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Classic finite elements for simulation of piezoelectric smart structures

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1. Introduction

Several authors focused their efforts on building adapted FEM that take into account the electromechanical coupling of piezoelectric materials. The first finite elements were two-dimensional or three-dimensional piezoelectric homogeneous plates. They designed by introducing the degrees of freedom (DOF) to take into consideration the potential [1, 2] and ultrasounds transducers (plates [3], shells [4]). Within this framework, several computation softwares were marketed (ANSYS, ABAQUS and ATTILA).

The insertion of these materials known as laminated piezoelectric or composite have required other approaches and other models of finite elements. Several techniques were adapted to take into consideration the layers or the piezoelectric fibers in these structures. We present here a non exhaustive list of work in this field [5 - 19].

The two-dimensional models of thin structures are defined directly on both surfaces (neutral fiber) of the structures. It is not easy to impose the boundary conditions of the Dirichlet type on both surfaces of the structure. In order to solve this problem, several techniques of modeling were used.

1.2 . Three-dimensional formulation

Several authors used three-dimensional isoparametric FEM to study the behavior of a thin piezoelectric structure. To improve their results, some authors [5 - 7] introduced incompatible modes by the incorporation of the DOF. These DOF associated with the potential are sometimes eliminated by condensation before the assembly. For the laminated structures, Kellers and Chang [7], Heyliger and Ramirez [8] have used a structure which is equivalent to multilayer. They used a three-dimensional geometry to take into consideration the potentials imposed on both surfaces and considered that transverse displacement is constant or linear into the thickness. In our work, we consider a two-dimensional shape.

Recently, Lučinskis and Mažeika [20, 21], have used the three-dimensional FEM) to study the behaviour of a beam piezoactuator or beam with piezoelectric wafer. Lučinskis considered SOLID5 ANSYS. He has modele of the electrodes with grouping surface nodes of the FEM model, on which is applied an electric voltage. Mažeika has employed a three dimensional FEM (ANSYS) to study vibration shapes and trajectories of contact point motion through the modal and harmonic response analysis.

1.3 . Degenerated three-dimensional formulation

Tzou and Ye [3] developed a quadratic triangular finite elements in the plan coupled to a linear interpolation in the thickness of one laminated. Thus they considered a three-dimensional shape, while keeping a two-dimensional modeling of the piezoelectric behaviour.

1.4 . Two-dimensional formulation with degrees of electric freedom

For a piezoelectric shell, Lammering [9] proposed a quadrilateral element with 7 DOF nodales, which 5 are mechanical traditional DOF (three translations and two rotations (model of Reisner Midlin)) and 2 electrical DOF which represent the potentials imposed on the upper and lower surfaces. Suleman and Venkayya [10] developed a quadrangle element for a composite plate whose electrical DOF are supposed to be constant in each piezoelectric layer of multilayer and linear in the thickness of the total structure. A similar representation of the electric potential was used by Shen [11] to model a smart piezoelectric beam. The finite elements (2D or 3D) used by this latter were the same as those used by Suleman for each layer (actuator, plate or beam, sensor) of the smart structure.

1.5 Mixed formulations of three-dimensional and two-dimensional

Recently, Kim [12] has proposed a mixed approach 2D/3D. The 3D elements were employed for the piezoelectric layers to take into account the imposed potentials and the 2D elements for the elastic layers. The connections between 3D and 2D finite elements are done using the methods of transition.

1.6 Formulation by decoupling the mechanical and electric effects

a. Some authors have used the electric behaviour of a piezoelectric structure with a nodal representation of the potential that it can be uncoupled from the mechanical behaviour using “compensation by condensation” of the electrical DOF to the profit of displacement [2, 5 - 7].
b. An iterative technique suggested by Gaudenzi and Bathe [13, 14] consists in uncoupling the electrical and mechanical behaviour. Firstly, they resolved the indirect problem by considering the known potential. Secondly they determined the electric parameters, essentially the potential by using the law of the direct behaviour. This approach is valid for the nonlinear behaviour of a structure, as well.
c. Recent applications of the piezoelectric finite elements were directed towards techniques of post processing. Precisely by recommended simplifications, they consist

on using standard finite elements to calculate mechanical displacements, then to deduce the electric entities (potential, load) by post processing. [15, 22 - 24] have neglected electrostatic energy in the formulation of a thin plate with a 4 nodes element (resp. a 9 nodes element) of a thick plate. They supposed that the potential and the electric field are constant. The induced load is calculated in post processing and the potential is introduced as external forces. An element of plate with 9 nodes was also proposed by Shah, Joshi and Chan [16] for the actuator aspect only. Baz and Ro [17] have presented an element of a plate with 9 DOF per node for hybrid passive/active control of the plate vibrations. Polit and Bruant [25] have presented new decoupled finite elements of multilayered piezoelectric plates. For taking into account the shear-bending plate, they have used 8 node quadrilateral finite elements with 5 DOF per node. For the electrical part, they have used independently other finite elements for electric potential, linear and quadratic into thickness.

Prakah-Asante and Craig [18] have used, to analyze the pure bending of an Euler-Bernoulli beam, the finite standard elements by modifying flexional rigidity.

A more complete synthesis on the piezoelectric finite elements was carried out by Benjeddou [26].

In our previous work [27-30], we have proved, with asymptotic methods (mathematical techniques), that we obtained a two-dimensional model from a three-dimensional model. The two-dimensional model presents implicit decoupling of electrical and mechanical behaviour of thin piezoelectric plate. We have modified the standard flexional rigidity. We have introduced the same modification used by "compensation by condensation" of the electrical DOF. Our modification is natural without introducing the electrical DOF.

Here, in our work, we confirm that we do not need specific finite elements for thin piezoelectric plates in particular, and for thin piezoelectric laminated plates in general. The standard finite elements of composite plates are sufficient for the simulation of the behaviour of piezoelectric plates or laminated plates with piezoelectric films or wafers. So as to correctly calculate mechanical displacement, it is necessary to introduce the modified elasticity matrix and the external force induced by the imposed electric potential. The modification of elasticity matrix allows taking into account implicitly the induced electric behaviour. The electric variables (electric potential, electric load) are calculated in post processing in the same way as the mechanical strains and stresses. However, it is better to create a "macro" for the calculation of the modified elasticity matrix and the force induced by the imposed potential.

2. Theoretical three-dimensional formulation of piezoelectric smart structure

The constitutive equation of the piezoelectric field can be expressed as

$$\sigma_i^{p,l} = A_{ijkl}^{p,l}\varepsilon_{kl} - e_{k,ik}^{p,l}E_k \tag{1}$$

$$D_i^{p,l} = e_{i,kl}^{p,l}\varepsilon_{kl} - c_{ik}^{p,l}E_k \tag{2}$$

where $\sigma_{ij}^{p,l}$ and $D_i^{p,l}$ are stresses and electric displacements of piezoelectric wafer (p) of layer (l), respectively. $A_{ijkl}^{p,l}$ are elastic constants, $e_{k,ij}^{p,l}$ are piezoelectric constants and $c_{ik}^{p,l}$ are dielectric constants, ε_{kl} is the strain vector and E_k is the electric field related to the electric potential V.

For metal elastic layers, the constitutive Eqs. (1) and (2) can be described by the following form

$$\sigma_{ij}^{l} = A_{ijkl}^{l}\varepsilon_{kl} \tag{3}$$

$$D_i^{l} = -\,c_{ik}^{l}E_k \tag{4}$$

Based on the principle of variation, the mechanical and electrical response of piezoelectric smart structure can be expressed as

$$\left.\begin{array}{c}
\int_{\Omega}\sigma_{ij}\left(u,\phi\right)\varepsilon_{ij}(v)\,d\Omega+\int_{\bigcup_p^N\Omega_p}D_i(u,\phi)E_i\left(\psi\right)d\Omega_p=\int_{\Omega}f_iv_i\,d\Omega+\int_{\Sigma^{\pm}}g_iv_i\,d\Sigma^{\pm}+\int_{\bigcup_p^N\Gamma^p}q\psi\,d\Gamma^p \\
\forall\left(v,\psi\right)\in V_0\left(\Omega\right) \\
V_0\left(\Omega\right)=\left\{\left(v,\psi\right)\in\left[H^1\left(\Omega\right)\right]^3\times H^1\left(\bigcup_p^N\Omega_p\right),v=0/\Gamma,\psi=0/\bigcup_p^N\Sigma\right.
\end{array}\right\} \tag{5}$$

where $\Omega=\Omega_e\bigcup_{p=1}^{N}\Omega_p$ is the volume including piezoelectric composites and wafers, Ω_P is the volume of each piezoelectric wafer p, $\Sigma_P^{\pm}$ and $\bigcup_p^N\Gamma_p$ represent respectively the surfaces on which the electric potential is imposed and the surfaces on which the electric load is imposed, $\Sigma^{\pm}$ and Γ represent respectively the surfaces on which the surface forces are imposed and the surface on which the displacement is imposed, f_i, g_i are the body and surface forces respectively and q is the electric load per unit area of the piezoelectric actuators, in practice $q=0$.

3. Theoretical two-dimensional formulation of the action and the detection of piezoelectric smart structure

To be able to act or detect with precision displacements of bending and membrane of a smart plate, the symmetrical provision of films or piezoelectric layers is necessary. Orthotropy and symmetry of piezoelectric films or layers make it possible to uncouple the bending and the membrane. To simplify the study, we consider in our work a three-layer plate, knowing that the study can easily be generalized for a multilayer.

We have proved in our previous work [29-32] that bending of the symmetrical smart orthotropic structure is made up of piezoelectric layers of wafers and defined by

$$\begin{cases} \text{Find } \xi_3 \in W_{KL} \\ \int_\omega \tilde{M}_{\alpha\beta}\partial_{\alpha\beta}\eta_3 = \sum_{s=1}^{M}\left[\int_{\omega_p^s} z_{2p+1}^s\left[\bar{e}_{3,\alpha\beta}\underline{\left(V_{2p+1}^s - V_{-(2p+1)}^s\right)}\right]\right] + \int_\omega p_3\eta_3 + m_\alpha\partial_\alpha\eta_3 \qquad \forall\,\eta \in W_{KL} \end{cases} \tag{6}$$

with

$$\bar{M}_{\alpha\beta} = \left[\sum_{s=1}^{M}\frac{1}{6}\sum_{p=1}^{N}\left[12h_{2p+1}^s {z_{2p+1}^s}^2\bar{a}_{\alpha\beta\gamma\delta} + {h_{2p+1}^s}^3\left(\bar{a}_{\alpha\beta\gamma\delta} + \underbrace{\frac{\bar{e}_{3,\alpha\beta}\bar{e}_{3,\gamma\delta}}{\bar{c}_{33}}}_{\text{terms of modification of elasticity matrix}}\right)\right]\chi_{2p+1}^s + \sum_{p=0}^{N-1}\frac{1}{6}\left(12h_{2p}^s\left(z_{2p}^s\right)^2 + \left(h_{2p}^s\right)^3\right)\bar{b}_{\alpha\beta\gamma\delta}\right]\partial_{\gamma\delta}\xi_3 \tag{7}$$

where h_{2p+1}^s (respectively z_{2p+1}^s) is the thickness (respectively the position) of the wafers layer $2p+1$; M is the total number of piezoelectric wafers; N is the total number of piezoelectric layers; χ_{2p+1}^s is a function indicator related to each wafer, in particular for the piezoelectric layer

$$\chi_{2p+1}^s = \chi_{2p+1} = \chi_\omega$$

For the piezoelectric actuator (converse effect), non null potentials are imposed on both surfaces of the wafer, then $\left(V_{2p+1}^s - V_{-(2p+1)}^s\right) \neq 0$.

For the piezoelectric sensor (direct effect), the wafer is a shorted-circuit, we can then measure the electric load induced by its flexion

$$Q_{2p+1}^s = \int_{\omega_p^s}\left[h_{2p+1}^s z_{2p+1}^s \bar{e}_{3,\gamma\delta}\partial_{\gamma\delta}\xi_3\right] \tag{8}$$

4. Numerical formulation of the action and the detection of piezoelectric smart structure

4.1 . Construction of standard finite elements

The approximation methods to determine the plane or the transverse displacement are similar. In practice, flexion is considered as an issue. We have thus focused our work on flexion simulation of a three-layer. This three-layer is made up of a metal plate (steel) and two films or two layers of piezoelectric distribution of wafers (ceramic PZT). This study is perfectly transposable for the simulation of the multilayer with membrane (extension).

For this purpose, we have used finite elements semi- C^1 traditional quadrangle (the continuity of the normal derivative is not assured). The arrow takes $w = \xi_3$, the elementary arrow w_q is defined by

$$w_q \approx \sum_{i=1}^{3}\sum_{j=1}^{4} p_i^j\left(x_q\right)\bar{w}^i\left(a_j^q\right) \tag{9}$$

with

$$p_i\left(a_j\right) = \delta_{ij}\,, \qquad \bar{w} = \left\{w, \partial_1 w, \partial_2 w\right\} \tag{10}$$

By operating the substitution of Eq. (9) into Eqs. (6) and (8), we obtain the discrete formulation of the behaviour of a three-layer smart plate

$$[K]\{W\} = \{L\} \tag{11}$$

with

$$\begin{cases} [K] = \sum_{k=1}^{N}\omega_{k,t}\left[B\left(b_{t,k}\right)\right]^T[A]\left[B\left(b_{t,k}\right)\right]\det(J) \\ \{L\} = \sum_{k=1}^{N}\omega_{k,t}\left[B\left(b_{t,k}\right)\right]^T[e]E + \left\{N\left(b_{t,k}\right)\right\}F + \sum_{r=1}^{R}\omega'_{s,r}\left\{N\left(b_{s,r}\right)\right\}G \end{cases} \tag{12}$$

where $\{\omega_{k,t}\},\{b_{k,t}\}$ are respectively weights and coordinates of the points of Gauss of the quadrangle t; $\{\omega_{s,r}\},\{b_{s,r}\}$ are respectively weights and coordinates of the points of Gauss of segment edges $[e] = \left(e_{3,\gamma\delta}z_{2p+1}^s\right)$.

For the sensor, we determine the value of the elementary electric load, from mechanical displacements in post treatment by the following relationship

$$q_f^t = \sum_{k=1}^{N}\omega_{k,t}\left[B\left(b_{t,k}\right)\right]^T[e]\{W\}\,det\left(J\left(b_{t,k}\right)\right) \tag{13}$$

The contribution of this study consists in the use of a code of standard finite elements that takes into account:

- modification of the mechanical law of piezoelectric material the behavior in order to take implicitly into consideration the electromechanical coupling;
- introduction of electric stress which represents the

indirect piezoelectric effect;
- computation a posteriori of the load induced by the direct piezoelectric effect.

A standard code of finite elements that introduces at least an orthotropic elasticity matrix is sufficient to determine the mechanical displacement of a piezoelectric smart structure. It introduces the modified elasticity matrix, which takes into account the electromechanical coupling. The development of macro is enough to calculate the electric load a posteriori from obtained displacements.

4.2 . Validation

To validate our finite elements, we have implemented an analytic solution of Pagano [31] for a simply supported three-layer. Then we have compared the analytical results and numerical results.

In this purpose, we have considered a steel plate, covered on both surfaces, by ceramic films. The three-layer is simply supported. It is a square shape of side L. L is equal to 100 mm. Thickness of the metal plate is h_m, and thickness of each film is h_p. The structure is subject to sinusoidal potentials ($V=V_0 \sin(p x) \sin(p y)$) on a surface and to potential of opposed sign on the other surface of each film.

We will compare the average values of the exact arrow (Pagano) of the three-layer ceramics-steel-ceramics W_e (Fig. 1), in the center of average surface and the arrow W_{kl} obtained by the numeric computation at the same point. The ratio of these two quantities W_{kl}/W_e tends towards the unit when ($h = hm + 2hp$) tends towards zero. Fig. 1 represents the evolution of this reduced arrow ratio W_{kl}/W_e according to the logarithm of the thickness.

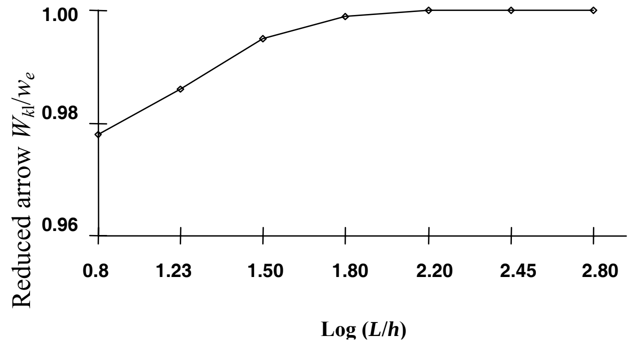

Fig. 1 Comparison of the analytic displacement w_e and the numercic displacement W_{kl} for elastic and piezoelectric multi-layers plate

4.3 . Sensitivity of the model to piezoelectric thickness of films or wafers

To determine the sensitivity results compared to the modified elasticity matrix of the piezoelectric films, we have evaluated the error of the arrow of three-layer compared to the thickness films. We have kept thickness of the steel layer constant. This error is about 1%, if thicknesses of the films are lower than 2% of the thickness of steel layer. This proves why some studies have neglected the electromechanical coupling for the piezoelectric actuator. The error made by traditional computation (without modifying the law of behaviour) on the values of the arrow is negligible for some configurations

Table 1

Electromechanical characteristics of PZT-ceramic film and steel layer

Steel	PZT
$E = 21.e^{10}$ N/m^2 $v = 0.31$	$a_{11} = a_{22} = 1322.10$ N/m^2 $a_{13} = a_{23} = 837.10$ N/m^2 $a_{44} = a_{55} = 295.10$ N/m^2 $a_{66} = 250.10$ N/m^2
	$e_{13} = e_{23} = -4.3$ C/m^2 $e_{33} = 16.7$ C/m^2 $e_{15} = e_{24} = 11.8$ C/m^2
	$c_{11} = c_{22} = 1.440\, c_0$ $c_{12} = 837\, c_0$ $c_0 = 8.8410^{12}$F/m^2

4.4 . Experiment devices: Plate with two piezoceramic wafers

We present also a simulation of an experimental device made up of a steel plate and two piezoelectric wafers (Fig. 2). The numerical results are similar to the experimental results (Table2). This experiment consists on measuring the arrow of neutral fiber of the steel structure under the action of two ceramics PZT wafers of on its two surfaces.

The numerical and experimental results (Table 2) are proven to be of the same order of magnitude (with 18% error between experimental and numerical displacement). We note that we have used the face values of the characteristic coefficients of material provided by the manufacturers.

Table 2

Comparison numerical and experimental results at three measure points (Fig. 2)

Position of measure	Experimental Displacement	Numerical displacement
150 mm	440 µm	350 µm
200 mm	720 µm	600 µm

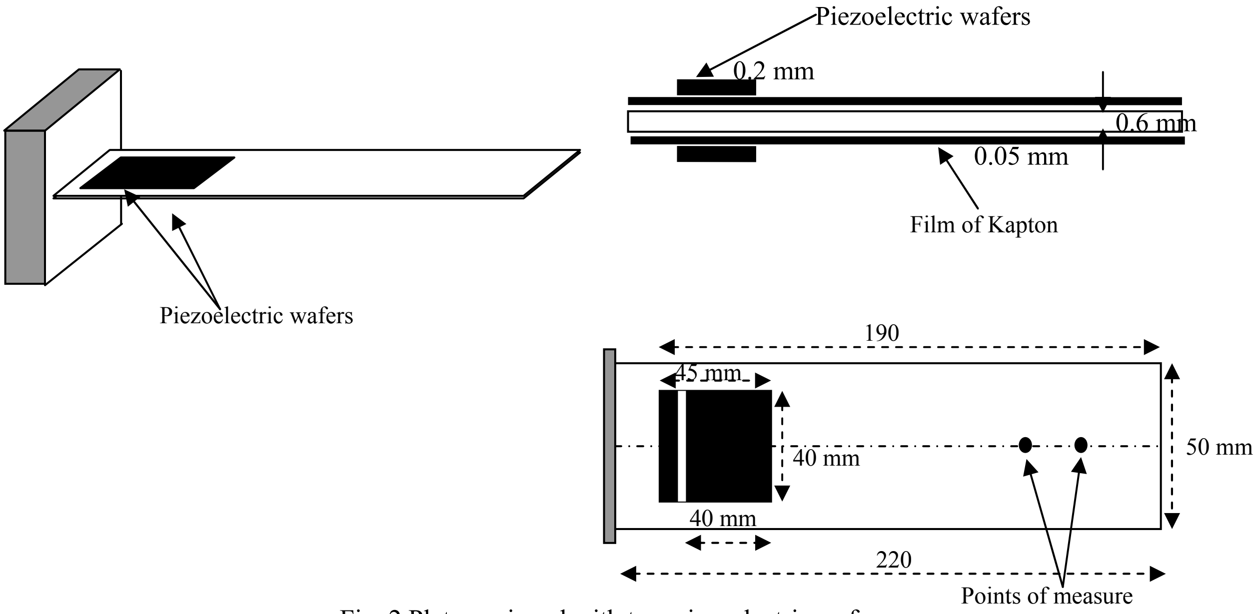

Fig. 2 Plate equipped with two piezoelectric wafers

4.5 Deformation simulation of the plate on the action of a eight piezoelectric wafers

We present in this section, the simulation of the steel plate deformation under the action of two eight wafers stuck in a symmetrical manner (Fig. 3). The wafers are applied alternatively by a tension ΔV of 100 V and - 100 V.

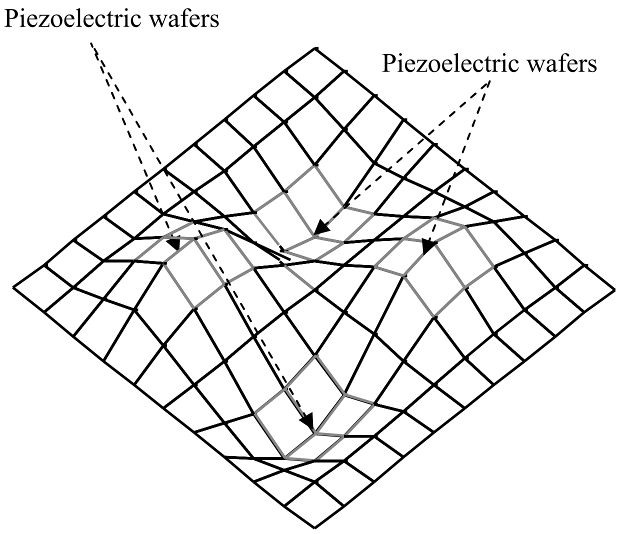

Fig. 3 Deformation shape of an eight wafers actuated plate: Qualitative eigenmode obtained "the bending mode" vibration (mode 7)

The deformation obtained is quantitatively identical to the eigenmode of the bending vibration (mode 7) of the structure. This shows that we can generate an anti-vibration in order to control vibrations of the structure, then to stabilize it.

5. Results and discussion

We built finite elements for multilayer with piezoelectric films or wafers, whose DOF are mechanical only, contrary to many authors who added a node with its potential DOF. We avoided any step of computation by condensation. We have so reduced sensibly to the computation times (that could be expensive). The terms used for our modification of elasticity matrix are the same as the terms used for "compensation by condensation" method. In our case, these terms are globally introduced into the elasticity matrix. In case the "compensation by condensation" method, these same terms are locally introduced into elementary stiffness. We have determined the displacement, by modification of the elasticity matrix and by introducing external forces induced by imposed potential. For the role "actuator" of piezoelectric films or wafers, the induced potential is considered in the modified elasticity matrix. For their role "sensor", we have determined firstly the displacement then we have calculated the induced load from the obtained displacement. In this case, the potential imposed is null and the charge is measured [13].

Table 3

Comparison between our model and model "compensation by condensation"

	Our model	Model "compensation by condensation"
Mathematic formulation	$\bar{M}_{\alpha\beta} = \left[\bar{A}_{\alpha\beta\gamma\delta} + \frac{\bar{e}_{3,\alpha\beta} * \bar{e}_{3,\delta\gamma}}{c_{33}}\right]\partial_{\gamma\delta} w$	$\bar{M}_{\alpha\beta} = \bar{A}_{\alpha\beta\gamma\delta}\partial_{\gamma\delta} w - \bar{e}_{3,\alpha\beta} E_3$ $q = \bar{e}_{3,\delta\gamma} + c_{33} E_3$
Finite element model	 Only DOF displacement is considered	 DOF Displacement and DOF potentien in midle point
Computation	Actuator: $[K]\{W\} = \{F\}$	Actuator: $\begin{bmatrix} K_{uu} & K_{u\varphi} \\ K_{u\varphi} & K_{\varphi\varphi} \end{bmatrix} \begin{Bmatrix} W \\ \varphi \end{Bmatrix} = \begin{Bmatrix} F \\ Q \end{Bmatrix}$ "compensation of potential by condensation" He considered $\{Q\} = 0$, $\{\varphi\} = -[K_{\varphi\varphi}]^{-1}[K_{u\varphi}]\{W\}$ $[K] = [K_{uu}] - [K_{u\varphi}][K_{\varphi\varphi}]^{-1}[K_{u\varphi}]$ $[K]\{W\} = \{L\}$

It Tables 3 $\{W\}$ is displacement vector , $\{\varphi\}$ is potential vector, $\{F\}$ is force vector and $\{Q\}$ is electric charge vector, $[K]$ is mechanic elementary rigidity, $[K_{uu}]$ is elementarymechanic rigidity, $[K_{u\varphi}]$ is elementary coupled electro-mechanic rigidity and $[K_{\varphi\varphi}]$ is elementary electric rigidity.

6. Conclusion

Two-dimensional variational formulation was used for multi-layer smart plates with distributed piezo-electric wafers, using the mathematical asymptotic analysis. The modification of rigidity matrix permitted to take into account implicitly the potential induced and the problem is considered as purely mechanical one. This potential induced, is often neglected in literature.

A finite element model was proposed to solve both actuator and sensor problems without using the electrical DOFs. A standard of four-node rectangular is considered. It permits to introduce the elastic matrix modified and the external forces induced by imposed potential. It was implemented to compute mechanical nodal displacements. There are then used to compute "unknowns" electrical.

We have proved that the results of the standard finite elements are of the same order as the experimental and analytical ones.

We have shown that the model without the modified elasticity matrix is valid when the piezoelectric films are very thin (1% of thickness of the steel layer). This is due to the fact of neglecting the bending of piezoelectric films.

Also, we have shown that if the thickness film is 1% higher than the thickness of non piezoelectric layer, the numerical results of the model with modified elasticity matrix, are of the same order as the experimental and analytical ones.

We note that a computation of retiming can improve the results. We used the face values of the coefficients characteristic of material provided by the manufacturers. It remains to using this model for the computation of active control could improve the results.

It is important also to be able to determine the positions of a given number of wafers for active control or control health of a piezoelectric smart structure.

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PJEZOELEKTRINIŲ ŠIUOLAIKINIŲ KONSTRUKCIJŲ IMITAVIMAS KLASIKINIAIS BAIGTINIAIS ELEMENTAIS

R e z i u m ė

Įvairūs autoriai kuria patobulintus arba specialius baigtinius elementus pjezoelektrinių medžiagų elektromechaniniam ryšiui ir elektriniams laisvės laipsniams nustatyti. Kiti autoriai laisvės laipsnių potencialą pašalina „kompensuojant kondensacija“. Taip sutaupoma skaičiavimo laiko.

Šis darbas rodo, kad klasikinis baigtinių elementų metodas (be elektrinio DOF) gali būti taikomas kuriant plonas šiuolaikines pjezoelektrines konstrukcijas. Klasikinio baigtinių elementų (tik mechaninių) metodo pakanka, tačiau turime korektiškai įvertinti modifikuotą elastingumo matricą ir panaudoto potencialo indukuotą elektros jėgą. Elektros krūvis yra apskaičiuotas pagal nustatytą poslinkį. Parodyta, kad modifikuoti lygiai yra tokie patys kaip ir taikant „kompensuojant kondensacija“ metodą. Todėl negaištamas skaičiavimo laikas „kompensavimo kondensacija“ tyrimams ir skaičiavimai gerokai paspartėja.

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CLASSIC FINITE ELEMENTS FOR SIMULATION OF PIEZOELECTRIC SMART STRUCTURES

S u m m a r y

Several authors have focused on the construction of the adapted or specific finite elements to take into account the electromechanical coupling of piezoelectric materials, in adding the electric degrees of freedom DOF. The computation times can be expensive in this case. Some authors have eliminated the electric potential DOF by “compensation by condensation”. It can justify a gain computation time.

This work shows that the classic finite elements (without electrical DOF) are sufficient for solving a thin piezoelectric smart structure. The classic finite elements (mechanical only) are sufficient, we must just introduce correctly the modified elasticity matrix and the electric force induced from imposed potential. The electric load is computed from the obtained displacement. We have shown that the modified terms are the same as those used in the “compensation by condensation” method. We have avoided the computation times by “compensation by condensation” steps and sensibly reduced the computation time.

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ИМИТАЦИЯ СОВРЕМЕННЫХ ПЬЕЗОЭЛЕКТРИЧЕСКИХ КОНСТРУКЦИЙ ПРИ ПОМОЩИ КЛАССИЧЕСКИХ КОНЕЧНЫХ ЭЛЕМЕНТОВ

Р е з ю м е

Некоторые авторы создают усовершенствованные или специальные конечные элементы для оценки электромеханических связей и электрических степеней свободы пьезоэлектрических материалов. В этом случае снижается расчетное время. Другие авторы исключают потенциал степеней свободы, применяя „компенсацию конденсацией“. Это оправдается сокращением расчетного времени.

Настоящая работа показывает, что достаточно классических конечных элементов (без электрического DOF) для создания тонких современных пьезоэлектрических конструкций. Классических конечных элементов (только механических) достаточно, но при этом необходимо использовать модифицированную матрицу упругости и электрическую силу индуцированную премененным потенциалом. Электрическая нагрузка опеделяется по полученному перемещению. Показано, что модифицированные уровни являются такими же, которые расчитаны при использовании метода »компенсации конденсацией». Таким образом можно избежать расчетного времени для изучения „компенсации конденсацией“ и значительно сократить общее время расчета.

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