

# Dynamics of Motion of the System «Electric Motor – Mechanism of Class IV with Dwell Driven Links» During Acceleration

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**crossref** <http://dx.doi.org/10.5755/j01.mech.23.6.16261>

## 1. Introduction

Since the 70's headed by Academician U.A. Dzholdasbekov Kazakh school of TMM (Theory of mechanisms and machines) was created, where first began to solve problems associated to the analysis of high classes mechanisms (MHC). For more than four decades' scientists-mechanics carried fundamental research in the creation of high classes mechanisms theory. In practice of mechanical engineering to carry out the dwell often applied cam mechanisms and mechanisms with curvilinear coulisse. Their disadvantage is that the law of motion of the output link is totally dependent on profile of the cam or the coulisse, change the cam when it is worn out leads to downtime of mechanism and tangible material losses. Because of friction in higher kinematic pairs and rapid wear of the cam profile and a roller driven link of the cam mechanism appear inaccuracies and dynamic errors in the implementation of the desired motion law. High classes mechanisms with dwell link have high reliability, durability and low material and energy consumption. The analyzed plane lever mechanisms of high classes (MHC) contain group of Assur with variable closed circuits. Because of the structural features the mechanisms of high classes have broad functional capabilities. Foreign publications on the dynamics of high classes mechanisms existing analogues for this article is limited. This mechanism can be used to replace the cam mechanisms.

In the monography of Academician U.A. Dzholdasbekov sets out the new graphic analytical, analytical and numerical methods for the analysis and synthesis of plane lever mechanisms. When solving various problems of the kinematics of high classes mechanisms used methods of replacing the driving link, instant centers of speeds [1].

In the paper the dynamic response of a DC separately excited motor coupled to a general four-bar linkage is investigated. The constraint equations are based on the velocity ratio equations rather than the commonly used loop equations [2]. The method of solving the problem we have used in this article to the above fourth class mechanism to determine the current, the angular velocity of motor and the velocity of mechanism links. Calculation of Lagrange multipliers will be shown in the second part of this article.

The article is considered the slider-crank mechanism driven by a servo motor for which the dynamics equations are obtained by using the new identification method based on real encoded genetic algorithm. In the work the corners and the speeds of links by numerical and experimental methods are obtained [3].

The article presents the kinematics model of slider-crank mechanism with a stepping motor that has been integrated using MATLAB Simulink software. The proposed method of proportional integral - differentiating regulation has been effective for position control of slider-crank mechanism with good accuracy. The graphs of motion of the mechanism slide are obtained [4].

In the article is considered the model of the mechanism with rigid links and the motor having an ideal characteristic. In the case of one motor and of one mechanism the dynamic model of an ideal machine can be described by equations  $\varphi=f(U)$ ,  $\theta=\Pi(\varphi)$ , of which are obtained the dependence of the motion law of the mechanism output link from the change law of the input parameter. Differential equations of motion of the motor electromechanical model obtained using Kirchhoff's and Newton's laws. According to the equation of the drive defined the angular velocity of motor [5].

This article describes the motion of one electro-mechanical system with feedback. Differential equations of motion of this system is in the form of Lagrange - Maxwell and Kirchhoff equations. Differential equations that allow to fully study the process of inhibition and disinhibition are made up, their solutions under certain initial values of electrical circuits currents and of rotation angles of the electromagnet anchor are obtained [6].

The article presents a unified dynamic modeling environment for the "differential drive - mobile robots" system. Dynamic model of the "differential drive - mobile robot" using the methods of Lagrange and Newton - Euler is constructed, it allows you to track the trajectory of motion of the system. It is also shown that both formulations are mathematically equivalent to their conformity [7].

In the book sets the basis of mathematical modeling of motion of the mechanical systems, mechanisms and machines. The methods of compiling the differential equations for mechanisms with rigid links, which can be represented in the form of one reduced mass, moving under the action of given force, are proposed and their general solutions are obtained [8].

The article considers the principle of the direct analogy of constructing a simple electric model of the simplest linear mechanical system with translational motion. Differential equations of electrical and mechanical circuit are obtained and the motor speed of mechanical system in the transitional process with the help of classical or operator method is defined [9].

The research of the dynamic model of single-acting crank pump with damless hydro turbine drive has been made in the present work. Compiled dynamic and

mathematical models of single-acting crank pump with damless hydro drive have been constructed. The graph of dependence between angular velocity and rotation angle of hydro turbine for different values of rotor radius and at different values of flow velocity of the watercourse have been constructed. Been obtained, that the average speed of rotation depends only on the hydro turbine parameters and on flow rate of the watercourse. The average angular velocity of hydro turbine in a steady state equal to the angular velocity of hydro turbine at idle and does not depend on the mode of operation of piston pumps [10].

## 2. Differential equations of motion of the system «electric motor – mechanism of class IV with dwell driven links»

Equivalent systems of different physical nature, described by ideal differential equations, allow us to establish a correspondence between the various physical quantities and parameters [1]. The equation of motion of a mechanical system with one degree of freedom, which is determined by the deviation of mass  $m$  from the equilibrium position, the vertical movement of the force  $p$ , the elastic restoring force, characterized by a stiffness  $c$ , and damping force of viscous friction with coefficient  $r$  (mechanical resistance) has the form:

$$m\ddot{\varphi} + r\dot{\varphi} + c\varphi = p. \quad (1)$$

The equation of the current  $i$ , permeates all elements of an electrical circuit consisting of serially connected elements - the source of EMF  $e$ , inductance  $L$ , resistance and the inverse capacitance  $S$  ( $S=C^{-1}$ ), is written as follows:

$$L \frac{di}{dt} + Ri + S \int i dt = e. \quad (2)$$

To obtain the equation of motion of the machine, the expression (1) should be considered in conjunction with equation (2). The problem of dynamics mechanisms with non-linear function of the position (especially in multimass systems, in systems that form branched and closed circuits) the expression of kinematic and potential energies through independent generalized coordinates leads to complex functional relations.

Necessity to express all the coordinates system through the generalized coordinates can be removed using a special form of Lagrange equations with «unnecessary» coordinates.

Lagrange equations in the presence of holonomic constraints are of the form [2]:

$$\frac{d}{dt} \left( \frac{dL}{dq_i} \right) - \frac{\partial L}{\partial q_i} - \sum_{j=1}^m \lambda_j \frac{\partial \Phi_j}{\partial q_i} = 0; \quad i=1, \dots, n, \quad (3)$$

$m$  constraint equations are as follows:  $\Phi_j(q_i, t) = 0$ ;  $j=1, \dots, m$ .

Euler-Lagrange equation with first order constraint  $\Phi_j(q_i, t) = 0$ ;  $j=1, \dots, m$ , where  $\lambda_j$  is indefinite multiplier;  $q_i$  is generalized coordinate of mechanism;  $\dot{q}_i$  is the first derivative of the generalized coordinates and the

angular velocity of the links mechanism.  $\Phi_j$  are a functions of  $q_i$  and  $\dot{q}_i$ , then:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} + \sum_{j=1}^m \left( \dot{\lambda}_j \frac{\partial \Phi_j}{\partial \dot{q}_i} + \lambda_j \left( \frac{d}{dt} \frac{\partial \Phi_j}{\partial \dot{q}_i} - \frac{\partial \Phi_j}{\partial q_i} \right) \right) = F_i, \quad (4)$$

$$i=1, \dots, n$$

*The driving motor.* The selected driving motor is a direct current motor with independent excitation. Motor speed is regulated by choosing terminal voltage of the anchor at constant current of the stator. If the stator current is constant, it can be assumed that the back EMF is proportional to the motor speed with a coefficient  $kp$ . The equations of the motor have the form:

$$\begin{cases} L_a \frac{di_a}{dt} + r_a i_a = U_T - kp \omega_{mot} \\ J \frac{d\omega_{mot}}{dt} + f \omega_{mot} = T_{mot} - T_L \end{cases}, \quad (5)$$

where  $L_a$  is inductance of the anchor;  $r_a$  is resistance of the anchor;  $U_T$  is the voltage at the terminals;  $i_a$  is current of the anchor;  $\omega_{mot}$  is motor speed;  $J$  is moment of inertia of the rotor;  $f$  is damping coefficient of the rotor;  $T_L$  is load torque. Motor torque is proportional to the current of the anchor  $T_{mot} = kp' i_a$ , where  $kp'$  expressed in electro-mechanical units, N·m/A;  $kp$  is in electrical units, V·s ( $E = c \cdot \Phi_E \cdot n$ ,  $\Phi_E$  – magnetic flux). Choosing the angle of rotation the driving link of the mechanism for the generalized coordinate  $q_1$ , current of the anchor for  $q_8$ , the equation (5) can be rewritten as:

$$\begin{cases} \dot{q}_8 = \frac{1}{L_a} [U_T - r_a q_8 - kp \dot{q}_1] \\ T_L = kp' q_8 - J \ddot{q}_1 - f \dot{q}_1 \end{cases}, \quad (6)$$

where  $T_L$  is the moment corresponding to the coordinate  $q_1$ .

*Driven mechanism.* As the driven mechanism is selected the mechanism of class IV with dwell slave units (Fig. 1).

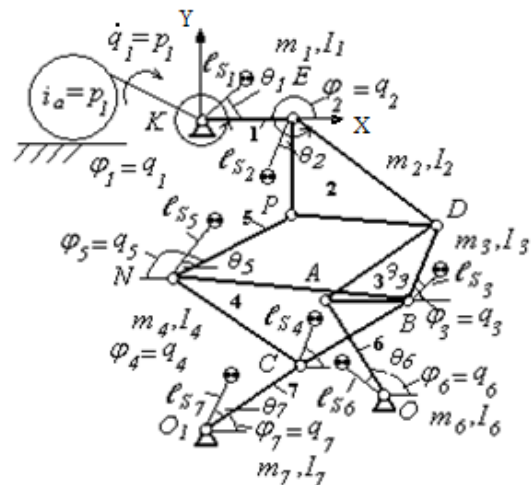


Fig. 1 The electromechanical system «motor-mechanism»

Closing circuits KEDAO, KEPNCO<sub>1</sub>, OABCO<sub>1</sub>, the projections we obtain the following dependence of velocity mechanism links:

$$\begin{cases} \frac{d\varphi_2}{dt} = \frac{l_1 \sin(\varphi_5 - \varphi_1)}{l_2 \sin(\varphi_2 - \varphi_5)} \times \frac{d\varphi_1}{dt}, \\ \frac{d\varphi_3}{dt} = \frac{l_1 \sin(\varphi_2 - \varphi_1) \sin(\varphi_5 - \varphi_6)}{l_3 \sin(\varphi_2 - \varphi_5) \sin(\varphi_6 - \varphi_3)} \times \frac{d\varphi_1}{dt}, \\ \frac{d\varphi_4}{dt} = \frac{l_1 \sin(\varphi_2 - \varphi_1) \sin(\varphi_5 - \varphi_7)}{l_4 \sin(\varphi_2 - \varphi_5) \sin(\varphi_4 - \varphi_7)} \times \frac{d\varphi_1}{dt}, \\ \frac{d\varphi_5}{dt} = \frac{2l_1 \sin(\varphi_1 - \varphi_2)}{l_5 \sin(\varphi_2 - \varphi_5)} \times \frac{d\varphi_1}{dt}, \\ \frac{d\varphi_6}{dt} = \frac{l_1 \sin(\varphi_2 - \varphi_1) \sin(\varphi_3 - \varphi_5)}{l_6 \sin(\varphi_2 - \varphi_5) \sin(\varphi_6 - \varphi_3)} \times \frac{d\varphi_1}{dt}, \\ \frac{d\varphi_7}{dt} = \frac{l_1 \sin(\varphi_2 - \varphi_1) \sin(\varphi_4 - \varphi_5)}{l_7 \sin(\varphi_2 - \varphi_5) \sin(\varphi_4 - \varphi_7)} \times \frac{d\varphi_1}{dt}. \end{cases} \quad (7)$$

Hereinafter, will be used the following selected generalized coordinates:

$$\begin{cases} q_1 = \varphi_1; \quad q_2 = \varphi_2; \quad q_3 = \varphi_3; \quad q_4 = \varphi_4; \\ q_5 = \varphi_5; \quad q_6 = \varphi_6; \quad q_7 = \varphi_7; \quad q_8 = i_n. \end{cases} \quad (8)$$

The energy mechanism. In accordance with Fig. 1 can be written kinetic energy of the class IV mechanism as follows:

$$\begin{aligned} T = & \frac{1}{2} \left[ I_1 \dot{q}_1^2 + I_2 \dot{q}_2^2 + I_3 \dot{q}_3^2 + I_4 \dot{q}_4^2 + I_5 \dot{q}_5^2 + I_6 \dot{q}_6^2 + I_7 \dot{q}_7^2 + \right. \\ & + m_2 \left( \ell_1^2 \dot{q}_1^2 + \ell_2^2 \dot{q}_2^2 + 2\ell_1 \ell_2 \dot{q}_1 \dot{q}_2 \cos(-q_1 + q_2 + \theta_2) \right) + \\ & + m_3 \left( \ell_2^2 \dot{q}_2^2 + \ell_3^2 \dot{q}_3^2 + 2\ell_2 \ell_3 \dot{q}_2 \dot{q}_3 \cos(-q_2 + q_3 + \theta_3) \right) + \\ & + m_4 \left( \ell_3^2 \dot{q}_3^2 + \ell_4^2 \dot{q}_4^2 + 2\ell_3 \ell_4 \dot{q}_3 \dot{q}_4 \cos(-q_3 + q_4 + \theta_4) \right) + \\ & \left. + m_5 \left( \ell_5^2 \dot{q}_5^2 + \ell_2^2 \dot{q}_2^2 + 2\ell_5 \ell_2 \dot{q}_5 \dot{q}_2 \cos(-q_2 + q_5 + \theta_5) \right) \right]. \quad (9) \end{aligned}$$

The equation of the potential energy of the class IV mechanism has the form:

$$\begin{aligned} V = & m_1 g \ell_{s_1} \sin(q_1 + \theta_1) + \\ & + m_2 g \left( \ell_1 \sin q_1 + \ell_{s_2} \sin(q_2 + \theta_2) \right) + \\ & + m_3 g \left( \ell_2 \sin q_2 + \ell_{s_3} \sin(q_3 + \theta_3) \right) + \\ & + m_4 g \left( \ell_3 \sin q_3 + \ell_{s_4} \sin(q_4 + \theta_4) \right) + \\ & + m_5 g \left( \ell_2 \sin q_2 + \ell_{s_5} \sin(q_5 + \theta_5) \right) + \end{aligned}$$

where  $K_1 = \sum_{j=1}^6 \left( \dot{\lambda}_j \frac{\partial \Phi_j}{\partial \dot{q}_1} + \lambda_j \left( \frac{d}{dt} \frac{\partial \Phi_j}{\partial \dot{q}_1} - \frac{\partial \Phi_j}{\partial q_1} \right) \right) = \dot{\lambda}_1 \ell_1 \sin(q_5 - q_1) + \lambda_1 \ell_1 \cos(q_5 - q_1) \dot{q}_5 + \ell_1 \sin(q_2 - q_1) \cdot$

$$\begin{aligned} & \cdot \left[ \dot{\lambda}_2 \sin(q_5 - q_6) + \lambda_2 \cos(q_5 - q_6) (\dot{q}_5 - \dot{q}_6) + \dot{\lambda}_3 \sin(q_5 - q_7) + \lambda_3 \cos(q_5 - q_7) (\dot{q}_5 - \dot{q}_7) - 2\dot{\lambda}_4 + \right. \\ & + \dot{\lambda}_5 \sin(q_3 - q_5) + \lambda_5 \cos(q_3 - q_5) (\dot{q}_3 - \dot{q}_5) + \dot{\lambda}_6 \sin(q_4 - q_5) + \lambda_6 \cos(q_4 - q_5) (\dot{q}_4 - \dot{q}_5) \left. \right] + \\ & + \ell_1 \cos(q_2 - q_1) \cdot \left[ \lambda_2 \sin(q_5 - q_6) + \lambda_3 \sin(q_5 - q_7) - 2\lambda_4 + \lambda_5 \sin(q_3 - q_5) + \lambda_6 \sin(q_4 - q_5) \right] \dot{q}_2, \end{aligned}$$

$$\begin{aligned} & + m_6 g \left[ \ell_{s_6} \sin(q_6 + \theta_6) + \ell_{k_0} \sin \Theta_1 \right] + \\ & + m_7 g \left[ \ell_{s_7} \sin(q_7 + \theta_7) + \ell_{k_0} \sin \Theta_2 + \ell_{o_0} \sin \Theta_3 \right]. \quad (10) \end{aligned}$$

Euler – Lagrange equations. For the mechanism of class IV with dwell driven link construct the Euler-Lagrange equations, substitute equations (9) and (10) in equation (4), will also assume that  $F_1 = T_L$  and  $L = T - V$ . After determination of derivatives and association members we obtain the equation of the system:

$$\begin{aligned} i = 1 & \\ & (I_1 + m_2 \ell_1^2 + J) \ddot{q}_1 + m_2 \ell_1 \ell_{s_2} \cos(-q_1 + q_2 + \theta_2) \ddot{q}_2 - \\ & - m_2 \ell_1 \ell_{s_2} \sin(-q_1 + q_2 + \theta_2) \dot{q}_2^2 + D \dot{q}_1 - k p' q_8 + \\ & + m_1 g \ell_{s_1} \cos(q_1 + \theta_1) + m_2 g \ell_1 \cos q_1 + K_1 = 0. \\ i = 2 & \\ & (I_2 + m_2 \ell_1^2 + m_3 \ell_2^2 + m_5 \ell_2^2) \ddot{q}_2 + m_2 \ell_1 \ell_{s_2} \cdot \\ & \cdot \cos(-q_1 + q_2 + \theta_2) \ddot{q}_1 + m_3 \ell_2 \ell_{s_3} \cdot \cos(-q_2 + q_3 + \theta_3) \ddot{q}_3 + \\ & + m_5 \ell_5 \ell_{s_2} \cos(-q_2 + q_5 + \theta_5) \ddot{q}_5 + m_2 \ell_1 \ell_{s_2} \cdot \\ & \cdot \sin(-q_1 + q_2 + \theta_2) \dot{q}_1^2 - m_3 \ell_2 \ell_{s_3} \cdot \sin(-q_2 + q_3 + \theta_3) \dot{q}_3^2 - \\ & - m_5 \ell_5 \ell_{s_2} \cdot \sin(-q_2 + q_5 + \theta_5) \dot{q}_5^2 + m_2 g \ell_{s_2} \cos(q_2 + \theta_2) + \\ & + m_3 g \ell_2 \cos q_2 + m_5 g \ell_2 \cos q_2 + K_2 = 0. \\ i = 3 & \\ & (I_3 + m_3 \ell_2^2 + m_4 \ell_3^2) \ddot{q}_3 + m_3 \ell_2 \ell_{s_3} \cos(-q_2 + q_3 + \theta_3) \ddot{q}_2 + \\ & + m_4 \ell_3 \ell_{s_4} \cos(-q_3 + q_4 + \theta_4) \ddot{q}_4 + m_3 \ell_2 \ell_{s_3} \cdot \\ & \cdot \sin(-q_2 + q_3 + \theta_3) \dot{q}_2^2 - m_4 \ell_3 \ell_{s_4} \sin(-q_3 + q_4 + \theta_4) \dot{q}_4^2 + \\ & + m_3 g \ell_{s_3} \cos(q_3 + \theta_3) + m_4 g \ell_3 \cos q_3 + K_3 = 0. \\ i = 4 & \\ & (I_4 + m_4 \ell_3^2) \ddot{q}_4 + m_4 \ell_3 \ell_{s_4} \cos(-q_3 + q_4 + \theta_4) \ddot{q}_3 + \\ & + m_4 \ell_3 \ell_{s_4} \cdot \sin(-q_3 + q_4 + \theta_4) \dot{q}_3^2 + \\ & + m_4 g \ell_{s_4} \cos(q_4 + \theta_4) + K_4 = 0. \\ i = 5 & \\ & (I_5 + m_5 \ell_5^2) \ddot{q}_5 + m_5 \ell_5 \ell_{s_2} \cos(-q_2 + q_5 + \theta_5) \ddot{q}_2 + \\ & + m_5 \ell_5 \ell_{s_2} \sin(-q_2 + q_5 + \theta_5) \dot{q}_2^2 + m_5 g \ell_{s_5} \cdot \\ & \cdot \cos(q_5 + \theta_5) + K_5 = 0. \\ i = 6 & \\ & I_6 \ddot{q}_6 + m_6 g \ell_{s_6} \cos(q_6 + \theta_6) + K_6 = 0. \\ i = 7 & \\ & I_7 \ddot{q}_7 + m_7 g \ell_{s_7} \cos(q_7 + \theta_7) + K_7 = 0, \quad (11) \end{aligned}$$

$$\begin{aligned}
K_2 &= \sum_{j=1}^6 \left( \dot{\lambda}_j \frac{\partial \Phi_j}{\partial \dot{q}_2} + \lambda_j \left( \frac{d}{dt} \frac{\partial \Phi_j}{\partial \dot{q}_2} - \frac{\partial \Phi_j}{\partial q_2} \right) \right) = \dot{\lambda}_1 \ell_2 \sin(q_5 - q_2) + \cos(q_5 - q_2) \cdot [(\lambda_1 \ell_2 + \lambda_4 \ell_5) \dot{q}_5 + \\
&+ (\lambda_2 \ell_3 \dot{q}_3 + \lambda_5 \ell_6 \dot{q}_6) \sin(q_6 - q_3) + (\lambda_3 \ell_4 \dot{q}_4 + \lambda_6 \ell_7 \dot{q}_7) \sin(q_4 - q_7)] - \ell_1 \cos(q_2 - q_1) \cdot \\
&\cdot [\lambda_2 \sin(q_5 - q_6) + \lambda_3 \sin(q_5 - q_7) - 2\lambda_4 + \lambda_5 \sin(q_3 - q_5) + \lambda_6 \sin(q_4 - q_5)] \dot{q}_1, \\
K_3 &= \sum_{j=1}^6 \left( \dot{\lambda}_j \frac{\partial \Phi_j}{\partial \dot{q}_3} + \lambda_j \left( \frac{d}{dt} \frac{\partial \Phi_j}{\partial \dot{q}_3} - \frac{\partial \Phi_j}{\partial q_3} \right) \right) = \sin(q_5 - q_2) \cdot [\dot{\lambda}_2 \ell_3 \sin(q_6 - q_3) + (\lambda_2 \ell_3 + \lambda_5 \ell_6) \cdot \\
&\cdot \cos(q_6 - q_3) \dot{q}_6] + \lambda_2 \ell_3 \cos(q_5 - q_2) \sin(q_6 - q_3) (\dot{q}_5 - \dot{q}_2) - \lambda_5 \ell_1 \sin(q_2 - q_1) \cos(q_3 - q_5) \dot{q}_1, \\
K_4 &= \sum_{j=1}^6 \left( \dot{\lambda}_j \frac{\partial \Phi_j}{\partial \dot{q}_4} + \lambda_j \left( \frac{d}{dt} \frac{\partial \Phi_j}{\partial \dot{q}_4} - \frac{\partial \Phi_j}{\partial q_4} \right) \right) = \sin(q_5 - q_2) \cdot [\dot{\lambda}_3 \ell_4 \sin(q_4 - q_7) - (\lambda_3 \ell_4 + \lambda_6 \ell_7) \cdot \\
&\cdot \cos(q_4 - q_7) \dot{q}_7] + \lambda_3 \ell_4 \cos(q_5 - q_2) \sin(q_4 - q_7) (\dot{q}_5 - \dot{q}_2) - \lambda_6 \ell_1 \sin(q_2 - q_1) \cos(q_4 - q_5) \dot{q}_1, \\
K_5 &= \sum_{j=1}^6 \left( \dot{\lambda}_j \frac{\partial \Phi_j}{\partial \dot{q}_5} + \lambda_j \left( \frac{d}{dt} \frac{\partial \Phi_j}{\partial \dot{q}_5} - \frac{\partial \Phi_j}{\partial q_5} \right) \right) = \dot{\lambda}_4 \ell_5 \sin(q_5 - q_2) - \cos(q_5 - q_2) \cdot [(\lambda_1 \ell_2 + \lambda_4 \ell_5) \dot{q}_2 + \\
&+ (\lambda_2 \ell_3 \dot{q}_3 + \lambda_5 \ell_6 \dot{q}_6) \sin(q_6 - q_3) + (\lambda_3 \ell_4 \dot{q}_4 + \lambda_6 \ell_7 \dot{q}_7) \sin(q_4 - q_7)] - \lambda_1 \ell_1 \cos(q_5 - q_1) \dot{q}_1 - \\
&- \ell_1 \sin(q_2 - q_1) \cdot [\lambda_2 \cos(q_5 - q_6) + \lambda_3 \cos(q_5 - q_7) - \lambda_5 \cos(q_3 - q_5) - \lambda_6 \cos(q_4 - q_5)] \dot{q}_1, \\
K_6 &= \sum_{j=1}^6 \left( \dot{\lambda}_j \frac{\partial \Phi_j}{\partial \dot{q}_6} + \lambda_j \left( \frac{d}{dt} \frac{\partial \Phi_j}{\partial \dot{q}_6} - \frac{\partial \Phi_j}{\partial q_6} \right) \right) = \sin(q_5 - q_2) \cdot [\dot{\lambda}_5 \ell_6 \sin(q_6 - q_3) - (\lambda_2 \ell_3 + \lambda_5 \ell_6) \cdot \\
&\cdot \cos(q_6 - q_3) \dot{q}_3] + \lambda_5 \ell_6 \cos(q_5 - q_2) \sin(q_6 - q_3) (\dot{q}_5 - \dot{q}_2) + \lambda_2 \ell_1 \sin(q_2 - q_1) \cos(q_5 - q_6) \dot{q}_1, \\
K_7 &= \sum_{j=1}^6 \left( \dot{\lambda}_j \frac{\partial \Phi_j}{\partial \dot{q}_7} + \lambda_j \left( \frac{d}{dt} \frac{\partial \Phi_j}{\partial \dot{q}_7} - \frac{\partial \Phi_j}{\partial q_7} \right) \right) = \sin(q_5 - q_2) \cdot [\dot{\lambda}_6 \ell_7 \sin(q_4 - q_7) + (\lambda_3 \ell_4 + \lambda_6 \ell_7) \cdot \\
&\cdot \cos(q_4 - q_7) \dot{q}_4] + \lambda_6 \ell_7 \cos(q_5 - q_2) \sin(q_4 - q_7) (\dot{q}_5 - \dot{q}_2) + \lambda_3 \ell_1 \sin(q_2 - q_1) \cos(q_5 - q_7) \dot{q}_1. \tag{12}
\end{aligned}$$

The equation of current  $q_8$

$$\dot{p}_1 - \frac{1}{G_i} \cdot \dot{p}_i = -\frac{\dot{G}_i}{G_i} p_1 = -H_i, \quad i = \overline{2, 7}, \tag{16}$$

$$L_a \dot{q}_8 + r_a q_8 + k p \dot{q}_1 = U_T.$$

where

*Equations of state.* We introduce seven new state variables defining the motion of the system in phase space:

$$\begin{cases} p_1 = \dot{q}_1; & p_2 = \dot{q}_2; & p_3 = \dot{q}_3; & p_4 = \dot{q}_4, \\ p_5 = \dot{q}_5; & p_6 = \dot{q}_6; & p_7 = \dot{q}_7 \end{cases}, \tag{13}$$

what gives:

$$\begin{cases} \ddot{q}_1 = \dot{p}_1; & \ddot{q}_2 = \dot{p}_2; & \ddot{q}_3 = \dot{p}_3; & \ddot{q}_4 = \dot{p}_4; \\ \ddot{q}_5 = \dot{p}_5; & \ddot{q}_6 = \dot{p}_6; & \ddot{q}_7 = \dot{p}_7. \end{cases} \tag{14}$$

Equation (7) can be written as:

$$\begin{cases} p_1 - \frac{1}{G_2} \cdot p_2 = 0; & p_1 - \frac{1}{G_3} \cdot p_3 = 0, & p_1 - \frac{1}{G_4} \cdot p_4 = 0; \\ p_1 - \frac{1}{G_5} \cdot p_5 = 0; & p_1 - \frac{1}{G_6} \cdot p_6 = 0; & p_1 - \frac{1}{G_7} \cdot p_7 = 0, \end{cases} \tag{15}$$

where  $G_2 \div G_7$  – the coefficients in the equations (7).

Differentiating equations (15) we obtain:

$$\begin{aligned}
\dot{G}_2 &= \text{ctg}(q_5 - q_1)(p_5 - p_1) + \text{ctg}(q_2 - q_5)(p_5 - p_2); \\
\dot{G}_3 &= \text{ctg}(q_2 - q_1)(p_2 - p_1) + \text{ctg}(q_5 - q_6)(p_5 - p_6) + \\
&+ \text{ctg}(q_2 - q_5)(p_5 - p_2) + \text{ctg}(q_6 - q_3)(p_3 - p_6); \\
\dot{G}_4 &= \text{ctg}(q_2 - q_1)(p_2 - p_1) + \text{ctg}(q_5 - q_7)(p_5 - p_7) + \\
&+ \text{ctg}(q_2 - q_5)(p_5 - p_2) + \text{ctg}(q_4 - q_7)(p_7 - p_4); \\
\dot{G}_5 &= \text{ctg}(q_1 - q_2)(p_1 - p_2) + \text{ctg}(q_2 - q_5)(p_5 - p_2); \\
\dot{G}_6 &= \text{ctg}(q_2 - q_1)(p_2 - p_1) + \text{ctg}(q_3 - q_5)(p_3 - p_5) + \\
&+ \text{ctg}(q_2 - q_5)(p_5 - p_2) + \text{ctg}(q_6 - q_3)(p_3 - p_6); \\
\dot{G}_7 &= \text{ctg}(q_2 - q_1)(p_2 - p_1) + \text{ctg}(q_4 - q_5)(p_4 - p_5) + \\
&+ \text{ctg}(q_2 - q_5)(p_5 - p_2) + \text{ctg}(q_4 - q_7)(p_7 - p_4). \tag{17}
\end{aligned}$$

The first seven of equations (11) can be written:

$$\begin{cases} A_1 \dot{p}_1 + B_1 \dot{p}_2 + C_1 \dot{\lambda}_1 + D_1 \dot{\lambda}_2 + F_1 \dot{\lambda}_3 + \\ + L_1 \dot{\lambda}_4 + M_1 \dot{\lambda}_5 + N_1 \dot{\lambda}_6 + E_1 = 0, \\ B_1 \dot{p}_1 + A_2 \dot{p}_2 + B_2 \dot{p}_3 + B_5 \dot{p}_5 + C_2 \dot{\lambda}_1 + E_2 = 0, \\ B_2 \dot{p}_2 + A_3 \dot{p}_3 + B_4 \dot{p}_4 + D_3 \dot{\lambda}_2 + E_3 = 0, \\ B_4 \dot{p}_3 + A_4 \dot{p}_4 + F_4 \dot{\lambda}_3 + E_4 = 0, \\ B_5 \dot{p}_2 + A_5 \dot{p}_5 + L_5 \dot{\lambda}_4 + E_5 = 0, \\ A_6 \dot{p}_6 + M_6 \dot{\lambda}_5 + E_6 = 0, \\ A_7 \dot{p}_7 + N_7 \dot{\lambda}_6 + E_7 = 0. \end{cases} \quad (18)$$

Together with the equations (16) equations (18) form a linear system of equations for the variables  $\dot{p}_1 \div \dot{p}_7$ ,  $\dot{\lambda}_1 \div \dot{\lambda}_6$ .

We introduce the following notations:

$$\begin{aligned} \mu_1 &= \frac{\sin(q_5 - q_1)}{\sin(q_2 - q_1)}, & \mu_2 &= \frac{\sin(q_5 - q_6)}{\sin(q_6 - q_3)}, \\ \mu_3 &= \frac{\sin(q_5 - q_7)}{\sin(q_4 - q_7)}, & \mu_4 &= -2, & \mu_5 &= \frac{\sin(q_3 - q_5)}{\sin(q_6 - q_3)}, \\ \mu_6 &= \frac{\sin(q_4 - q_5)}{\sin(q_4 - q_7)}, & \mu &= \frac{\sin(q_2 - q_5)}{\sin(q_2 - q_1)}, \end{aligned} \quad (19)$$

Adding the motor equation, we get the twenty-one first order differential equation (thirteen of which are non-linear) describing the system «motor – linkage»:

$$\dot{x} = f(x),$$

where  $x^T = [q_1, q_2, q_3, q_4, q_5, q_6, q_7, p_1, p_2, p_3, p_4, p_5, p_6, p_7, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, q_8]$ . (20)

The first seven equations of the system are as follows:

$$\begin{aligned} E_1 &= -m_2 \ell_1 \ell_{s_2} \sin(-q_1 + q_2 + \theta_2) p_2^2 + D p_1 - k p' q_8 + m_1 g \ell_{s_1} \cos(q_1 + \theta_1) + m_2 g \ell_1 \cos q_1 + \lambda_1 \ell_1 \cos(q_5 - q_1) p_5 + \\ &+ \ell_1 \cos(q_2 - q_1) \cdot [\lambda_2 \sin(q_5 - q_6) + \lambda_3 \sin(q_5 - q_7) - 2\lambda_4 + \lambda_5 \sin(q_3 - q_5) + \lambda_6 \sin(q_4 - q_5)] p_2 + \ell_1 \sin(q_2 - q_1) \cdot \\ &\cdot [\lambda_2 \cos(q_5 - q_6)(p_5 - p_6) + \lambda_3 \cos(q_5 - q_7)(p_5 - p_7) + \lambda_5 \cos(q_3 - q_5)(p_3 - p_5) + \lambda_6 \cos(q_4 - q_5)(p_4 - p_5)]; \\ E_2 &= m_2 \ell_1 \ell_{s_2} \sin(-q_1 + q_2 + \theta_2) p_1^2 - m_3 \ell_2 \ell_{s_3} \sin(-q_1 + q_3 + \theta_3) p_3^2 - m_5 \ell_5 \ell_{s_2} \sin(-q_2 + q_5 + \theta_5) p_5^2 + \\ &+ m_2 g \ell_{s_2} \cos(q_2 + \theta_2) + m_3 g \ell_2 \cos q_2 + m_5 g \ell_2 \cos q_2 - \ell_1 \cos(q_2 - q_1) \cdot [\lambda_2 \sin(q_5 - q_6) + \lambda_3 \sin(q_5 - q_7) - \\ &- 2\lambda_4 + \lambda_5 \sin(q_3 - q_5) + \lambda_6 \sin(q_4 - q_5)] p_1 + \cos(q_5 - q_2) \cdot [(\lambda_1 \ell_2 + \lambda_4 \ell_5) p_5 + (\lambda_2 \ell_3 p_3 + \lambda_5 \ell_6 p_6) \sin(q_6 - q_3) + \\ &+ (\lambda_3 \ell_4 p_4 + \lambda_6 \ell_7 p_7) \sin(q_4 - q_7)]; \\ E_3 &= m_3 \ell_2 \ell_{s_3} \sin(-q_2 + q_3 + \theta_3) p_2^2 - m_4 \ell_3 \ell_{s_4} \sin(-q_3 + q_4 + \theta_4) p_4^2 + m_3 g \ell_{s_3} \cos(q_3 + \theta_3) + m_4 g \ell_3 \cos q_3 - \\ &- \lambda_5 \ell_1 \sin(q_2 - q_1) \cos(q_3 - q_5) p_1 + \lambda_2 \ell_3 \cos(q_5 - q_2) \sin(q_6 - q_3) (p_5 - p_2) + (\lambda_2 \ell_3 + \lambda_5 \ell_6) \sin(q_5 - q_2) \cdot \\ &\cdot \cos(q_6 - q_3) p_6; \\ E_4 &= m_4 \ell_3 \ell_{s_4} \sin(-q_3 + q_4 + \theta_4) p_3^2 + m_4 g \ell_{s_4} \cos(q_4 + \theta_4) - \lambda_6 \ell_1 \sin(q_2 - q_1) \cos(q_4 - q_5) p_1 + \lambda_3 \ell_4 \cos(q_5 - q_2) \cdot \\ &\cdot \sin(q_4 - q_7) (p_5 - p_2) - (\lambda_3 \ell_4 + \lambda_6 \ell_7) \sin(q_5 - q_2) \cos(q_4 - q_7) p_7; \end{aligned}$$

$$\begin{cases} \dot{q}_1 = p_1, & \dot{q}_2 = p_2, & \dot{q}_3 = p_3, & \dot{q}_4 = p_4, \\ \dot{q}_5 = p_5, & \dot{q}_6 = p_6, & \dot{q}_7 = p_7, \end{cases} \quad (21)$$

we add to them the equations for the angular velocity of the mechanism  $\dot{p}_1 \div \dot{p}_7$ , the equations for the Lagrange multipliers  $\dot{\lambda}_1 \div \dot{\lambda}_6$  given below, and the equation for the current.  $\dot{q}_8 = [U_a - r_a q_8 - k p p_1] / L_a$ .

The coefficients in the system of differential equations are given by the expressions:

$$\begin{aligned} A_1 &= I_1 + m_2 \ell_1^2 + J; & A_2 &= I_2 + m_2 \ell_{s_2}^2 + m_3 \ell_2^2 + m_5 \ell_{s_2}^2; \\ A_3 &= I_3 + m_3 \ell_{s_3}^2 + m_4 \ell_3^2; & A_4 &= I_4 + m_4 \ell_{s_4}^2; \\ A_5 &= I_5 + m_5 \ell_{s_5}^2; & A_6 &= I_6; & A_7 &= I_7; \\ B_1 &= m_2 \ell_1 \ell_{s_2} \cos(-q_1 + q_2 + \theta_2); \\ B_2 &= m_3 \ell_2 \ell_{s_3} \cos(-q_2 + q_3 + \theta_3); \\ B_4 &= m_4 \ell_3 \ell_{s_4} \cos(-q_3 + q_4 + \theta_4); \\ B_5 &= m_5 \ell_5 \ell_{s_2} \cos(-q_2 + q_5 + \theta_5); \\ K_1 &= \sin(q_2 - q_1); & K_2 &= \sin(q_2 - q_1) \sin(q_6 - q_3); \\ K_3 &= \sin(q_2 - q_1) \sin(q_4 - q_7); \\ C_1 &= \ell_1 \sin(q_5 - q_1) = \ell_1 \mu_1 K_1; \\ D_1 &= \ell_1 \sin(q_2 - q_1) \sin(q_5 - q_6) = \ell_1 \mu_2 K_2; \\ F_1 &= \ell_1 \sin(q_2 - q_1) \sin(q_5 - q_7) = \ell_1 \mu_3 K_3; \\ L_1 &= -2\ell_1 \sin(q_2 - q_1) = \ell_1 \mu_4 K_1; \\ M_1 &= \ell_1 \sin(q_2 - q_1) \sin(q_3 - q_5) = \ell_1 \mu_5 K_2; \\ N_1 &= \ell_1 \sin(q_2 - q_1) \sin(q_4 - q_5) = \ell_1 \mu_6 K_3; \\ C_2 &= \ell_2 \sin(q_5 - q_2) = -\ell_2 \mu K_1; \\ D_3 &= \ell_3 \sin(q_5 - q_2) \sin(q_6 - q_3) = -\ell_3 \mu K_2; \\ F_4 &= \ell_4 \sin(q_5 - q_2) \sin(q_4 - q_7) = -\ell_4 \mu K_3; \\ L_5 &= \ell_5 \sin(q_5 - q_2) = -\ell_5 \mu K_1; \\ M_6 &= \ell_6 \sin(q_5 - q_2) \sin(q_6 - q_3) = -\ell_6 \mu K_2; \\ N_7 &= \ell_7 \sin(q_5 - q_2) \sin(q_4 - q_7) = -\ell_7 \mu K_3. \end{aligned}$$

$$E_5 = m_5 \ell_5 \ell_{s2} \cos(-q_2 + q_5 + \theta_3) p_2^2 + m_5 g \ell_{s5} \cos(q_5 + \theta_3) - \lambda_1 \ell_1 \cos(q_5 - q_1) p_1 - \ell_1 \sin(q_2 - q_1) \cdot$$

$$\cdot [\lambda_2 \cos(q_5 - q_6) + \lambda_3 \cos(q_5 - q_7) - \lambda_5 \cos(q_3 - q_5) - \lambda_6 \cos(q_4 - q_5)] p_1 - \cos(q_5 - q_2) \cdot [(\lambda_1 \ell_2 + \lambda_4 \ell_5) p_2 +$$

$$+ (\lambda_2 \ell_3 p_3 + \lambda_5 \ell_6 p_6) \sin(q_6 - q_3) + (\lambda_3 \ell_4 p_4 + \lambda_6 \ell_7 p_7) \sin(q_4 - q_7)];$$

$$E_6 = m_6 \ell_6 \ell_{s6} \sin(q_6 + \theta_6) + \lambda_2 \ell_1 \sin(q_2 - q_1) \cos(q_5 - q_6) p_1 + \lambda_5 \ell_6 \cos(q_5 - q_2) \sin(q_6 - q_3) (p_5 - p_2) -$$

$$- (\lambda_2 \ell_3 + \lambda_5 \ell_6) \sin(q_5 - q_2) \cos(q_6 - q_3) p_3;$$

$$E_7 = m_7 \ell_6 \ell_{s7} \cos(q_7 + \theta_7) + \lambda_3 \ell_1 \sin(q_2 - q_1) \cos(q_5 - q_7) p_1 + \lambda_6 \ell_7 \cos(q_5 - q_2) \sin(q_4 - q_7) (p_5 - p_2) +$$

$$+ (\lambda_3 \ell_4 + \lambda_6 \ell_7) \sin(q_5 - q_2) \cos(q_4 - q_7) p_4.$$

$$H_2 = \frac{\dot{G}_2}{G_2} p_1 = [\text{ctg}(q_5 - q_1)(p_5 - p_1) +$$

$$+ \text{ctg}(q_2 - q_5)(p_5 - p_2)] \cdot p_1;$$

$$H_3 = \frac{\dot{G}_3}{G_3} p_1 = [\text{ctg}(q_2 - q_1)(p_2 - p_1) +$$

$$+ \text{ctg}(q_5 - q_6)(p_5 - p_6) + \text{ctg}(q_2 - q_5)(p_5 - p_2) +$$

$$+ \text{ctg}(q_6 - q_3)(p_3 - p_6)] \cdot p_1;$$

$$H_4 = \frac{\dot{G}_4}{G_4} p_1 = [\text{ctg}(q_2 - q_1)(p_2 - p_1) +$$

$$+ \text{ctg}(q_5 - q_7)(p_5 - p_7) + \text{ctg}(q_2 - q_5)(p_5 - p_2) +$$

$$+ \text{ctg}(q_4 - q_7)(p_7 - p_4)] \cdot p_1;$$

$$H_5 = \frac{\dot{G}_5}{G_5} p_1 = [\text{ctg}(q_1 - q_2)(p_1 - p_2) +$$

$$+ \text{ctg}(q_2 - q_5)(p_5 - p_2)] \cdot p_1;$$

$$H_6 = \frac{\dot{G}_6}{G_6} p_1 = [\text{ctg}(q_2 - q_1)(p_2 - p_1) +$$

$$+ \text{ctg}(q_3 - q_5)(p_3 - p_5) + \text{ctg}(q_2 - q_5)(p_5 - p_2) +$$

$$+ \text{ctg}(q_6 - q_3)(p_3 - p_6)] \cdot p_1;$$

$$H_7 = \frac{\dot{G}_7}{G_7} p_1 = [\text{ctg}(q_2 - q_1)(p_2 - p_1) +$$

$$+ \text{ctg}(q_4 - q_5)(p_4 - p_5) + \text{ctg}(q_2 - q_5)(p_5 - p_2) +$$

$$+ \text{ctg}(q_4 - q_7)(p_7 - p_4)] \cdot p_1.$$

### 3. Example

The equations of integrated using the Runge-Kutta method of fourth order (ode45) during acceleration. An analysis of the coefficients of equations (18) show that at  $t = 0$  in view of  $p_1(0) = \omega_1(0) = 0$  all the members containing undefined multipliers are zero.

Physical characteristics of the mechanism. Example of the mechanism shown in Fig.1. The calculations changed damping and the total moment of inertia of the rotor and the driving link. The initial parameters of the mechanism are:

$$\ell_1 = 10,040 \text{ m}; r_1 = 0 \text{ m}; \ell_2 = 0,090 \text{ m}; \ell_{21} = 0,041 \text{ m};$$

$$\ell_{22} = 0,072 \text{ m}; r_2 = 0,045 \text{ m}; \ell_3 = 0,060 \text{ m};$$

$$\ell_{31} = 0,030 \text{ m};$$

$$\ell_{32} = 0,040 \text{ m}; r_3 = 0,03 \text{ m}; \ell_4 = 0,047 \text{ m}; \ell_{41} = 0,100 \text{ m};$$

$$\ell_{42} = 0,115 \text{ m}; r_4 = 0,3 \text{ m}; \ell_5 = 0,72 \text{ m}; r_5 = 0,36 \text{ m};$$

$$\ell_6 = 0,50 \text{ m}; r_6 = 0,25 \text{ m}; \ell_7 = 0,65 \text{ m}; r_7 = 0,325 \text{ m};$$

$$L_1 = 0,148 \text{ m}; L_2 = 0,126 \text{ m}; L_3 = 0,082 \text{ m};$$

$$I_1 = 1,28 \cdot 10^{-3} \text{ N}\cdot\text{m}\cdot\text{s}^2; I_2 = 80 \cdot 10^{-3} \text{ N}\cdot\text{m}\cdot\text{s}^2;$$

$$I_3 = 21,97 \cdot 10^{-3} \text{ N}\cdot\text{m}\cdot\text{s}^2; I_4 = 176 \cdot 10^{-3} \text{ N}\cdot\text{m}\cdot\text{s}^2;$$

$$I_5 = 3,73 \cdot 10^{-3} \text{ N}\cdot\text{m}\cdot\text{s}^2; I_6 = 2,5 \cdot 10^{-3} \text{ N}\cdot\text{m}\cdot\text{s}^2;$$

$$I_7 = 5,49 \cdot 10^{-3} \text{ N}\cdot\text{m}\cdot\text{s}^2; m_1 = 0,4 \text{ kg}; m_2 = 2 \text{ kg};$$

$$m_3 = 1,3 \text{ kg};$$

$$m_4 = 2,6 \text{ kg}; m_5 = 0,72 \text{ kg}; m_6 = 0,5 \text{ kg}; m_7 = 0,65 \text{ kg};$$

$$TH_1 = TH_2 = TH_3 = TH_4 = TH_5 = TH_6 = TH_7 = 0.$$

The physical characteristics of the motor.  $kp = 0,678 \text{ N}\cdot\text{m}/\text{A} = 0,678 \text{ V}\cdot\text{s}$ ;  $r_a = 0,4 \Omega$ ;  $r = 0,125 \text{ m}$ ;  $L_a = 0,05 \text{ H}$ ;  $J = 0,0565 \text{ N}\cdot\text{m}\cdot\text{s}^2$ ;  $D = 0,226 \text{ N}\cdot\text{m}\cdot\text{s}$  (damping coefficients of the rotor and the driving link are combined). Operating voltage  $U_a = 15 \text{ V}$  with zero initial current of anchor  $i_a(0) = 0$ .

### 4. Conclusions

*Results.* There are two variants of the example. The total moment of inertia of the rotor and the driving link is increased from  $0,0565 \text{ N}\cdot\text{m}\cdot\text{s}^2$  to  $0,346 \text{ N}\cdot\text{m}\cdot\text{s}^2$ . In both cases, the motion starts from rest.

Fig. 2 shows graphs of operating points of the angular velocity of the mechanism of class IV with dwell driven links. Set the speed of all the links is achieved by 1.5 seconds. The angular velocity of the second link mechanism is 50 rad/s, the third is 79 rad/s, the fourth is 23 rad/s, the fifth is 120 rad/s, the sixth is 40 rad/s and seventh is 18 rad/s. The negative sign of the angular velocity of the mechanism links indicate a change in direction of rotation of links.

Fig. 3 shows the reaction of the system described with given above data. The graph represents the dependence of the angular velocity of the driving link or the motor on time. The motor overlocks the mechanism is similar to the reaction of weak damping system. Set the speed of close to 10 rad/s and achieved by 0,4 seconds. In this case, increasing the damping leads to only an increase of consumed current and a similar but somewhat slower reaction with decreased maximum speed, in addition to energy losses. Is much more effective in excluding of overlocking is to increase the dimensions of the flywheel (moment of inertia the driving link). Established fluctuations constitute about 30% of the average velocity.

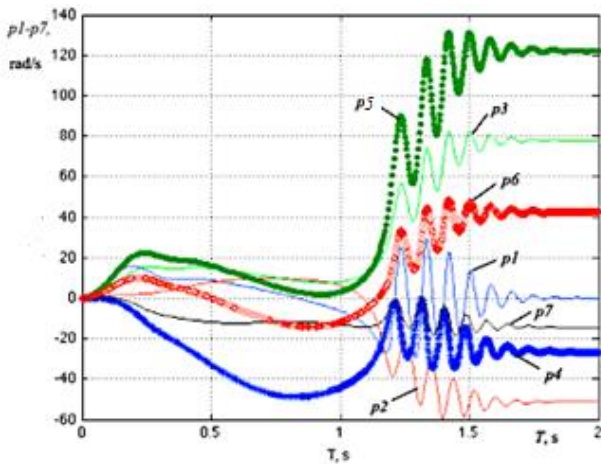


Fig. 2 The graph of operating points velocities of the class IV mechanism

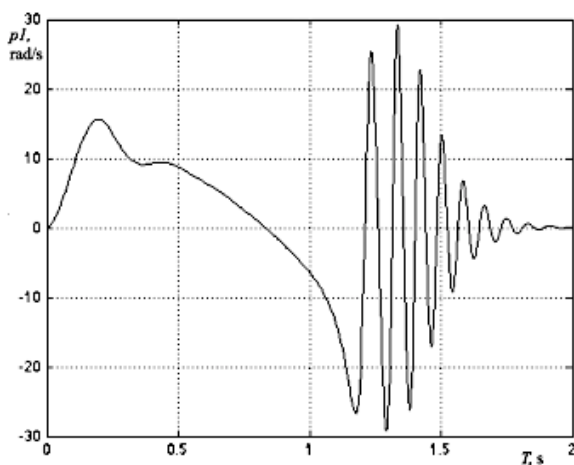


Fig. 3 The graph of the velocity of the class IV mechanism driving link at  $J+I=0,0565+1,28 \cdot 10^{-3} \text{N} \cdot \text{m} \cdot \text{s}^2$

Fig. 4 illustrates the effect of increasing the dimensions of the flywheel on the angular velocity of rotation of the driving link. Acceleration time increases, but the fluctuations increased with the above mentioned moment of inertia of the rotor and the driving link.

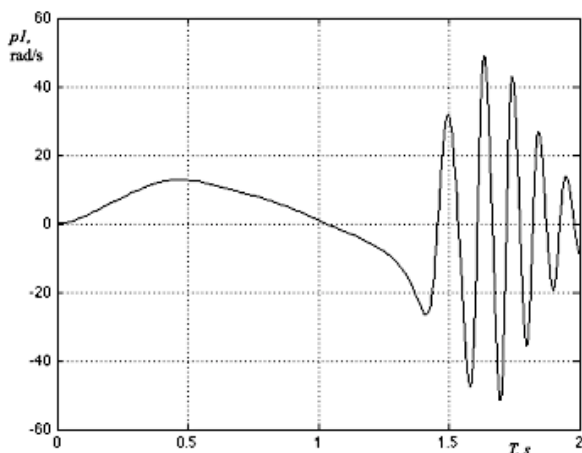


Fig. 4 The graph of the velocity of the class IV mechanism driving link at  $J+I=0,346+1,28 \cdot 10^{-3} \text{N} \cdot \text{m} \cdot \text{s}^2$

Fig. 5 shows the corresponding graph of consumed current during acceleration. High initial current is valid during the time equal to 1.5 seconds and is equal to

62 A. Since the motor torque is proportional to the current of anchor, we can see that torque is almost constant, but varies by almost 3% from the steady average value. High initial current corresponding to a special short time of overlocking, may be undesirable in practice, but is an example of the extreme behavior of the system (like "weakly damped" reaction).

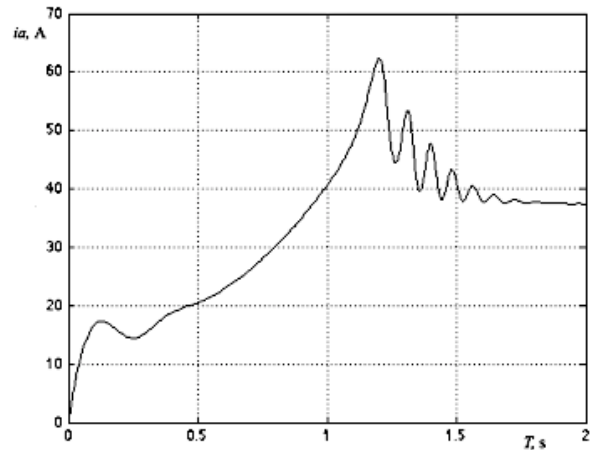


Fig. 5 The graph of the motor current

Investigation of graphs showed that when different applications torque to the drive link, it changes the value of a steady rate for the same time. Solution, as used here in, is easily reproduced, obtained quickly, it is stable and exact, as the multiple cycles are carried out without the accumulation of errors. Choice of optimal parameters, that is, the motor must conform to the mechanism, and modes or calculation of the driving motors of mechanisms can be facilitated using the analysis of similar type which will reduce energy consumption.

## References

1. **Dzholdasbekov, U.A.** 2001. Theory of mechanisms of high classes, Almaty: Gylym. 427 p. (in Russian).
2. **Myklebust, A.** 1982. Dynamic Response of an Electric Motor-Linkage System During Start, *J. Mech. Des.* 104(1): 137-142. <http://dx.doi.org/10.1115/1.3256303>
3. **Jih-Lian Ha; Rong-Fong Fung; Kun-Yung Chen; Shao-Chien Hsien.** 2006. Dynamic modeling and identification of a slider-crank mechanism, *Journal of Sound and Vibration* 289: 1019–1044. Available from Internet: <https://sites.ualberta.ca/~yousefim/bfiles/Crank%20Dynamics.pdf>
4. **Fauzi Ahmad; Ahmad Lukman Hitam; Khisbullah Hudha; Hishamuddin Jamaluddin.** 2011. Position Tracking of Slider Crank Mechanism using PID Controller Optimized by Ziegler Nichol's Method, 3(2). [http://jmet.utem.edu.my/index2.php?option=com\\_docman&task=doc\\_view&gid=66&Itemid=40](http://jmet.utem.edu.my/index2.php?option=com_docman&task=doc_view&gid=66&Itemid=40)
5. **Ualiev, G.** 2008. Mathematical models of mechanical systems considering characteristics of the motor, *Bulletin of the Moscow City Pedagogical University. Series: Informatics and informatization of education*, 14: 97-102. <http://elibrary.ru/download/63688485.pdf>.



6. **Ualiev, Z. G.** 2010. Dynamics of one electromechanical system with feedback, Bulletin of the Kazakh National Pedagogical University named after Abai, Series of physical and mathematical sciences Almaty 2(30): 237-241 (in Russian). Available from Internet: [http://kaznpu.kz/docs/vestnik/fizika\\_matematika/2302010.pdf](http://kaznpu.kz/docs/vestnik/fizika_matematika/2302010.pdf).
7. **Dhaouadi, R.; Hatab, A. A.** 2013. Dynamic Modeling of Differential-Drive Mobile Robots using Lagrange and Newton-Euler Methodologies: A Unified Framework. *Adv Robot Autom*, 2: 107. <http://dx.doi.org/10.4172/2168-9695.1000107>.
8. **Ualiev, G.; Ualiev, Z. G.** 2006. Mathematical modeling of the mechanical systems dynamics with variable characteristics. Almaty: Kazakh National Pedagogical University. 275 p. (in Russian).
9. **Sergashova, N.A.** 2013. Electromechanical analogy in mechanical circuits and electromechanical devices. *Science and education transport*. P.374-376. Available from Internet: <http://elibrary.ru/download/14771859.pdf>
10. **Bissembayev, K.; Kinzhebayeva, D.** 2014. Research on the of dynamic model of single-acting crank pump with damless hydro turbine drive, *Mechanika* 20(6): 560-572. <http://dx.doi.org/10.5755/j01.mech.20.6.7432.D>.

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DYNAMICS OF MOTION OF THE SYSTEM  
«ELECTRIC MOTOR – MECHANISM OF CLASS IV  
WITH DWELL DRIVEN LINKS» DURING  
ACCELERATION

S u m m a r y

In this article the dynamics of electromechanical system "electric motor – mechanism IV class with dwell driven links" is investigated. Numerical values of the lengths of mechanism links and physical parameters of the electric motor are given. Differential equations of motion of the electromechanical system using the second type Lagrange equations with indefinite multipliers is compiled. The equations of motion are solved using the method described in the paper by A. Myklebust, where the constraint equations are based on the equations of velocity relations. It was revealed that the value of the fluctuations depends on the value of the applied torque to the driving link of mechanism.

The results obtained in the theoretical research can be successfully used for the calculation and selection of the drive motors, as well in the study of the dynamics of high classes mechanisms.

**Keywords:** the electric motor, IV class mechanism with dwell driven links, differential equations of motion of IV class mechanism, the second type Lagrange equations, constraint equations, angular speeds.

Received September 20, 2016

Accepted December 07, 2017