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Elastic analysis of pressurized thick hollow cylindrical shells with clamped-clamped ends

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1. Introduction

There are many engineering applications of commonly used structures, such as rods, annular disks, cylindrical and spherical shells when subjected to different loading and boundary conditions. Deformation and stress analysis of thick-walled cylinders subjected to either internal or external pressure is an important topic in engineering because of their rigorous applications in industry as well as in daily life. For this reason, the classical problem of a pressurized thick hollow cylinder has been the topic of a variety of theoretical investigations.

Naghdi and Cooper [1], assuming the cross shear effect, formulated the shear deformation theory (SDT). Mirsky and Hermann [2], derived the solution of thick cylindrical shells of homogenous and isotropic materials, using the first shear deformation theory (FSDT). Greenspon [3], opted to make a comparison between the findings regarding the different solutions obtained for cylindrical shells. Making use of Mirsky-Hermann theory and the finite difference method (FDM), Ziv and Perl [4] obtained the vibration response for semi-long cylindrical shells. Using SDT and Frobenius series, Suzuki *et. al.* [5], obtained the solution of the free vibration of cylindrical shells with variable thickness, and Takashaki *et. al.* [6] obtained the same solution for conical shells. A paper was published by Kang and Leissa [7] where equations of motion and energy functional were derived for a three-dimensional coordinate system. The field equations are utilized to express such energy functional in terms of displacement components . The stress state of two-layer hollow bars in which they are exposed to axial load is analyzed [8]. The layers are made of isotropic, homogeneous, linearly elastic material, and they are considered as concentric cylinders.

Assuming that the material properties vary nonlinearly in the radial direction and the Poisson's ratio is constant, Zamani Nejad and Rahimi [9] obtained closed form solutions for one-dimensional steady-state thermal stresses in a rotating functionally graded pressurized thick-walled hollow circular cylinder. A complete and consistent 3D set of field equations has been developed by tensor analysis to characterize the behavior of FGM thick shells of revolution with arbitrary curvature and variable thickness along the meridional direction [10]. Using the analytical method for stress strain state of two-layer mechanically inhomogeneous pipe subjected to internal pressure at elastic plastic loading are analyzed by Brazenas and Vaiciulis [11].

This article presents the general method of derivation and the analysis of an internally pressurized thick-walled cylinder shell with clamped-clamped ends, taking into account the effect of shear stresses and strains.

2. Classical theory

The plane elasticity theory (PET) or classical theory is based on the assumption that the straight lines perpendicular to the central axis of the cylinder remain unchanged after loading and deformation. According to this theory, the deformations are axisymmetric and do not change along the longitudinal cylinder. In other words, the elements do not have any rotation, and the shear strain is assumed to be zero. Thus, equilibrium equations are independent of one another, and the coupling of the equations is deleted. Therefore,

$$\left.\begin{array}{l} u_r(r), \qquad u_x(x) \\ \gamma_{xz} = \dfrac{\partial u_r}{\partial x} + \dfrac{\partial u_x}{\partial r} \Rightarrow \tau_{xz} = 0 \end{array}\right\} \tag{1}$$

and

$$\left.\begin{array}{l} \dfrac{d\sigma_r}{dr} + \dfrac{\sigma_r - \sigma_\theta}{r} = 0 \\ \dfrac{d\sigma_x}{dx} = 0 \end{array}\right\} \tag{2}$$

The differential equation based on the Navier Solution is

$$\frac{d^2u_r}{dr^2} + \frac{1}{r}\frac{du_r}{dr} - \frac{1}{r^2}u_r = 0 \tag{3}$$

The solution of the Eq. (3) is

$$u_r(r) = C_1 r + \frac{C_2}{r} \tag{4}$$

This method is applicable in problems in which shear stresses and strains are considered zero. However, to solve the problems such as the following it is not possible to use the PET

$$\left.\begin{array}{l} u_r(r,x), \qquad u_x(r,x) \\ \gamma_{xz} = \dfrac{\partial u_r}{\partial x} + \dfrac{\partial u_x}{\partial r} \Rightarrow \tau_{xz} \neq 0. \end{array}\right\} \tag{5}$$

3. Shear deformation theory (SDT)

In SDT, the straight lines perpendicular to the central axis of the cylinder do not necessarily remain unchanged after loading and deformation, suggesting that the deformations are axial axisymmetric and change along the longitudinal cylinder. In other words, the elements have rotation, and the shear strain is not zero.

In Fig. 1, the location of a typical point m (r), within the shell element may be determined by R and z, as

$$r = R(x) + z \tag{6}$$

where R represents the distance of middle surface from the axial direction, and z is the distance of typical point from the middle surface.

In Eq. (6), x and z must be as follows

$$-\frac{h}{2} \le z \le \frac{h}{2}, \qquad 0 \le x \le L \tag{7}$$

where h and L are the thickness and the length of the cylinder.

$R(x)$ and inner and outer radii (r_i, r_o) of the cylinder are as follows

$$r_i = R - \frac{h}{2} = const., \qquad r_o = R + \frac{h}{2} = const. \tag{8}$$

Based on PET, the radial displacement of the cylinder is

$$u_r(r) = C_1(R+z) + \frac{C_2}{R+z} \tag{9}$$

Using the Taylor's expansion for $\left|\frac{z}{R}\right| < 1$,

$$u_r(r) = C_1(R+z) + \frac{C_2}{R}\left(1 - \frac{z}{R} + \frac{z^2}{R^2} + ...\right) \tag{10}$$

Thus,

$$u_r(r) = u_0 + u_1 z + u_2 z^2 + ... \tag{11}$$

According to Eq. (11), the radial displacement is written in the form of a polynomial function of z. When $z = 0$, it shows the displacement of the mid-plane.

The general axisymmetric displacement field (U_x, U_z), in the first-order Mirsky-Hermann's theory could be expressed on the basis of axial and radial displacements, as follows

$$\left.\begin{aligned} U_x(x,z) &= u(x) + \phi(x) z \\ U_\theta &= 0 \\ U_z(x,z) &= w(x) + \psi(x) z \end{aligned}\right\} \tag{12}$$

where $u(x)$ and $w(x)$ are the displacement components of the middle surface. Also, $\phi(x)$ and $\psi(x)$ are the functions used to determine the displacement field.

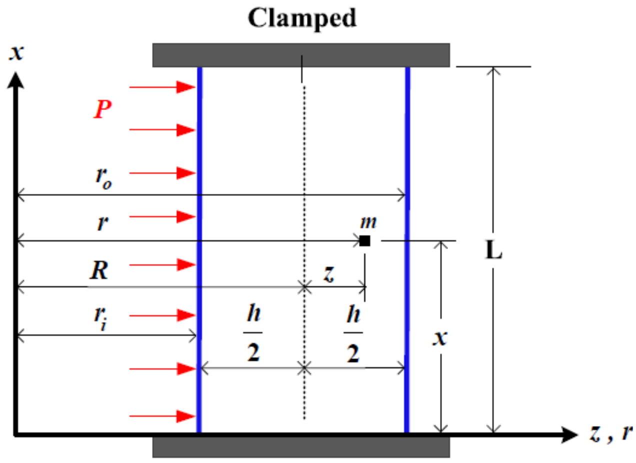

Fig. 1 Cross section of the thick cylinder with clamped-clamped ends

The strain-displacement relations in the cylindrical coordinates system are

$$\left.\begin{aligned} \varepsilon_x &= \frac{\partial U_x}{\partial x} = \frac{du}{dx} + \frac{d\phi}{dx} z \\ \varepsilon_\theta &= \frac{U_z}{r} = \frac{w}{R+z} + \frac{\psi}{R+z} z \\ \varepsilon_z &= \frac{\partial U_z}{\partial z} = \psi \\ \gamma_{xz} &= \frac{\partial U_x}{\partial z} + \frac{\partial U_z}{\partial x} = \\ &= \left(\phi + \frac{dw}{dx}\right) + \frac{d\psi}{dx} z \end{aligned}\right\} \tag{13}$$

In addition, the stresses on the basis of constitutive equations for homogenous and isotropic materials are as follows

$$\left.\begin{aligned} \sigma_i &= \lambda E\left[(1-\upsilon)\varepsilon_i + \upsilon(\varepsilon_j + \varepsilon_k)\right], \qquad i \ne j \ne k \\ \tau_{xz} &= \frac{1-2\upsilon}{2} \lambda E \gamma_{xz} \\ \lambda &= \frac{1}{(1+\upsilon)(1-2\upsilon)} \end{aligned}\right\} \tag{14}$$

where σ_i and ε_i are the stresses and strains in the axial (x), circumferential (θ), and radial (z) directions; υ and E are Poisson's ratio and Young's modulus, respectively.

The normal forces (N_x, N_θ, N_z), shear force (Q_x), bending moments (M_x, M_θ, M_z), and the torsional moment (M_{xz}) in terms of stress resultants are

$$\begin{Bmatrix} N_x \\ N_\theta \\ N_z \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x\left(1+\frac{z}{R}\right) \\ \sigma_\theta \\ \sigma_z\left(1+\frac{z}{R}\right) \end{Bmatrix} dz \tag{15}$$

$$\begin{Bmatrix} M_x \\ M_\theta \\ M_z \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x\left(1+\frac{z}{R}\right) \\ \sigma_\theta \\ \sigma_z\left(1+\frac{z}{R}\right) \end{Bmatrix} zdz \tag{16}$$

$$Q_x = \int_{-h/2}^{h/2} \tau_{xz}\left(1+\frac{z}{R}\right)dz \tag{17}$$

$$M_{xz} = \int_{-h/2}^{h/2} \tau_{xz}\left(1+\frac{z}{R}\right)zdz \tag{18}$$

On the basis of the principle of virtual work, the variations of strain energy are equal to the variations of the external work as follows

$$\delta U = \delta W \tag{19}$$

where U is the total strain energy of the elastic body and W is the total external work due to internal pressure. The strain energy is

$$\left.\begin{aligned} &U = \iiint_V U^* dV \\ &dV = rdrd\theta dx = (R+z)dxd\theta dz \\ &U^* = \frac{1}{2}\left(\sigma_x\varepsilon_x + \sigma_\theta\varepsilon_\theta + \sigma_z\varepsilon_z + \tau_{xz}\gamma_{xz}\right) \end{aligned}\right\} \tag{20}$$

and the external work is

$$\left.\begin{aligned} &W = \iint_S \left(\vec{f}.\vec{u}\right)dS \\ &dS = r_i d\theta dx = \left(R-\frac{h}{2}\right)d\theta dx \\ &\vec{f}.\vec{u} = PU_z \end{aligned}\right\} \tag{21}$$

where P is internal pressure .

The variation of the strain energy is

$$\delta U = R\int_0^{2\pi}\int_0^L\int_{-h/2}^{h/2} \delta U^*\left(1+\frac{z}{R}\right)dzdxd\theta \tag{22}$$

The resulting Eq. (22) will be

$$\frac{\delta U}{2\pi} = R\int_0^L\int_{-h/2}^{h/2}\left(\sigma_x\delta\varepsilon_x + \sigma_\theta\delta\varepsilon_\theta + \right. \\ \left. +\sigma_z\delta\varepsilon_z + \tau_{xz}\delta\gamma_{xz}\right)\left(1+\frac{z}{R}\right)dzdx \tag{23}$$

and the variation of the external work is

$$\delta W = \int_0^{2\pi}\int_0^L \left(P\delta U_z\right)\left(R-\frac{h}{2}\right)dxd\theta \tag{24}$$

The resulting Eq. (24) will be

$$\frac{\delta W}{2\pi} = P\int_0^L \delta U_z\left(R-\frac{h}{2}\right)dx \tag{25}$$

Substituting Eqs. (13) and (14) into Eq. (19), and drawing upon calculus of variation and the virtual work principle, we will have

$$\left.\begin{aligned} &R\frac{dN_x}{dx} = 0 \\ &R\frac{dM_x}{dx} - RQ_x = 0 \\ &R\frac{dQ_x}{dx} - N_\theta = \\ &= -P\left(R-\frac{h}{2}\right) \\ &R\frac{dM_{xz}}{dx} - M_\theta - RN_z = \\ &= P\frac{h}{2}\left(R-\frac{h}{2}\right) \end{aligned}\right\} \tag{26}$$

and the boundary conditions are

$$R\left[N_x\delta u + M_x\delta\phi + Q_x\delta w + M_{xz}\delta\psi\right]_0^L = 0 \tag{27}$$

Eq. (27) states the boundary conditions which must exist at the two ends of cylinder. In order to solve the set of differential equations (26), forces and moments need to be expressed in terms of the components of displacement field, using Eqs. (15) to (18). Thus, set of differential equations (26) could be derived as follows

$$\left.\begin{aligned} &\left[\bar{A}_1\right]\frac{d^2}{dx^2}\{\bar{y}\} + \left[\bar{A}_2\right]\frac{d}{dx}\{\bar{y}\} + \\ &+\left[\bar{A}_3\right]\{\bar{y}\} = \{\bar{F}\} \\ &\{\bar{y}\} = \{u \quad \phi \quad w \quad \psi\}^T \end{aligned}\right\} \tag{28}$$

The set of equations (28) is a set of linear non-homogenous equations with constant coefficients. The coefficients matrices $\left[\bar{A}_i\right]_{4\times4}$, and force vector $\{\bar{F}\}$ are

$$\left[\bar{A}_1\right] = \begin{bmatrix} (1-\upsilon)Rh & (1-\upsilon)\frac{h^3}{12} & 0 & 0 \\ (1-\upsilon)\frac{h^3}{12} & (1-\upsilon)\frac{Rh^3}{12} & 0 & 0 \\ 0 & 0 & \mu Rh & \mu\frac{h^3}{12} \\ 0 & 0 & \mu\frac{h^3}{12} & \mu\frac{Rh^3}{12} \end{bmatrix} \tag{29}$$

$$\left[\bar{A}_2\right]=\begin{bmatrix} 0 & 0 & \upsilon h & \upsilon Rh \\ 0 & 0 & -\mu Rh & -(\mu-2\upsilon)\dfrac{h^3}{12} \\ -\upsilon h & \mu Rh & 0 & 0 \\ -\upsilon Rh & (\mu-2\upsilon)\dfrac{h^3}{12} & 0 & 0 \end{bmatrix} \quad (30)$$

$$\left[\bar{A}_3\right]=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\mu Rh & 0 & 0 \\ 0 & 0 & -(1-\upsilon)\alpha & -h+(1-\upsilon)R\alpha \\ 0 & 0 & -h+(1-\upsilon)R\alpha & -(1-\upsilon)R^2\alpha \end{bmatrix} \quad (31)$$

$$\{\bar{F}\}=\frac{P}{\lambda E}\left(R-\frac{h}{2}\right)\left\{0 \quad 0 \quad -1 \quad \frac{h}{2}\right\}^T \quad (32)$$

The parameters μ and α are as follows

$$\left.\begin{aligned} \mu &= K\left(\frac{1-2\upsilon}{2}\right) \\ \alpha &= ln\left(\frac{R+\frac{h}{2}}{R-\frac{h}{2}}\right) \end{aligned}\right\} \quad (33)$$

where K is the shear correction factor that is embedded in the shear stress term.

It is assumed that in the static state, for cylindrical shells $K=5/6$ [12].

$\left[\bar{A}_3\right]$ is irreversible and its reverse is needed in the next calculations. In order to make $\left[\bar{A}_3\right]^{-1}$, the first equation in the set of Eqs. (26) is integrated.

$$RN_x = C_0 \quad (34)$$

In Eq. (28), it is apparent that u does not exist, but du/dx does. Taking du/dx as v,

$$u=\int v dx + C_7 \quad (35)$$

Thus, set of differential Eqs. (28) could be derived as follows

$$\left.\begin{aligned} &[A_1]\frac{d^2}{dx^2}\{y\}+[A_2]\frac{d}{dx}\{y\}+ \\ &+[A_3]\{y\}=\{F\} \\ &\{y\}=\begin{Bmatrix} v \\ \phi \\ w \\ \psi \end{Bmatrix} \end{aligned}\right\} \quad (36)$$

where, the coefficients matrices $[A_i]_{4\times4}$, and force vector $\{F\}$ are

$$[A_1]=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & (1-\upsilon)\dfrac{Rh^3}{12} & 0 & 0 \\ 0 & 0 & \mu Rh & \mu\dfrac{h^3}{12} \\ 0 & 0 & \mu\dfrac{h^3}{12} & \mu\dfrac{Rh^3}{12} \end{bmatrix} \quad (37)$$

$$[A_2]=\begin{bmatrix} 0 & (1-\upsilon)\dfrac{h^3}{12} & 0 & 0 \\ (1-\upsilon)\dfrac{h^3}{12} & 0 & -\mu Rh & -(\mu-2\upsilon)\dfrac{h^3}{12} \\ 0 & \mu Rh & 0 & 0 \\ 0 & (\mu-2\upsilon)\dfrac{h^3}{12} & 0 & 0 \end{bmatrix} \quad (38)$$

$$[A_3]=\begin{bmatrix} (1-\upsilon)Rh & 0 & \upsilon h & \upsilon Rh \\ 0 & -\mu Rh & 0 & 0 \\ -\upsilon h & 0 & -(1-\upsilon)\alpha & -h+(1-\upsilon)R\alpha \\ -\upsilon Rh & 0 & -h+(1-\upsilon)R\alpha & -(1-\upsilon)R^2\alpha \end{bmatrix} \quad (39)$$

$$\{F\}=\frac{1}{\lambda E}\begin{Bmatrix} C_0 \\ 0 \\ -P\left(R-\dfrac{h}{2}\right) \\ P\dfrac{h}{2}\left(R-\dfrac{h}{2}\right) \end{Bmatrix} \quad (40)$$

The equations (36) are the set of nonhomogenous linear differential equations with constant coefficients.

4. Analytical solution

Defining the differential operator $P(D)$, Eq. (36) is written as

$$\left.\begin{aligned} &P(D)=[A_1]D^2+[A_2]D+[A_3] \\ &D=\frac{d}{dx} \quad , \quad D^2=\frac{d^2}{dx^2} \end{aligned}\right\} \quad (41)$$

Thus

$$P(D)\{y\}=\{F\} \tag{42}$$

The differential Eq. (42) has the general solution including general solution for homogeneous case $\{y\}_g$ and particular solution $\{y\}_p$, as follows

$$\{y\}=\{y\}_g+\{y\}_p \tag{43}$$

For the general solution for homogeneous case, $\{y\}_g=\{V\}e^{mx}$ is substituted in $P(D)\{y\}=0$.

$$e^{mx}\left[m^2[A_1]+m[A_2]+[A_3]\right]\{V\}=\{0\} \tag{44}$$

Given that $e^{mx}\neq 0$, the following eigenvalue problem is created.

$$\left[m^2[A_1]+m[A_2]+[A_3]\right]\{V\}=\{0\} \tag{45}$$

To obtain the eigenvalues, the determinant of coefficients must be considered zero.

$$\left|m^2[A_1]+m[A_2]+[A_3]\right|=0 \tag{46}$$

Thus,

$$\begin{vmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{vmatrix}=0 \tag{47}$$

where

$$\left.\begin{aligned}
&A_{11}=(1-\upsilon)Rh \\
&A_{12}=A_{21}=m(1-\upsilon)\frac{h^3}{12} \\
&A_{13}=-A_{31}=(1/R)A_{14}=-(1/R)A_{41}=\upsilon h \\
&A_{22}=m^2(1-\upsilon)\frac{Rh^3}{12}-\mu Rh \\
&A_{23}=-A_{32}=-m\mu Rh \\
&A_{24}=-A_{42}=-m(\mu-2\upsilon)\frac{h^3}{12} \\
&A_{33}=m^2\mu Rh-(1-\upsilon)\alpha \\
&A_{34}=A_{43}=m^2\mu\frac{h^3}{12}-h+(1-\upsilon)R\alpha \\
&A_{44}=m^2\mu\frac{Rh^3}{12}-(1-\upsilon)R^2\alpha
\end{aligned}\right\} \tag{48}$$

The result of the determinant above is a six-order polynomial which is a function of m, the solution of which is a 6 eigenvalues m_i. The eigenvalues are 3 pairs of conjugated root. Substituting the calculated eigenvalues in Eq. (45), the corresponding eigenvectors $\{V\}_i$ are obtained. Therefore, the general solution for homogeneous Eq. (42) is

$$\{y\}_g=\sum_{i=1}^{6}C_i\{V\}_i e^{m_i x} \tag{49}$$

The constants C_1 to C_6 are obtained by applying boundary conditions. Given that $\{F\}$ is comprised of constant parameters, the particular solution is obtained as follows.

$$\{y\}_p=[A_3]^{-1}\{F\} \tag{50}$$

Therefore, the general solution for Eq. (42) is

$$\{y\}=\sum_{i=1}^{6}C_i\{V\}_i e^{m_i x}+[A_3]^{-1}\{F\} \tag{51}$$

In general, the problem consists of 8 unknown values of C_i, including C_0 (Eq. 34), C_1 to C_6 (Eq. 51), and C_7 (Eq. 35). By applying boundary conditions, one can obtain the constants of C_i.

Given that the two ends of the cylinder are clamped-clamped, then

$$\begin{Bmatrix} u \\ \phi \\ w \\ \psi \end{Bmatrix}_{x=0}=\begin{Bmatrix} u \\ \phi \\ w \\ \psi \end{Bmatrix}_{x=L}=\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{52}$$

Based on PET in the plane strain state, radial stress, circumferential stress and radial displacement are as follows [13]:

$$\sigma_r=\frac{P}{k^2-1}\left[1-\frac{k^2}{(\bar{r})^2}\right] \tag{53}$$

$$\sigma_\theta=\frac{P}{k^2-1}\left[1+\frac{k^2}{(\bar{r})^2}\right] \tag{54}$$

$$\sigma_x=\frac{2\upsilon P}{k^2-1} \tag{55}$$

$$u_r=\frac{Pr_i\bar{r}(1+\upsilon)}{E(k^2-1)}\left[1-2\upsilon+\frac{k^2}{(\bar{r})^2}\right] \tag{56}$$

where $\bar{r}=r/r_i$ and $k=r_o/r_i$.

5. Results and discussion

In this section, we present the results for a homogeneous and isotropic thick hollow cylindrical shell with $r_i=40$ mm, $h=20$ mm and $L=800$ mm. The Young's modulus and Poisson's ratio, respectively, have the values of $E=200$ GPa and $\upsilon=0.3$. The applied internal pressure is 80 MPa.

The analytical solution is carried out by writing the program in MAPLE 12. The numerical solution is obtained through finite element method (FEM).

Table

Numerical results of the different solutions

	σ_r, MPa	σ_θ, MPa	σ_x, MPa	u_r, mm
FSDT	-27.56	155.62	36.24	0.03826
FEM	-28.16	156.11	36.22	0.03843
PET	-28.16	156.16	38.40	0.03827

Table presents the results of the different solutions for the middle of the cylinder ($x = L/2$) and mid-layer ($z = 0$). The results suggest that in points further away from the boundary it is possible to make use of PET.

Fig. 2 shows the distribution of axial displacement at different layers. At points away from the boundaries, axial displacement does not show significant differences in different layers, while at points near the boundaries, the reverse holds true. The distribution of radial displacement at different layers is plotted in Fig. 3. The radial displacement at points away from the boundaries depends on radius and length. According to Figs. 2 and 3, the greatest axial and radial displacement occurs in the internal surface $(z = -h/2)$. Distribution of circumferential stress in different layers is shown in Fig. 4. The circumferential

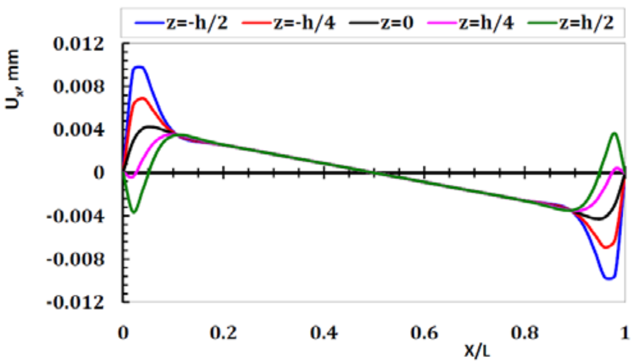

Fig. 2 Axial displacement distribution in different layers

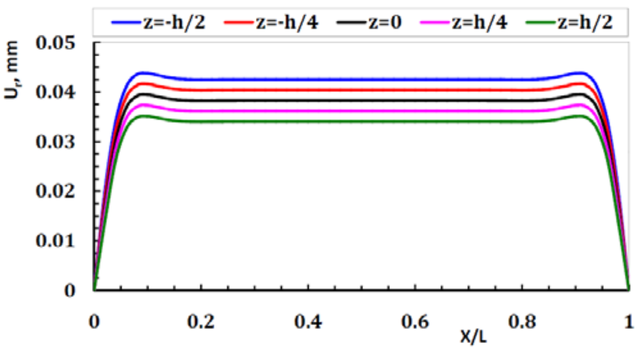

Fig. 3 Radial displacement distribution in different layers

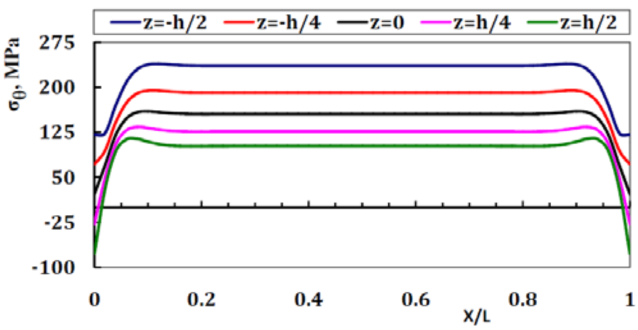

Fig. 4 Circumferential stress distribution in different layers

stress at all points depends on radius and length. The circumferential stress at layers close to the external surface at points near boundary is negative, and at other layers positive. The greatest circumferential stress occurs in the internal surface $(z = -h/2)$.

Fig. 5 shows the distribution of shear stress at different layers. The shear stress at points away from the boundaries at different layers is the same and trivial. However, at points near the boundaries, the stress is significant, especially in the internal surface, which is the greatest.

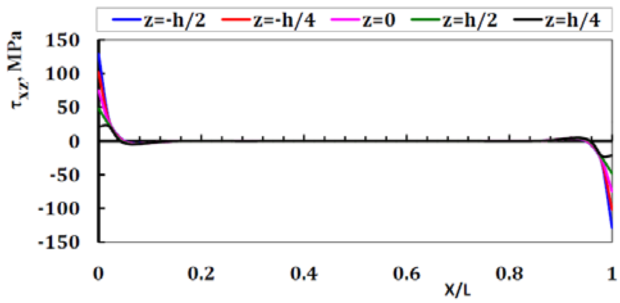

Fig. 5 Shear stress distribution in different layers

In the Figs. 6-10, displacement and stress distributions are obtained using FSDT are compared with the solutions of FEM and are presented in the form of graphs.

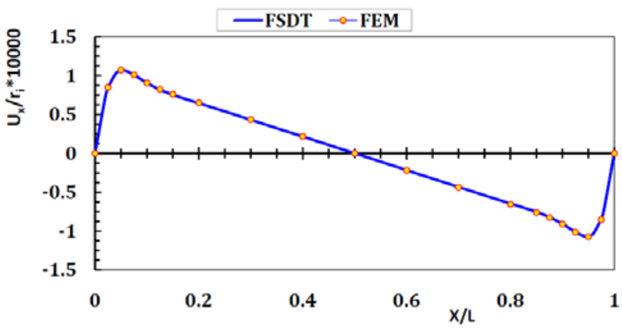

Fig. 6 Axial displacement distribution in internal layer

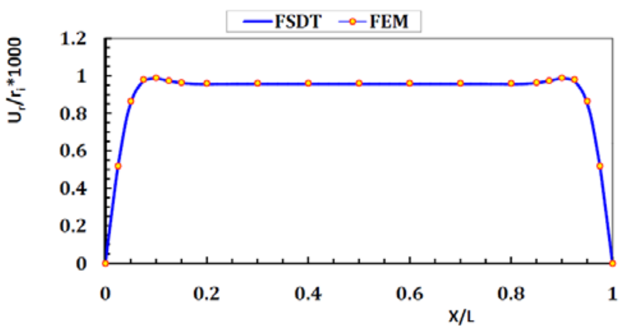

Fig. 7 Radial displacement distribution in internal layer

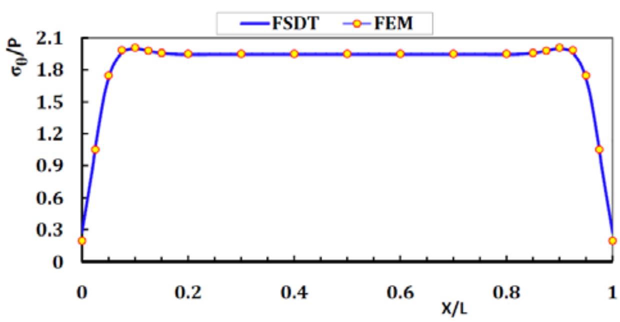

Fig. 8 Circumferential stress distribution in internal layer

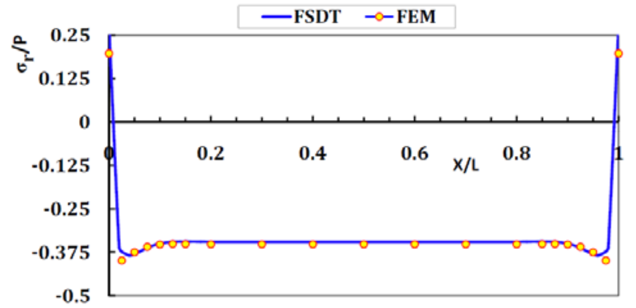

Fig. 9 Radial stress distribution in internal layer

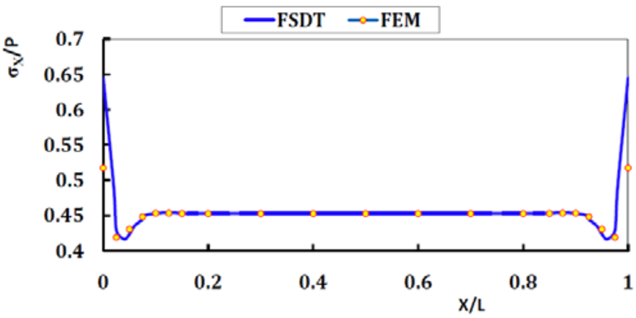

Fig. 10 Axial stress distribution in middle layer

6. Conclusions

In the present study, the advantages as well as the disadvantages of the PET (Lame´ solution) for hollow thick-walled cylindrical shells with different boundary conditions at the two ends were indicated. Regarding the problems which could not be solved through PET, the solution based on the FSDT is suggested. At the boundary areas (20 percent of the length of the cylinder) of a thick-walled cylinder with clamped-clamped ends, having constant thickness and uniform pressure, given that displacements and stresses are dependent on radius and length, use cannot be made of PET, and FSDT must be used. In the areas further away from the boundaries (80% of the length of the cylinder), as the displacements and stresses along the cylinder remain constant and dependent on radius, PET ought to be used. The shear stress in boundary areas cannot be ignored, but in areas further away from the boundaries, it can be ignored. Therefore, the PET can be used, provided that the shear strain is zero. The maximum displacements and stresses in all the areas of the cylinder occur on the internal surface. The analytical solutions and the solutions carried out through the FEM show good agreement.

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PLONASIENIŲ HERMETINIŲ CILINDRŲ SU UŽSANDARINTAIS GALAIS TAMPRIOJI ANALIZĖ

R e z i u m ė

Šiame straipsnyje pasiūlytos diferencialinės lygtys, išreiškiančios homogeninių ir izotropinių asimetrinių plonasienių cilindrų, kurių galuose sudarytos skirtingos ribinės sąlygos, elgsena, taikant pirmos eilės šlyties deformacijos teoriją ir virtualaus darbo principą. Naudojantis šiomis sąlygomis buvo išspręsta sistema nehomogeninių tiesinių diferencialinių lygčių, sudarytų skaičiuoti cilindrui su užsandarintais galais, ištirtas krūvio ir atramų efektas įtempiams ir poslinkiams. Problemai spręsti taip pat buvo taikomas baigtinių elementų metodas, o gauti rezultatai palyginti su analitiniais.

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ELASTIC ANALYSIS OF PRESSURIZED THICK HOLLOW CYLINDRICAL SHELLS WITH CLAMPED-CLAMPED ENDS

S u m m a r y

In this paper, the differential equations governing the homogenous and isotropic axisymmetric thick-walled cylinders with different boundary conditions at the two ends were generally derived, making use of first-order shear deformation theory (FSDT) and the virtual work principle. Following that, the set of nonhomogenous linear differential equations for the cylinder with clamped-clamped ends was solved, and the effect of loading and supports on the stresses and displacements was investigated. The problem was also solved, using the finite element method (FEM), the results of which were compared with those of the analytical method

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УПРУГИЙ АНАЛИЗ ГЕРМЕТИЧНЫХ ТОНКОСТЕННЫХ ЦИЛИНДРОВ С ЗАЩЕМЛЕННЫМИ КОНЦАМИ

Р е з ю м е

В статье предложены дифференциальные уравнения, описывающие поведение гомогенных и изотропных ассиметричных тонкостенных цилиндров с различными предельными условиями в концах используя теорию деформации сдвига и принцип виртуальной работы. При использовании перечисленных условий была решена система негомогенных линейных дифференциальных уравнений для цилиндра с защемленными концами и исследовано влияние нагрузки и опор на напряжение и перемещение. Для решения проблемы также использовался метод конечных элементов, а полученные результаты сопоставлены с аналитическими.

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