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Mathematic modelling and the deep drawing force simulation with the wall thickness thinning experiment application

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1. Abstract

Deep drawing with wall-thickness thinning has a broad application in product fabrication with the height higher than diameter and the base thickness higher than the product wall thickness. Some of them are given in Fig. 1.

Fig. 1 Product selection obtained by deep drawing with wall- thickness thinning

The principal technological process parameters are, amongst others, made of: speed, deformation rate (dependent of weight and of matrix cone angle), the states of material in contact (area topography, tribological conditions, physical and chemical material characteristics), tool geometry and working part.

The process projecting of deep-drawing with wall-thickness thinning for its natural processes and many influential parameters demands a detailed analysis of all influential parameters where the basic aim is the cheaper, more qualitative and profitable fabrication [1-3]. Hence, with the election of optimal values of the influential parameters the demanded product quality can be obtained by the mi-nimum energy consumption.

Theoretically, with the application of the analytical models it is difficult to determine the optimal processing conditions whereas in every processing process more influential factors and their interactions are present.

For the process known as the stochastic processes, when experiment performing, certain approximations which are the constituent part of analytical modelling are eliminated. With the application of the statistic methods and obtained experimental results, the more accurate data that determines the real process parameters are obtained.

The mathematical modelling and optimization methods represent the basic methods in analysis of projecting process with the basic aim to innovate the existing processes, their modernisation and rising to a higher technological level.

Modelling of the analysis process is the foundation of the optimization and defining of optimal analysis conditions which is impossible without preliminary definition of the reliable mathematical model.

In this respect, as a subject of this experimental process research of deep-drawing with wall-thickness thinning is made. The main goal is to obtain the exact and accurate data to serve defining the mathematical model for the deep-drawing force. Thereafter, with the simulation of the obtained mathematical model, the determination of the dimension and the character of the drawing force, depending on input independent variables of the process parameters is included in experiment.

2. Election of the input-output process parameters

A successful performance of the experiment demands the identification and limitation of certain influential process parameters to a concrete number. It refers to defining of only a certain input variables, as the independent variables x_i, as an input into the process , and function defining of output process y_i that are variable dependent dimensions. Such approach enables qualitative managing of the process and development towards modelling achievement [1,3].

The input-output process parameters included in the experiment are shown in Fig. 2 where are: ψ is deformation, s_1 is wall-thickness after drawing, mm, μ is abrasion coefficient and F_i is drawing force, kN, while the other variables were observed as the constants.

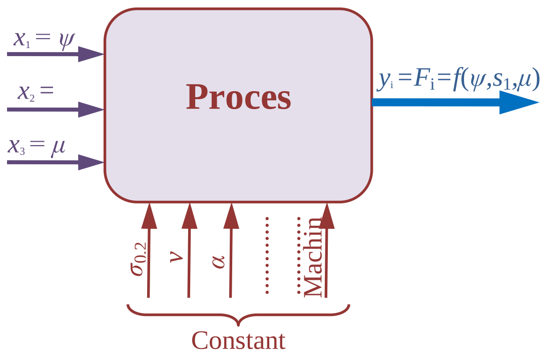


Fig. 2 Election of the input-output process parameters included into the experiment [1,3,4]

During the experiment, the variable-independent process parameters, ψ, s_1 and μ varied through three values,

i. e., the output function value F_i was measured for different parameter values and their combinations, Table 1.

Table 1
Value variation of the influential parameters

Variation levels	Influential process parameters		
	ψ	s_1, mm	μ
Minimum	0.278	1.90	0.10
Medium	0.403	2.38	0.15
Maximum	0.528	2.86	0.20

3. Description of the research equipment

With the experiment plan it was estimated the performance of entirely 12 tests, i.e., measuring of the drawing force F_i, the test of which eight tests were performed with different influential parameter values and four with the medium values Table 2.

The scheme of the deep-drawing process with wall-thickness thinning is shown in Fig. 3. The working parts of dimensions D_0, s_0, and h_0 are set up in the tool matrix, then operating of the lifting piece the working part draws through the matrix where the complete part of the new dimensions is obtained: D_1, s_1, h_1.

The structure of the research equipment utilized for the experiment is shown in Fig. 4. The experiment performance consists of: on the hydraulic strainer ¨1¨, on the tool place, a specially produced tool for deep-drawing with wall-thickness thinning ¨2¨ is fitted, then sensors for drawing force measurement and contact constrains are connected with the connector cables (which are placed o the tool) with the measuring apparatus ¨3¨. After that, the previously prepared working parts are fitted in the tool, and the process of drawing is preformed according to the defined experiment plan (number of tests, working part dimension, greasing manner, etc.).

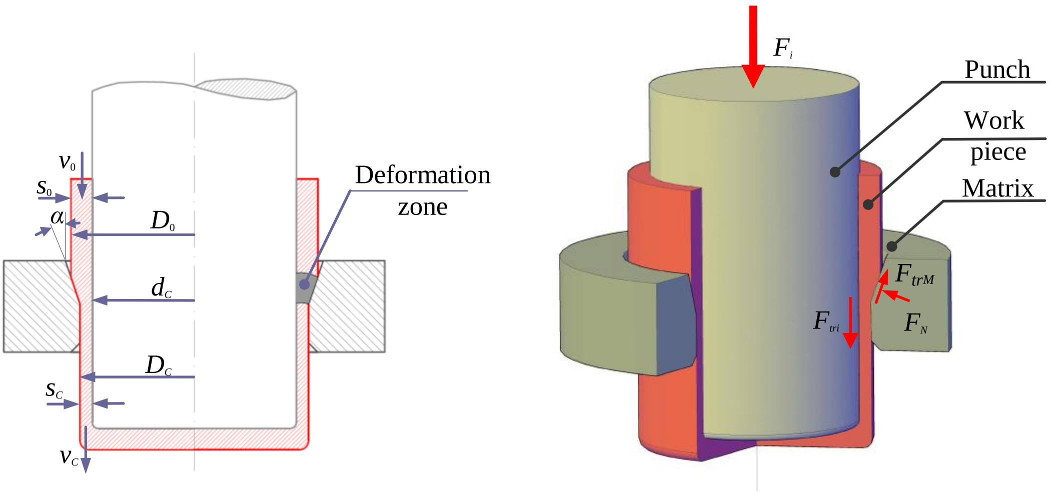
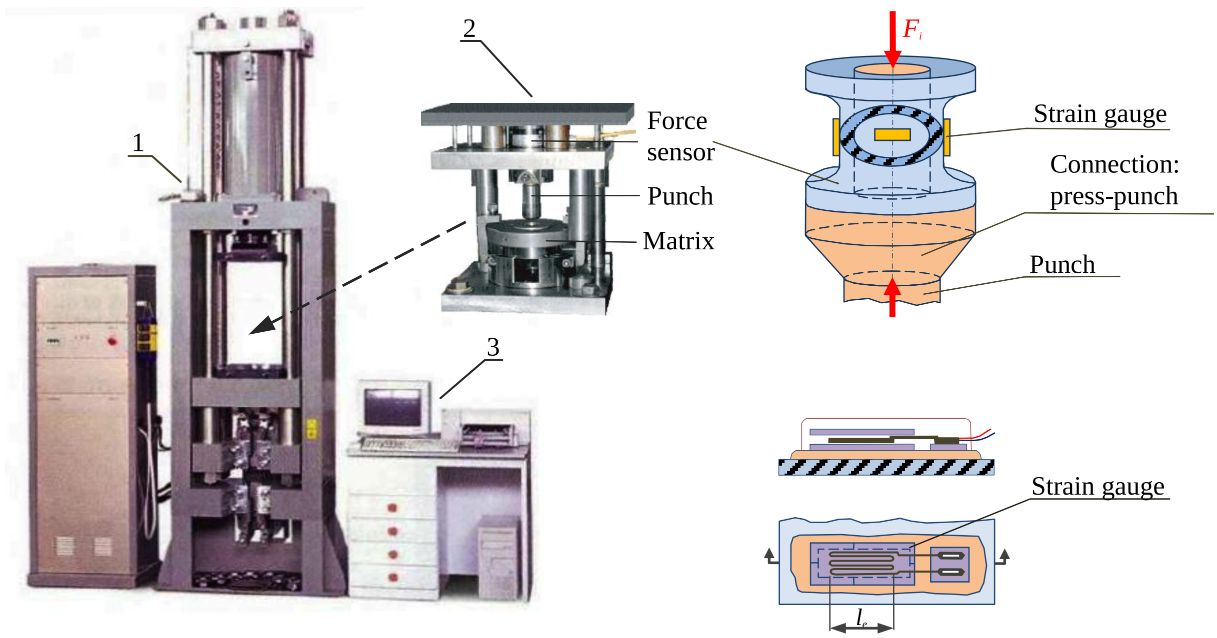


Fig. 3 Scheme process presentation



a b

Fig. 4 Research equipment: a) hydraulic strainer and the tool; b) construction of the force sensor [5, 6]

4. Homogeneity evaluation of experimental results

According to already defined input and output process variables (Fig. 2), as well as their values variations (Table 1.), 12 tests are performed with the repetition only in the central dot of the orthogonal plan ($n_0 = 4$) [1]. The test repetition is necessary in order to perform dispersion analyses of the experimental results (experiment homogeneity, experiment error estimation, model adequacy, etc.). The obtained experimental results for the drawing force F_i, with physical and coded values of the selected influential parameters x_i are given in the complete experiment matrix plan, Table 2.

The elected mathematical model for the drawing force modelling is linear mathematical model, and in the coded form along with $F_i = y_i = f(x_i)$ has the form

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_{12}X_1X_2 + \\ + b_{23}X_2X_3 + b_{13}X_1X_3 + b_{123}X_1X_2X_3 \quad (1)$$

where X_i are the influential parameters in coded form, b_i are the regression model coefficients.

Table 2

Complete experiment matrix plan

Number of experiments	Physical values of parameters			Coded values of parameters								Experimental results $Y_j = F_i$, kN
	$x_1 = \psi$	$x_2 = s_1$ (mm)	$x_3 = \mu$	X_0	X_1	X_2	X_3	X_1X_2	X_2X_3	X_1X_3	$X_1X_2X_3$	
1.	0.278	1.90	0.10	+1	-1	-1	-1	+1	+1	+1	-1	113
2.	0.528	1.90	0.10	+1	+1	-1	-1	-1	+1	-1	+1	230
3.	0.278	2.86	0.10	+1	-1	+1	-1	-1	-1	+1	+1	150
4.	0.528	2.86	0.10	+1	+1	+1	-1	+1	-1	-1	-1	290
5.	0.278	1.90	0.20	+1	-1	-1	+1	+1	-1	-1	+1	156
6.	0.528	1.90	0.20	+1	+1	-1	+1	-1	-1	+1	-1	272
7.	0.278	2.86	0.20	+1	-1	+1	+1	-1	+1	-1	-1	174
8.	0.528	2.86	0.20	+1	+1	+1	+1	+1	+1	+1	+1	330
9.	0.403	2.38	0.15	+1	0	0	0	0	0	0	0	200
10.	0.403	2.38	0.15	+1	0	0	0	0	0	0	0	195
11.	0.403	2.38	0.15	+1	0	0	0	0	0	0	0	190
12.	0.403	2.38	0.15	+1	0	0	0	0	0	0	0	198

After the performed experiment it is necessary to evaluate the single generic dispersions, i.e. to evaluate the experiment homogeneity in order to determine the difference of the obtained numerical values [1,4]. In this regard, Cohran's criteria for dispersion homogeneity evaluation is applied (specification in the literature [1]) with a form

$$K_h = \frac{max\, S_j^2}{\sum_{j=1}^{N} S_j^2} \le K_t\left(f_j, N\right) \quad (2)$$

where f_j is degree of freedom ($f_j = n - 1 = 3$), n_j is repetition number in the pattern ($n_j = n_0 = 4$), K_t is table values for Cohran's criteria, $S_j^2 = \frac{1}{n_j - 1}\sum_{i=1}^{3}\left(y_{ji} - \bar{y}_j\right)^2$ is pattern variance, $\sum_{j=1}^{N} S_j^2 = \frac{\sum_{j=1}^{N}\sum_{i=1}^{n}(y_{ji} - \bar{y}_j)^2}{\sum_{j=1}^{N}(n_j - 1)}$ is complete experiment variance.

With the measuring repetition performed was only in dots 9, 10, 11, and 12 tests, and after that obtained values for the variables S_j^2, i, $\sum_{j=1}^{N} S_j^2$ in were obtained the form (2), then it is $K_h = 0.583$.

According to the data for $K_t\left(f_j = 3, N = n_0\right) = 0.781.$ [1], with the elected statistic value ($P = 0.99$) and with the applied condition given in the form (2) it is

$$K_h = 0.583 \le K_t\left(3,4\right) = 0.781 \quad (3)$$

According to the form (3), Cohran's criteria condition given in the form (2) is achieved. The experimental result dispersion is homogenous and further modelling with data obtained by the experiment can be carried out.

5. Processing of the experiment results

The mathematical model of the process is given in the form (1), in order to be accepted, it is necessary to calculate and evaluate the regression coefficient signification of the model (b_i) and to examine the model adequacy. For the conditions of test repetitions in the central dot of the orthogonal plan, the regression coefficient values of the model are obtained by applying the following forms

$$b_0 = \frac{1}{N}\sum_{j=1}^{N} X_{0j}y_j = \frac{1}{N}\left(\sum_{j=1}^{N-n_0} X_{0j}y_j + \sum_{j=n_0}^{N} X_{0j}y_{0j}\right) \quad (4)$$

$$b_i = \frac{1}{N - n_0}\sum_{j=1}^{N} X_{ij}y_j, \text{ for } i = 1, 2, \ldots, k \quad (5)$$

$$b_{im} = \frac{1}{N - n_0}\sum_{j=1}^{N} X_{ij}X_{mj}y_j, \text{ for } 1 \le i < m \le k \quad (6)$$

where n_0 is number of repeated experiments in the central dot, y_{0j} is experiment results in the central plan dot, y_j is experiment results.

According to this, applying the forms (4), (5), (6) and a Table 2, the values of regression coefficient b_i are: $b_0 = 208.17$; $b_1 = 66.125$; $b_2 = 21.625$; $b_3 = 18.625$; $b_{12} = 7.875$; $b_{13} = 1.875$; $b_{23} = -2.625$; $b_{123} = 2.125$.

In addition to the obtained coefficient values, it is necessary to determine their signification in function model in the form (1), i. e. their single influence on model accuracy [1]. For the estimation of the coefficient signification Fisher's criteria is applied, i. e. F-test with the form

$$F_{ri} = \frac{S_{bi}^2}{S_0^2} \geq F_t(f_{bi}, f_2) = F_t(1, f_0),\ \text{for } i = 1, 2, \ldots, k \quad (7)$$

where are

error evaluation of coefficient:

$$S_{b0}^2 = \frac{Nb_0^2}{f_{b0}},\ S_{bi}^2 = \frac{(N-n_0)b_i^2}{f_{bi}} \quad (8)$$

$$F_{r0} = \frac{Nb_0^2}{S_0^2},\ F_{ri} = \frac{(N-n_0)b_i^2}{S_0^2},\ \text{for } i = 1, 2, \ldots, k \quad (9)$$

error evaluation in central dot of the test plan:

$$S_0^2 = \frac{\sum_{j=1}^{n_0}\left(y_{0j} - \bar{y}_0\right)^2}{f_0} \quad (10)$$

arithmetic mean of the value measurement result y_{0j} in zero point plan:

$$\bar{y}_0 = \frac{\sum_{j=1}^{n_0} y_{0j}}{n_0} \quad (11)$$

$f_{b0} = f_{b1} = \ldots f_{bi} = f_{bk} = 1$ – degree of freedom of the model coefficient, $f_0 = n_0 - 1 = 3$ – degree of freedom in central dot plan.

With applying the specified forms from (7) to (11), and with the additional value calculating, the estimation of the coefficient signification is shown in Table 3, where the 'Verification' is in the line, and by applying the symbol ¨*¨, their signification is determined, and those are the coefficients b_0, b_1, b_2, b_3, b_{12}.

The coefficients b_i, not significant for further process are excluded from the mathematical model in the form (1), and the new model form is obtained

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_{12}X_1X_2$$

Table 3

Coefficient signification (b_i)

Coeficients b_i b_{ij}	Degree of freedom f_i	Square S_{bi}^2	Calculate values, F-test $F_{ri} = \frac{S_{bi}^2}{S_0^2}$	Table value $F_t(f_1,f_2) = F_t(1,3)$	Verification
$b_0 = 208.17$	$f_0 = 1$	$S_{b0}^2 = N b_0^2 = 520001.999$	$F_{r0} = 27489.044$	10.1	*
$b_1 = 66.125$	$f_1 = 1$	$S_{b1}^2 = (N-n_0)b_1^2 = 34980.125$	$F_{r1} = 1849.166$	10.1	*
$b_2 = 21.625$	$f_2 = 1$	$S_{b2}^2 = (N-n_0)b_2^2 = 3741.125$	$F_{r2} = 197.768$	10.1	*
$b_3 = 18.625$	$f_3 = 1$	$S_{b3}^2 = (N-n_0)b_3^2 = 2775.125$	$F_{r3} = 146.702$	10.1	*
$b_{12} = 7.875$	$f_{12} = 1$	$S_{b12}^2 = (N-n_0)b_{12}^2 = 496.125$	$F_{r12} = 26.626$	10.1	*
$b_{13} = 1.875$	$f_{13} = 1$	$S_{b13}^2 = (N-n_0)b_{13}^2 = 28.125$	$F_{r13} = 1.487$	10.1	
$b_{23} = -2.625$	$f_{23} = 1$	$S_{b23}^2 = (N-n_0)b_{23}^2 = 55.125$	$F_{r23} = 2.914$	10.1	
$b_{123} = 2.125$	$f_{123} = 1$	$S_{b123}^2 = (N-n_0)b_{123}^2 = 36.125$	$F_{r123} = 1.909$	10.1	

Final mathematical model with the significant coefficient values b_i, now has the form

$$Y = 208.17 + 66.125X_1 + 21.625X_2 + +18.625X_3 + 7.875X_1X_2 \quad (12)$$

After defining the model function F_i, the next step in the experiment result processing is adequacy determination of the obtained model and calculation of the multiple regression coefficient R as an additional adequacy criteria.

In general case, the adequacy of the obtained mathematical model is evaluated by the comparison of the experimentally obtained values y_j^E and calculating values y_j^R obtained from the model, where the adequacy condition is determined by the F-criteria

$$F_a = \frac{S_a^2}{S_0^2} \leq F_t\left(f_a, f_0\right),\ \textit{for}\ S_a^2 > S_0^2 \quad (13)$$

or

$$F_a = \frac{S_0^2}{S_a^2} \leq F_t\left(f_0, f_a\right),\ \text{for}\ S_0^2 > S_a^2 \quad (14)$$

where are

adequacy dispersion:

$$S_a^2 = \frac{\sum_{j=1}^{N}\left(y_j^E - y_j^R\right)^2 - \sum_{j=1}^{n_0}\left(y_{0j} - \bar{y}_0\right)^2}{f_a} \qquad (15)$$

$f_a = N - k - 1 - f_0 = 12 - 3 - 1 - 3 = 5$, degree of freedom relating to the dispersion adequacy,

$F_t\left(f_a, f_0\right) = F_t\left(5{,}3\right) = 28.2$ – table value of F-criteria, square value

$$\sum_{j=1}^{n_0}\left(y_{0j} - \bar{y}_0\right)^2 = S_0 = S_0^2\ f_0 \qquad (16)$$

When the form values are calculated (13), (15) and (16), F-test criteria has the value

$$F_a = 11.04 \le F_t\left(5{,}3\right) = 28.2 \qquad (17)$$

which presents that the obtained mathematical model (12) adequately describes the deep drawing force F_i .

The second criteria for the mathematical model election is the multiple regression coefficient R, the value of is determined according to the form

$$R = \sqrt{1 - \frac{\sum_{j=1}^{N}\left(y_j^E - y_j^R\right)^2}{\sum_{j=1}^{N}\left(y_j^E - \bar{y}^E\right)^2}} \qquad (18)$$

The coefficient value $R = 0.99$ is obtained by applying data from Table 4 and the term (18).

Hence, in both cases of the adequacy model evaluation (Fisher's criteria and multiple regression coefficient R), the results show that the obtained mathematical model in coded form (12), adequately describes the deep drawing process with wall-thickness thinning, i.e the deep drawing force F_i.

Table 4

Calculating values for R

Number of experiments	Physical values of parameters			Coded values of parameters				y_j^E	y_j^R	$\left(y_j^E - \bar{y}^E\right)^2$	$\left(y_j^E - y_j^R\right)^2$
	Ψ	s_1	μ	X_0	X_1	X_2	X_3				
1.	0.278	1.90	0.10	+1	-1	-1	-1	113	109.7	9063.04	11.09
2.	0.528	1.90	0.10	+1	+1	-1	-1	230	226.2	475.24	14.44
3.	0.278	2.86	0.10	+1	-1	+1	-1	150	137.2	3387.24	163.84
4.	0.528	2.86	0.10	+1	+1	+1	-1	290	285.2	6691.24	23.04
5.	0.278	1.90	0.20	+1	-1	-1	+1	156	146.9	2724.84	82.81
6.	0.528	1.90	0.20	+1	+1	-1	+1	272	263.4	4070.44	73.96
7.	0.278	2.86	0.20	+1	-1	+1	+1	174	174.4	1169.64	0.16
8.	0.528	2.86	0.20	+1	+1	+1	+1	330	322.4	14835.24	57.76
9.	0.403	2.38	0.15	+1	0	0	0	200	208.2	67.24	67.24
10.	0.403	2.38	0.15	+1	0	0	0	195	208.2	174.24	174.24
11.	0.403	2.38	0.15	+1	0	0	0	190	208.2	331.24	331.24
12.	0.403	2.38	0.15	+1	0	0	0	198	208.2	104.04	104.04
									$\sum$	43093.70	1103.90

6. The experimental results presentation

After achieved evaluation of the mathematical model adequacy, it is necessary to compile the term (12) compile from the coded form with variables (X_i) to the physical model, with the real process parameters (ψ, s_1, μ), instead of the coded.

With the applied forms for transformation according to [1], and with the additional recalculating the equations are obtained

$$\left.\begin{aligned} &X_1 = 8\psi - 3.224;\ \ X_2 = 2.083 s_1 - 4.96;\\ &X_3 = 20\mu - 3;\\ &X_1 X_2 = 16.664\psi s_1 - 6.716 s_1 - 39.68\psi + 16 \end{aligned}\right\} \qquad (19)$$

with the insertion into the form (12) the final output function form is obtained $Y = F_i$:

$$F_i = -42.16 + 216.52\psi - 7.84 s_1 + 372.5\mu + 131.23\psi s_1 \qquad (20)$$

The form (20) presents the final mathematical model in physical form for the deep drawing force with wall-thickness thinning F_i, which was the main goal of this paper.

By the model simulation in the graphic package ¨Graphic 2. 9¨ with the parameter value variation ψ, s_1, μ, the 3D diagrams are obtained: presented in Figs. 5-7.

Thus, the insight into the character and the drawing force values F_i enables not only defining the gauge of the input parameter values (ψ, s_1, μ), but also out of them.

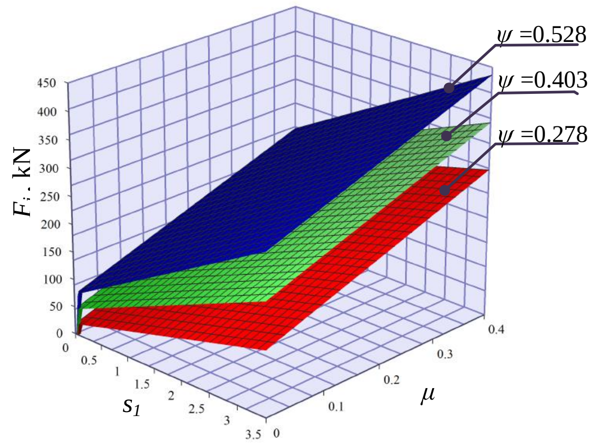


Fig. 5 The deep-drawing force $F_i = f(\psi)$

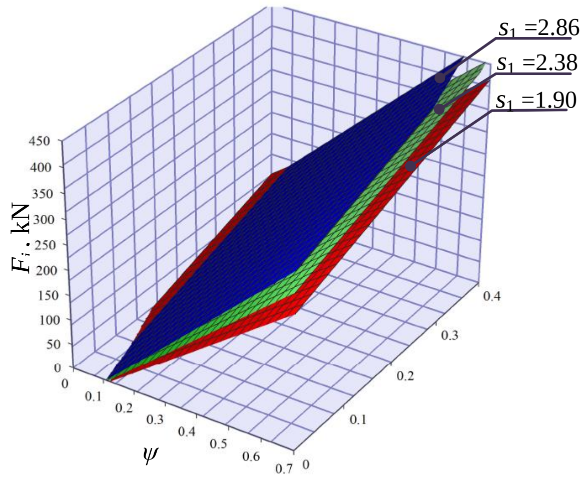


Fig. 6 The deep-drawing force $F_i = f(s_1)$

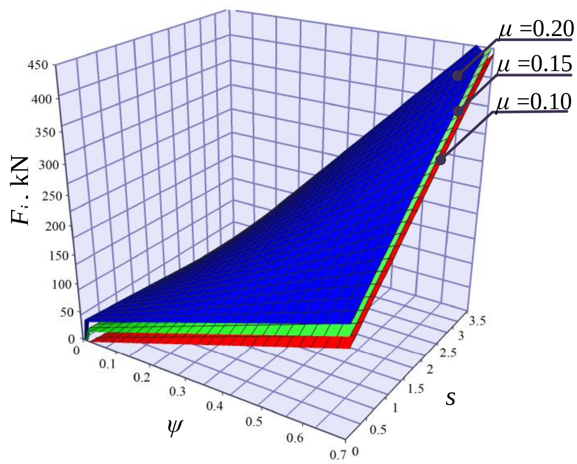


Fig. 7 The deep-drawing force $F_i = f(\mu)$

7. Conclusion

The research results and analysis showed:

- the obtained mathematical model for the drawing force F_i adequately describes the process, which is explicitly confirmed with the adequacy criteria (Fisher's criteria and criteria of the multiple coefficient regression *R*);
- the drawing-force function is completely linear;
- with the simulation of the obtained mathematical model for the drawing force F_i *i*n the programme package ¨Graphic 2.9¨, the monitoring of the character and drawing force values is enabled, not only for the defined gauge values of the input parameters ψ, s_1, μ, but also out of them.

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MATEMATINIS MODELIAVIMAS IR GILIOJO IŠTEMPIMO JĖGOS IMITAVIMAS ATLIEKANT SIENELĖS SUPLONINIMO BANDYMĄ

R e z i u m ė

Straipsnyje aprašomas sienelės suploninimo giliuoju ištempimu procesas. Naudojant atitinkamą matavimo įrangą, atlikus proceso analizę ir pritaikius stochastinį modeliavimo metodą, buvo sudarytas giliojo ištempimo jėgos, išreikštos funkcija $F_i = (\psi, s_1, \mu)$, fizinis matematinis modelis.

Straipsnyje pritaikytas modeliavimo ir imitavimo metodas taip pat gali būti taikomas kitų giliojo ištempimo sienelės suploninimo technologinių procesų parametrams nustatyti naudojant aukštesnės eilės matematinius modelius. Tai leis sumažinti detalių ištempimo jėgą ir užtikrinti reikalingą kokybę, sunaudojant kuo mažiau energijos.

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MATHEMATIC MODELLING AND THE DEEP DRAWING FORCE SIMULATION WITH THE WALL THICKNESS THINNING EXPERIMENT APPLICATION

S u m m a r y

This paper presents the research based on experiment performing of the deep-drawing process with wall-thickness thinning. The corresponding measurement equipment was applied, where with the process analysis and with the applied stochastic modelling method, the physical mathematical model for the deep-drawing force is obtained, in the form: $F_i = f(\psi, s_1, \mu)$.

The applied modelling and simulation methods in this paper, can also be used for defining the optimal values of bigger number of the technological process parameters of deep drawing, with wall-thickness thinning, by the applied higher order mathematical model. Principally, it would relate to the reduction of the drawing force with the obtained products, according to the demanded quality along with the minimum energy consumption.

Keywords: mathematic modelling, deep drawing force, wall thickness thinning.

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