, z_3),\ 4(x_4, y_4, z_4)$$

to the corner nodes

$$A(x_A, y_A, z_A),\ B(x_B, y_B, z_B),\ C(x_C, y_C, z_C),$$
$$D(x_D, y_D, z_D)$$

i.e. we shall express the coordinates of nodes *A, B, C, D* by the coordinates of nodal points *1, 2, 3, 4*. To solve this problem, the equations of lines going via given points, known from analytical geometry, are going to be used.

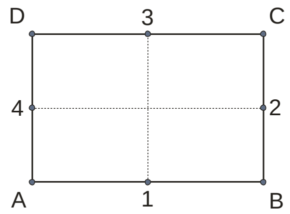

Fig. 2 Numbering of the nodes of the base of the element

Because the branched structure of a complex shape is analysed, the connective finite element can be in various positions in its finite elements scheme. That is why it is necessary to analyse a few cases of the positions of the nodes of the connective finite elements in space.

1. $z = c = \text{const},\ x_1 + x_3 = x_2 + x_4,\ y_1 + y_3 = y_2 + y_4$

In this case, the coordinates of the corner points are entered as follows

$$A\left(\frac{a_1}{d}, \frac{a_2}{d}, c\right), B\left(\frac{b_1}{d}, \frac{b_2}{d}, c\right), C\left(\frac{c_1}{d}, \frac{c_2}{d}, c\right), D\left(\frac{d_1}{d}, \frac{d_2}{d}, c\right)$$

where

$$x_{24} = x_2 - x_4,\ \ y_{24} = y_2 - y_4$$

$$x_{13} = x_1 - x_3,\ y_{13} = y_1 - y_3$$

$$a_1 = -x_{13}\left(x_1 y_{24} - y_1 x_{24}\right) + x_{24}\left(x_4 y_{13} - y_4 x_{13}\right)$$

$$a_2 = y_{24}\left(x_4 y_{13} - y_4 x_{13}\right) - y_{13}\left(x_1 y_{24} - y_1 x_{24}\right)$$

$$b_1 = -x_{13}\left(x_1 y_{24} - y_1 x_{24}\right) + x_{24}\left(x_2 y_{13} - y_2 x_{13}\right)$$

$$b_2 = y_{24}\left(x_2 y_{13} - y_2 x_{13}\right) - y_{13}\left(x_1 y_{24} - y_1 x_{24}\right)$$

$$c_1 = x_{24}\left(x_2 y_{13} - y_2 x_{13}\right) - x_{13}\left(x_3 y_{24} - y_3 x_{24}\right)$$

$$c_2 = -y_{13}\left(x_3 y_{24} - y_3 x_{24}\right) + y_{24}\left(x_2 y_{13} - y_2 x_{13}\right)$$

$$d_1 = -x_{13}\left(x_3 y_{24} - y_3 x_{24}\right) + x_{24}\left(x_4 y_{13} - y_4 x_{13}\right)$$

$$d_2 = y_{24}\left(x_4 y_{13} - y_4 x_{13}\right) - y_{13}\left(x_3 y_{24} - y_3 x_{24}\right)$$

For the points not to be in one line, a necessary conditiondiscriminant $d = \begin{vmatrix} y_{24} & -x_{24} \\ y_{13} & -x_{13} \end{vmatrix} \neq 0$.

2. $z = c = \text{const},\ x_1 = x_3,\ y_2 = y_4$

In this case, the coordinates of the corner nodes are as follows

$$A\left(x_4, y_1, c\right),\ B\left(x_2, y_1, c\right),\ C\left(x_2, y_3, c\right),\ D\left(x_4, y_3, c\right)$$

3. $z_1 = z_3,\ z_2 \neq z_4$

The coordinates of the corner nodes are calculated as follows

$$A\left(x_1 + x_{24} t_1, y_1 + y_{24} t_1, z_4\right)$$

$$B\left(x_1 + x_{24} t_2, y_1 + y_{24} t_2, z_2\right)$$

$$C\left(x_3 + x_{24} t_3, y_3 + y_{24} t_3, z_4\right)$$

$$D\left(x_3 + x_{24} t_4, y_3 + y_{24} t_4, z_4\right)$$

where

$$t_1 = \frac{z_{41}}{z_{24}},\ \ t_2 = \frac{z_{21}}{z_{24}},\ \ t_3 = \frac{z_{23}}{z_{24}},\ \ t_4 = \frac{z_{43}}{z_{24}}.$$

4. $z_1 \neq z_3,\ z_2 = z_4$

The coordinates of the corner points are entered as follows

$$A\left(x_4 + x_{13} t_{11}, y_4 + y_{13} t_{11}, z_1\right)$$

$$B\left(x_2 + x_{13} t_{22}, y_2 + y_{13} t_{22}, z_1\right)$$

$$C\left(x_2 + x_{13} t_{33}, y_2 + y_{13} t_{33}, z_3\right)$$

$$D\left(x_4 + x_{13} t_{44}, y_4 + y_{13} t_{44}, z_3\right)$$

where

$$t_{11} = \frac{z_{14}}{z_{13}},\ \ t_{22} = \frac{z_{12}}{z_{13}},\ \ t_{33} = \frac{z_{32}}{z_{13}},\ \ t_{44} = \frac{z_{34}}{z_{13}}.$$

5. $z \neq \text{const}$

$x_1 + x_3 = x_2 + x_4,\ \ y_1 + y_3 = y_2 + y_4,$

$z_1 + z_3 = z_2 + z_4,\ \ z_1 \neq z_3,\ \ z_2 \neq z_4$

The rank of matrix $M = \begin{bmatrix} x_{13} & y_{13} & z_{13} \\ x_{24} & y_{24} & z_{24} \end{bmatrix}$ – has to be equal 2.

Minors of second order are made of the elements of matrix M

$$M_1 = \begin{vmatrix} y_{13} & z_{13} \\ y_{24} & z_{24} \end{vmatrix}\ \ M_2 = \begin{vmatrix} x_{13} & z_{13} \\ x_{24} & z_{24} \end{vmatrix}\ \ M_3 = \begin{vmatrix} x_{13} & y_{13} \\ x_{24} & y_{24} \end{vmatrix}$$

which have to satisfy the condition $M_1^2 + M_2^2 + M_3^2 \neq 0$.

Consider the following cases:

If $M_1 \neq 0$, then

$$A\left(x_1 + x_{24} t_1, \frac{a_3}{M_1}, \frac{a_4}{M_1}\right),\ B\left(x_1 + x_{24} t_2, \frac{b_3}{M_1}, \frac{b_4}{M_1}\right),$$

$$C\left(x_3 + x_{24} t_3, \frac{c_3}{M_1}, \frac{c_4}{M_1}\right),\ D\left(x_3 + x_{24} t_4, \frac{d_3}{M_1}, \frac{d_4}{M_1}\right).$$

where

$$a_3 = y_{13}\left(y_1 z_{24} - z_1 y_{24}\right) - y_{24}\left(y_4 z_{13} - z_4 y_{13}\right)$$

$$a_4 = z_{13}\left(y_1 z_{24} - z_1 y_{24}\right) - z_{24}\left(y_4 z_{13} - z_4 y_{13}\right)$$

$$b_3 = x_{13}\left(y_1 z_{24} - z_1 y_{24}\right) - y_{24}\left(x_2 z_{13} - z_2 x_{13}\right)$$

$$b_4 = z_{13}\left(y_1 z_{24} - z_1 y_{24}\right) - z_{24}\left(x_2 z_{13} - z_2 x_{13}\right)$$

$$c_3 = y_{13}\left(y_3 z_{24} - z_3 y_{24}\right) - y_{24}\left(y_2 z_{13} - z_2 y_{13}\right)$$

$$c_4 = z_{13}\left(y_3 z_{24} - z_3 y_{24}\right) - z_{24}\left(y_2 z_{13} - z_2 y_{13}\right)$$

$$d_3 = y_{13}(y_3 z_{24} - z_3 y_{24}) - y_{24}(y_4 z_{13} - z_4 y_{13})$$

$$d_4 = z_{13}(y_3 z_{24} - z_3 y_{24}) - z_{24}(y_4 z_{13} - z_4 y_{13})$$

If $M_2 \neq 0$, then

$$A\left(\frac{a_{33}}{M_2}, y_1 + y_{24}t_1, \frac{a_{44}}{M_2}\right),\ B\left(\frac{b_{33}}{M_2}, y_1 + y_{24}t_2, \frac{b_{44}}{M_2}\right),$$

$$C\left(\frac{c_{33}}{M_2}, y_3 + y_{24}t_3, \frac{c_{44}}{M_2}\right),\ D\left(\frac{d_{33}}{M_2}, y_3 + y_{24}t_4, \frac{d_{44}}{M_2}\right)$$

where

$$a_{33} = x_{13}(x_1 z_{24} - z_1 x_{24}) - x_{24}(x_4 z_{13} - z_4 x_{13})$$

$$a_{44} = z_{13}(x_1 z_{24} - z_1 x_{24}) - z_{24}(x_4 z_{13} - z_4 x_{13})$$

$$b_{33} = y_{24}(x_1 z_{24} - z_1 x_{24}) - x_{24}(x_2 z_{13} - z_2 x_{13})$$

$$b_{44} = z_{13}(x_1 z_{24} - z_1 x_{24}) - z_{24}(x_2 z_{13} - z_2 x_{13})$$

$$c_{33} = -x_{24}(x_2 z_{13} - z_2 x_{13}) + x_{13}(x_3 z_{24} - z_3 x_{24})$$

$$c_{44} = z_{13}(x_1 z_{24} - z_3 x_{24}) - z_{24}(x_2 z_{13} - z_2 x_{13})$$

$$d_{33} = x_{13}(x_3 z_{24} - z_3 x_{24}) - x_{24}(x_4 z_{13} - z_4 x_{13})$$

$$d_{44} = z_{13}(x_3 z_{24} - z_3 x_{24}) - z_4(x_4 z_{13} - z_4 z_{13})$$

If $M_1 = M_2 = 0$, then $M_3 = 0$, because $z_1 \neq z_3$, $z_2 \neq z_4$. This contradicts the condition $M_1^2 + M_2^2 + M_3^2 \neq 0$.

4. The transformation formulae of the temperature values of the connective finite element

In the case when the scheme of the finite elements of the structure is composed only from connective quadrangular finite elements, it is not necessary to translate the temperature values of the corner nodes to the middle side nodes of bases. Otherwise, when connective finite elements in the structure join plane finite elements, it is necessary to translate the temperature values of the corner nodes to the middle side nodes of bases. Having evaluated previously formulated conditions (7), we get the following transformation matrix of temperature values

$$R = \begin{bmatrix} [R_1] & [0] \\ [0] & [R_1] \end{bmatrix}$$

where

$$[R_1] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Thus the thermal conductivity matrix and thermal capacity matrix of the connective finite element to calculate temperature values in the middle side nodes of the base of the finite element are found as follows

$$[K_1]^e = [R]^T [K]^e [R]$$

$$[C_1]^e = [R]^T [C]^e [R].$$

5. Numerical results

We shall analyse the temperature field in a structure of an anisotropic material in the case of a transient heat transfer process. The algorithm was written by FORTRAN.

A cooled branched structure is given [4]. The air moving across the upper surface has a temperature of 20ºC, convection coefficient is 8.1 W/(m^2 °C). The lower surface is cooled by a liquid of the temperature of –196ºC, convection coefficient is 3529 W/(m^2 °C). The solution schema of the finite elements of the structure with a quadrangular connective finite element is shown in Fig. 3.

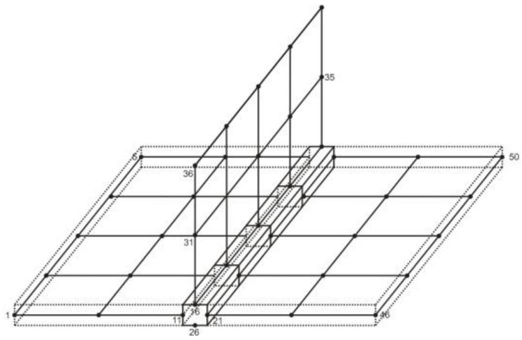

Fig. 3 The grid of the finite elements of the structure

To analyse the temperature convergence, two finite element schemes were adapted: with and without a connective quadrangular finite element. The convergence of the results is illustrated by observing the change of temperature of a certain point of the calculated scheme in time, e.g. 31 (Fig. 3).

The view of the temperature convergence of node 31 of the two calculated schemes is presented in Fig. 4.

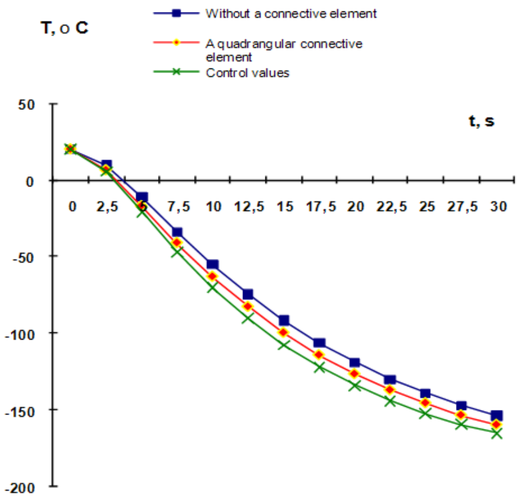

Fig. 4 The temperature in time of node 31 of the two calculated schemes

The correctness of the formulae of the transformation of node coordinates and temperatures values of the connective finite element is established by calculating temperature values in the connective nodes 11, 16, 21 and 26 (Fig. 3). The results of the calculation are presented in the table below.

The comparison of the temperature values

Numbers of the nodes (Fig. 3)	Obtained temperature values *T*, ºC	Control temperature values [4] *T*, ºC
11	-73.2	-73.5
16	-59.2	-59.8
21	-73.2	-73.5
26	-80.2	-80.6

6. Conclusions

1. The numerical solution of the problem of a transient temperature field converges.
2. In cases of the first and second sampling of the junction zone, the curves of the temperature values of node 31 differ. According to the calculated scheme with a connective finite element, the calculated temperature values (at node 31) are closer to the control values than the values obtained according to the scheme without a connective finite element.
3. The obtained transformation formulae of the node coordinates and temperature values allow making a finite element scheme of the analysed structure both with corner and middle side nodes of a base of a connective finite element. This allows a more efficient modelling of joining a few plates with different normals to a horizontal surface.

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PLONASIENIŲ IŠSIŠAKOJANČIŲ KONSTRUKCIJŲ BAIGTINIŲ ELEMENTŲ MODELIAI ŠILUMOS PERNEŠIMO UŽDAVINIUOSE

R e z i u m ė

Straipsnyje nagrinėjamas plonasienės išsišakojančios konstrukcijos, pagamintos iš anizotropinės medžiagos plokštelių sandūros zonos nestacionaraus temperatūros lauko analizės uždavinys. Tokių konstrukcijų plokštelių sandūros zonoje vyksta dideli temperatūrų svyravimai, kurie turi įtakos konstrukcijos įtempto ir deformuoto būvio analizei. Sandūros zonai diskretizuoti naudojamas jungiamasis erdvinis baigtinis elementas, kurio pagrindas yra keturkampis. Gauta jungiamojo baigtinio elemento šilumos laidumo matrica, kai medžiaga yra anizotropinė.

Išvestos koordinačių transformacijos ir temperatūros verčių transformacijos formulės, kurios įgalina sudaryti nagrinėjamo objekto baigtinių elementų tinklelį tiek su jungiamojo elemento pagrindo kampiniais, tiek su pagrindo kraštinių vidurių mazgais. Tai leidžia efektyviau modeliuoti, kai jungiamos kelios plokštelės, turinčios skirtingas horizontalaus paviršiaus normales. Išvestos formulės patikrintos sprendžiant šaldomos plokštelės temperatūros lauko uždavinį.

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THE FINITE ELEMENT MODELS OF THIN-WALLED BRANCHED STRUCTURES IN HEAT TRANSFER PROBLEMS

S u m m a r y

The paper deals with the problem of analysis of the transient temperature field in the plate junction zone of a thin-walled branched structure of an anisotropic material. In the junction zone of the plates of such structures, great temperature oscillations take place, which have an influence on the analysis of the stress-strain state of the structure. The junction zone of the plates was modelled by a connective finite element with a quadrangular base. The thermal conductivity matrix of the connective finite element was obtained in the case of an anisotropic material. The coordinate transformation and temperature value transformation formulae were derived, which allowed composing a grid of the finite elements of the examined object both with the corner and middle side nodes of the connective finite element. This allows a more efficient modelling of joining a few plates with different normals to a horizontal surface. The derived formulae were checked by the solution of a problem of the temperature field of a cooled plate.

Keywords: thin-walled branched structure, connective finite element, heat transfer.

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