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Investigation of stress-strain state of micro-corrugated cardboard components in the process of its laminating and manufacturing packaging

Svitlana HAVENKO*,, Volodymyr BERNATSEK*, Svitlana KHADZHYNOVA****
**Ukrainian Academy of Printing, Pidholosko 19, 79020 Lviv, Ukraine, E-mail: havenko@point.lviv.ua*
***Technical University of Lodz, Skorupki 10/12, r. 105, 90-924 Lodz, Poland, E-mail: svitlana.khadzhynowa@p.lodz.pl*

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1. Introduction

Analysis of development of the packaging industry around the world shows that in recent years has increased the need of consumers for quality container with profile-based micro-corrugated cardboard. This is primarily due to its ecological compatibility, convenience in operation, high durability and resistance to shock and vibration loads, the ability to combine with other materials, resistant to moisture, fat, etc., complete recycling after use. In addition, micro-corrugated cardboard luxury packaging are popular because their easily decoration by using printing technologies. The application of lamination process enhances the design possibilities of such packaging. Important in this production chain is the choice of cardboard, micro-corrugated cardboard and adhesives for laminating in view of their technological and operational characteristics. Therefore, actual for the packaging production is the use of micro-corrugated cardboard and other ecologically friendly materials. Significant place among them is laminated micro corrugated cardboard. Thanks to its ability to modeling in the packaging design and use the latest printing technology for decoration, it can be realize different design ideas. In addition, it can be used in creating small advertising forms and structures, windows and displays. Packaging combines the characteristics of consumer packaging (bright design) and corrugated container (durability and reliability) after laminated [1-6].

2. Experiment equipment and method

The object of research was laminated micro-corrugated cardboard, which is used for the liner (cardboard "Arktika" with gsm 180 g/m^{2}and 250 g/m^2) that stuck to micro corrugated cardboard mark 311E with fluting gsm 112 g/m^{2}and 140 g/m^2. It was used adhesives for bonding in industrial production CR (Poland), Devakol VK (Slovenia) and developed adhesive composition №3 (patent of Ukraine №20619). All materials are subject to acclimatization before carrying out experiments. Studies carried out at a temperature of 18-22°C and relative humidity of 65-75%.

Theoretical and experimental studies of micro-corrugated cardboard lamination process carried out on the basis of mathematical modeling (least squares method), the basics of the Hertz contact bodies' theory and Winkler hypotheses, electronic microscopy.

3. Results and discussion

Lamination process of micro-corrugated cardboard involves bonding wavy structure – fluting (cardboard blanks) to the flat bed – liner (printed sheets), followed by pressing and drying products. According to the literature sources it is assumed that the term "lamination" comes from the German word "kaschieren" – paste, duplicate, laminate, although there is a version of the French origin of the word - "cacher" – hide, obscure.

It should be noted that the main task in implementing of lamination process is to obtain good adhesion of the coating to the substrate. Since micro-corrugated cardboard relates to multilayer materials, in the manufacture of micro-corrugated cardboard products may have different kind of tension. Therefore it is important to research the stress-strain state of its components during lamination.

Investigation of mechanical factors influence on the restructuring of micro-corrugated cardboard made based on physical models. It was considered two specific cases of stress-strain state of micro-corrugated cardboard during lamination.

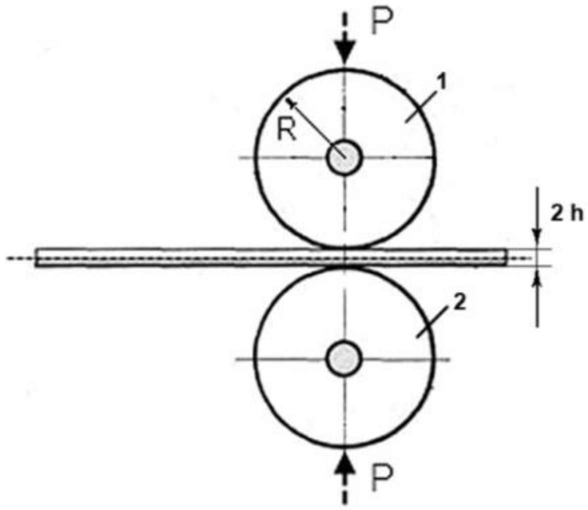


Fig. 1 Simulation of micro-corrugated cardboard compression, where h- height of corrugated wave (thickness of elastic layer), P - corrugated compression efforts by rigid cylinders *1*, *2* of the same radius R

In the first case, micro-corrugated cardboard (Fig. 1), as resilient layer with thickness $2h$, compressed by efforts P of two hard cylinders *1*, *2* with the same radius R. Deformation of micro-corrugated cardboard associated with changing of adhesive layer bandwidth application.

Because the task is symmetrical about the not deformed median plane of micro corrugated cardboard, it can be replaced by an equivalent in terms of stress-strain state by two-dimensional task of run-up rigid cylinder of radius R (Fig. 2) on the elastic layer of thickness h on a rigid base.

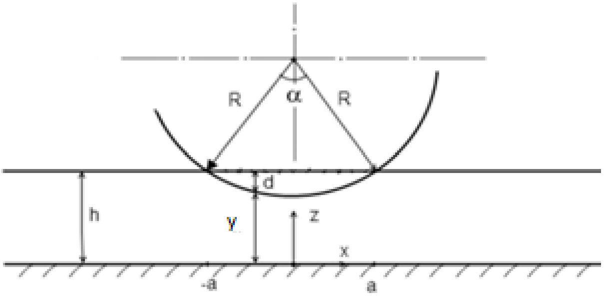


Fig. 2 Scheme of run-up cylinder of putting glue roller on micro corrugated cardboard, where *R* - radius of rigid cylinder, *h* - height of corrugated wave (thickness of elastic layer), [- *a*, *a*] - the length of the contact zone, *d* - the value of wave corrugations deflection

In this case, considering the hollow structure of micro corrugated cardboard and, consequently, practical absence of surface deformation outside the contact zone with the cylinder, it is sufficiently limited of Hertz contact bodies' theory. Under this theory, Winkler elastic layer model most adequately describes the elastic deformation of micro-corrugated cardboard compression [7-11], which is sometimes called "spring mattress" (Fig. 3). Winkler hypothesis is applied for expression $p(h) = p(d(x)) = p(x)$.

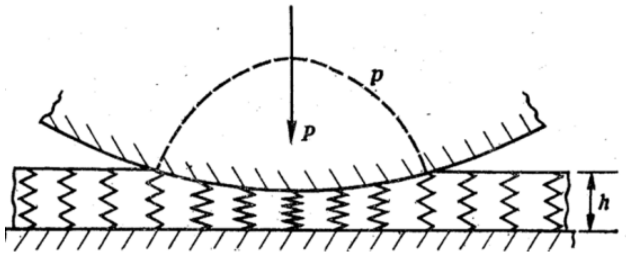


Fig. 3 Scheme of Winkler elastic layer, where *p* - linear efforts of compression, *ρ* - curvature, *h* - height of corrugated wave (thickness of elastic layer)

Deformation of micro-corrugated cardboard surface, according shown in Fig. 2 scheme of run-up cylinder of putting glue roller on it, described the relation:

$$d(x) = \begin{cases} h - y = d_{max} - R + \sqrt{R^2 - x^2}, & x \in [-a, a] \\ 0, & x \notin [-a, a] \end{cases}, \quad (1)$$

where [-*a*, *a*] are the length of the contact area:

$$a = R\sin\frac{\alpha}{2} = \sqrt{d_{max}(2R - d_{max})}\,, \quad (2)$$

where d_{max} is value of the maximum surface deflection of micro-corrugated cardboard at $X = 0$. Normal pressure in any point of contact, according to Winkler hypothesis, is given byrelation:

$$p(x) = -kd(x)\,, \quad (3)$$

where *k* is adjusted compressive modulus of elasticity of micro-corrugated cardboard. It is clear, that at $X = 0$ does not exceed the threshold value of the pressure, at which the elastic properties of the material is lost and starts not reverse crumpling. The parameters X=0 and Y=0 match specific values of parameters *x* and *y* used in the equations.

For fluting material value *P* limit is about 120 to 450 kPa, and for the cardboard – under from 350 to 1050 kPa. The effort that provides cylinder clamping to the corrugated layer surface, balanced by contact pressure:

$$P = -\int_{-a}^{a} p(x)dx = k\int_{-a}^{a}\left(d_{max} - R + \sqrt{R^2 - x^2}\right)dx\ . \quad (4)$$

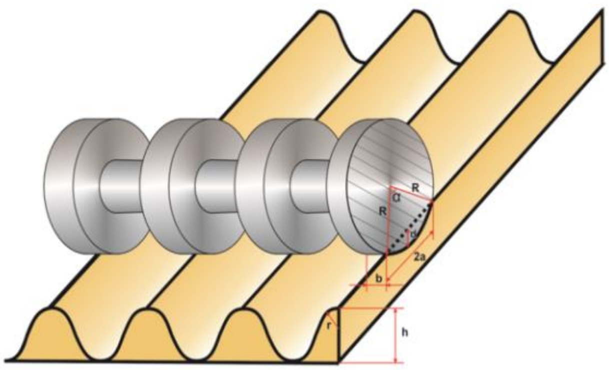


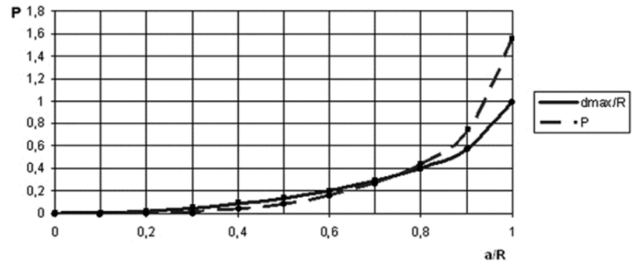
Fig. 4 3D model of waves corrugations deformation in the run-up of putting glue roller on the micro-corrugated cardboard, where *R* – cylinder radius of putting glue disk, *b* – width of the contact zone, *h* – wave corrugated height, r – wave corrugation radius, 2*a* – value of the contact zone, *d* – value of wave corrugated flexing, *α* – contact zone angle

Considering the dependence (3) can be obtained correlation that links linear force *P* with a maximum deflection d_{max} or maximum pressure p_{max}:

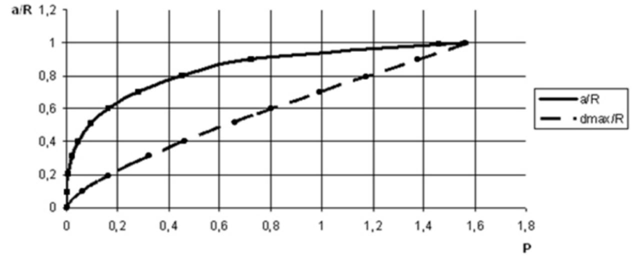
$$\begin{aligned} P &= 2ak(d_{max} - R) + ak\sqrt{R^2 - a^2} + kR^2 \arcsin\frac{a}{R} = \\ &= ak(d_{max} - R) + kR^2 \arcsin\frac{a}{R} = \\ &= k\left(\begin{aligned} &\sqrt{d_{max}(2R - d_{max})}(d_{max} - R) + \\ &+ R^2 \arcsin\frac{\sqrt{d_{max}(2R - d_{max})}}{R} \end{aligned} \right) = \\ &= \frac{1}{k}\left(\begin{aligned} &-\sqrt{-p_{max}(p_{max} + 2kR)}(kR + p_{max}) + \\ &+ kR^2 \arcsin\frac{\sqrt{-p_{max}(p_{max} + 2kR)}}{kR} \end{aligned} \right). \end{aligned} \quad (5)$$

A result of researches it was established that when the wave of corrugated has a sinusoidal original form, bandwidth 2b increases with the contact pressure increasing and, consequently, the surface deflection d, due to bending deformation of wave corrugated form and limited to loss of elastic properties at extreme deflection. Elastic deformation (compression) of trapezoidal wave corrugated form does not changing the width of the contact zone, as connected only with the phenomenon of its side faces loss stability.

Dependence of maximum deflection d_{max}/R and pressure - $p_{max}/(kR)$ and the relative value of contact zone *lengtha/R* from the relative linear effort $R/(kR^2)$ shown in Fig. 5.



a

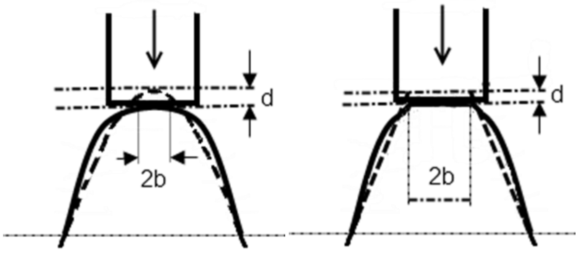


b

Fig. 5 Dependence of the contact zone length from relative linear effort (a) and the actual use of linear effort (b), where *a/R*-length of contact zone, d_{max}/R- maximum deflection of micro wave corrugation, *P* - linear efforts

For the real use values of parameters at *a/R* <<1 have a content, as in the case of *a/R* ≈1, d_{max} / *R*≈1 cylinders have to deform the layer material almost to the very axis that does not meet to technological standards.

Let us consider the influence of mechanical factors on micro-corrugated cardboard based on the research of model in which its deformation associated with changing bandwidth of applying adhesive layer.



a b

Fig. 6 Typical types of micro corrugations wave crumpling a) sinusoidal; b) trapezoid, where 2*b* - bandwidth of contact between putting glue roller and wave corrugation, *d* - value of wave corrugations deflection

If the corrugated wave has an initial sinusoidal shape (Fig. 6, a), then bandwidth 2*b* grows with increasing of contact pressure and, therefore, surface deflection *d*. This increase is due to bending deformation of wave corrugations form and limited to loss of elastic properties at extreme deflection. Elastic deformation (compression) of trapezoidal wave corrugations form (Fig. 6, b), in contrast to the sinusoidal, does not change the width of the contact zone. The reason is that the trapezoid is more rigid and its deformation associated primarily with the phenomenon of side's stability loss. In some cases if it is necessary for adjusting of contact bandwidth a convenient choose of wave corrugation form without angle kinks in the contact zone. The dependence of the contact bandwidth from deflection is determined in each case by the wave corrugations form.

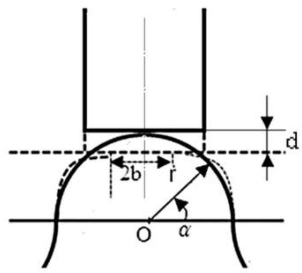


Fig. 7 Sinusoidal wave corrugations form, where 2*b* - bandwidth of contact between putting glue roller and wave corrugation, *d* - value of wave corrugations deflection, *r* - radius

We considered the case of sinusoidal wave corrugations form deformation (Fig. 7). Due to the axial symmetry of the problem, we considered the half of wave corrugation, which median line bends under load (Fig. 8).

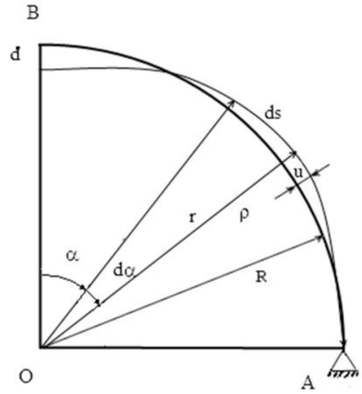


Fig. 8 The scheme of calculating of wave corrugated deflection under load, where *R*, *α* - polar coordinates, *ρ* - curvature of axial line AB to deformation, *u* - deflection

To construct a mathematical model of said tasks we used the theory of beams bending (rods) [8, 9] that adequately reflects the geometry and parameters of wave corrugations deformation. If the curvature of center line AB before deformation is described by the expression:

$$\frac{d\alpha}{ds} = \frac{1}{r}, \tag{6}$$

then after deformation, if we will use polar coordinates *R*, *α*, where *R* = *r* - *u* and the formula for determining the curvature, we will get:

$$\frac{1}{\rho} = \frac{R^2 + 2\left(\frac{dR}{d\alpha}\right)^2 - R\frac{d^2R}{d\alpha^2}}{\left[R^2 + \left(\frac{dR}{d\alpha}\right)^2\right]^{3/2}}. \tag{7}$$

In the latter depending after ignoring the values of the highest order little significance an expression for the curvature:

$$\frac{1}{\rho}=\frac{d\alpha}{ds}\left(1+\frac{u}{r}\right)+\frac{d^2u}{ds^2}\,. \qquad (8)$$

Substituting (8) into the equation of bend [7]:

$$\frac{1}{\rho}-\frac{1}{r}=\frac{M}{EJ} \qquad (9)$$

we receives Boussinesq equation [4, 7, 9] to determine the deflections *u*:

$$\frac{d^2u}{d\alpha^2}+u=\frac{Mr^2}{EJ}\,. \qquad (10)$$

Equation (10) is integrated and its general solution is:

$$u=\left(C_1-\int\frac{Mr^2}{EJ}\sin\alpha\, d\alpha\right)\cos\alpha+$$
$$+\left(C_2-\int\frac{Mr^2}{EJ}\cos\alpha\, d\alpha\right)\sin\alpha. \qquad (11)$$

Considering the flexural stiffness of micro corrugation like constant to solve the problem it is necessary to find an expression for the bending moment *M* in any section of micro corrugation. Let us consider the left half of wave corrugations (Fig. 9).

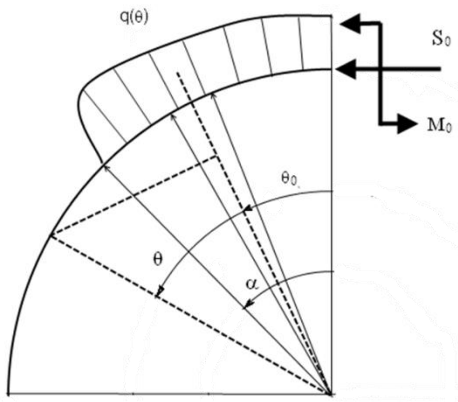


Fig. 9 The scheme of calculating of force bending moment in any section of the wave corrugations, where M_0 - bending moment and S_0 - force, replacing the action of the right half-wave, $q(\theta)$ - distributed load, generated by pressure of cylinder at the site of contact $[0, \alpha]$, θ - arbitrary coordinate

At Fig. 9. M_0 and S_0 - bending moment and force, that replacing the action of the right half-wave, $q(\theta)$ - distributed load generated by pressure of cylinder at the site of contact $[0, \alpha]$.

Then an expression for the bending moment in any coordinate θ is:

$$M(\theta)=-M_0-S_0r(1-\cos\theta)+$$
$$+\begin{cases} r^2\int_0^{\theta} q(\theta_0)\sin(\theta-\theta_0)d\theta_0, \ at\ \theta\le\alpha \\ r^2\int_0^{\alpha} q(\theta_0)\sin(\theta-\theta_0)d\theta_0, \ at\ \theta\ge\alpha \end{cases}. \qquad (12)$$

Substituting (12) to (11), we will get:
for $\theta<\alpha$

$$\frac{EJ}{r^2}u=C_1\cos\theta+C_2\sin\theta-M_0-$$
$$-S_0r\left(1-\frac{\theta}{2}\sin\theta\right)+r^2\int_0^{\theta} q(\theta_0)\sin(\theta-\theta_0)d\theta_0, \qquad (13)$$

for $\theta\ge\alpha$

$$\frac{EJ}{r^2}u=C_1\cos\theta+C_2\sin\theta-$$
$$-M_0-S_0r\left(1-\frac{\theta}{2}\sin\theta\right)+$$
$$+\frac{r^2}{2}\int_0^{\alpha} q(\theta_0)\left[\sin\ \theta\cos\theta_0-\theta\cos(\theta-\theta_0)\right]d\theta_0. \qquad (14)$$

To investigate the dependence of wave corrugated deformation it was selected two main dimensionless parameters: the radiuses ratio *R/r* and relative dipping of putting glue roller in corrugated wave. It was obtained the dependence of relative deflection $\tilde{u}=\frac{u}{r}$ from α (where - $\alpha=\arccos(1-\tilde{d})$, $\tilde{d}=\frac{d}{r}$ are relative value) and the angle θ in relative Cartesian coordinates (*X/r*, *Y/r*, *Z/r*) with using methods of modeling.

We assume that the value of α greater than 30° is inadmissible since the magnitude *d* greater than *r* on 14%. The dependence of the wave corrugated flexing from d_{max}/r at constant *R/r* = 10 and *Y* = 0 shown in Fig. 10. Analyzing the results it can be argued that the ratio of *R/r* does not change the maximum deflection of micro corrugation and only increases the length of the contact zone. Changing the maximum deflection dictated only by parameter d_{max}/r .

For a visual representation of deformed micro corrugation it was built a plot of dependence the deflection in the transverse (radial) section at constant *R/r* = 10 and *X* = 0 (Fig. 11) in polar coordinates (ρ, θ).

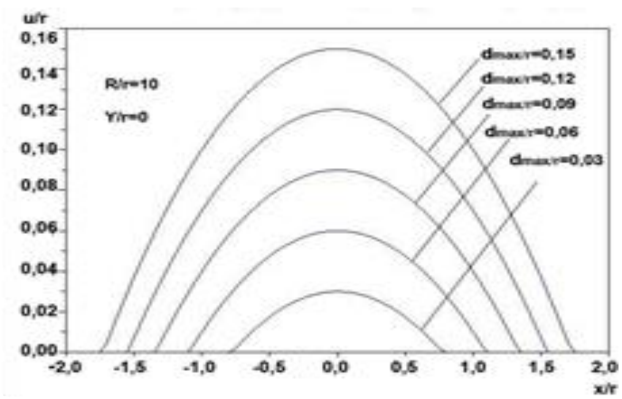


Fig. 10 The dependence of the wave corrugated relative deflection $\tilde{u}$ in longitudinal section at constant respect radiuses *R/r* = 10 and *Y* = 0, where $\tilde{u}$ - relative deflection of wave corrugations, *R/r* - ratio of radiuses and d_{max}/r - relative dipping of putting glue roller in wave corrugation, d_{max}- maximum deflection of wave corrugations

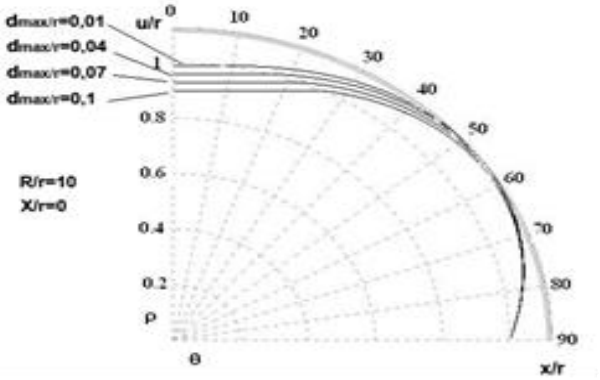


Fig. 11 The dependence of the wave corrugated relative deflection $\tilde{u}$ in the transverse section at constant R/r = 10 and X = 0 in polar coordinates (ρ, θ)

The character of wave corrugated deformation illustrates a three-dimensional model (Fig. 12), in which the value of R/r = 10 and d_{max} / r =0.15.

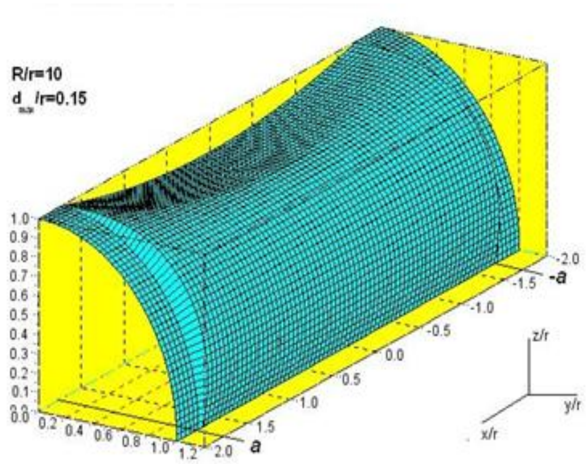


Fig. 12 3D model of wave corrugated deformation, where R/r - ratio of radiuses and d_{max}/r - relative dipping of putting glue roller in wave corrugation, which is dimensionless parameters

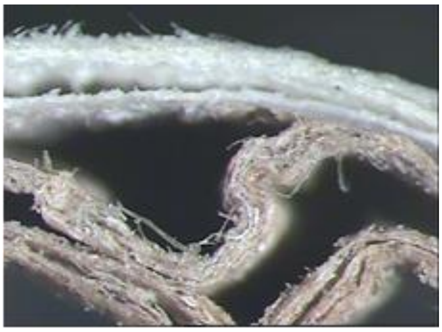
Therefore, it was established that the ratio of radiuses and maximum of wave corrugated deflection are major factors that influence on the deformation of micro-corrugated cardboard in longitudinal and transverse direction during lamination.

Samples deformation of laminated micro-corrugated cardboard confirmed the stability of its adhesive bond to multiple bends that clearly seen in Fig. 13, where the destruction is on the structure of liner and fluting.

During the research of deformation properties of laminated micro corrugated cardboard it was founded that its strength in the machine and transverse directions grows with the increase of the fluting thickness. So the machine direction should be kept perpendicular to the height of packaging construction during its design.

Experimental research established that the increase of the liner thickness on 112 units the bundle efforts of fibers increases by 10 N in the transverse direction for all samples of investigated micro-corrugated cardboard.

a



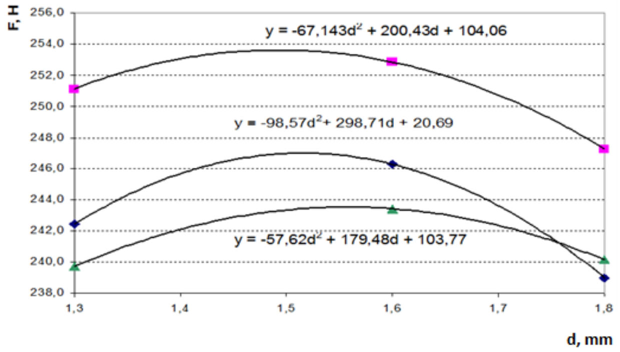
b

c

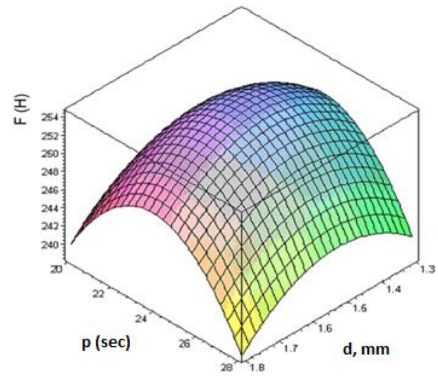
d

Fig. 13 Morphological structure of micro-corrugated cardboard components laminated with glue №3 [×100]: a - liner with gsm 180 g/m^2, fluting with gsm 112 g/m^2; c - liner with gsm 250 g/m^2, fluting with gsm 140 g/m^2; a, c - before and b, d - after tests on bending

For regimes optimization of micro-corrugated lamination process it is important to identify factors that influence the quality of laminated micro-corrugated cardboard. It was found that the viscosity of the adhesive and the distance between putting glue and clamping rollers of designed adhesive system of laminating machine significantly affects the quality of lamination. Therefore, to ensure a strong adhesive connection between the liner and fluting it is important to determine their optimal values. Mathematical and



a



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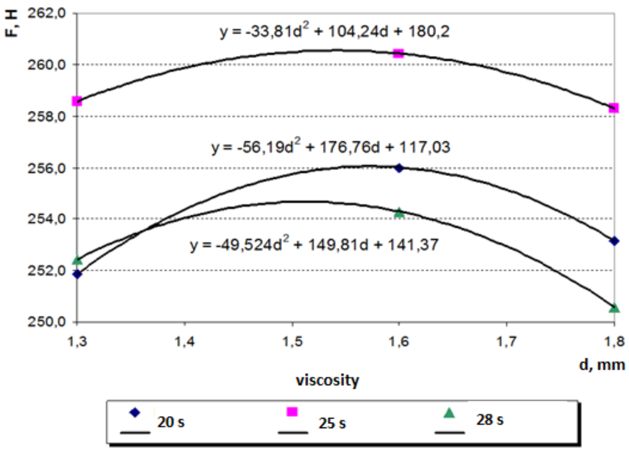

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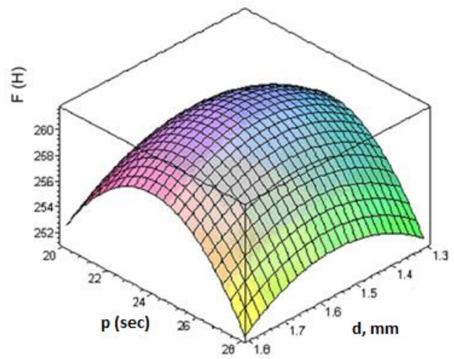

d

Fig. 14 Graphic dependences (a, c) and obtained 3D model (b, d) of bundle efforts of laminated micro-corrugated cardboard $F(H)$ on the viscosity of the adhesive $p(s)$ and the distance d (mm) between rollers in laminating devices: a,b) liner with gsm 180 g/m^2; c,d) liner with gsm 250 g/m^2

statistical processing of experimental results and their optimization was performed by mathematic modeling with method of least squares in program Original 7.0 and Excel. For the mathematical modeling were selected following parameters: distance between laminating rollers – 1.3; 1.6; 1.8 mm and the viscosity of the adhesive 25 sec.

It was established that at a distance of 1.3 mm there is a significant clamping force causing the corrugations deformation. Increase of the distance between rollers to 1.8 mm - reducing lamination strength on 2 - 4 units. Fig. 14 presented graphics depending and 3D models of stratification efforts of laminated cardboard, as a function of two arguments: viscosity and distance between the rollers in laminating device. As you can see, despite the increase in bulk density of micro-corrugated cardboard from 180 to 250 g/m^2, there is a similar relationship between these factors.

Thus, as a result of mathematical modeling and experimental researches it was found optimal regimes of lamination process (viscosity of adhesive 25 sec and the distance between the laminating rollers 1.6 mm of laminating device) that provide the greatest strength during lamination (252.9 and 260.4 N) of cardboard with gsm 180 g/m^2 and 250 g/m^2, respectively, to micro-corrugated cardboard 311E.

4. Conclusions

1. As a researches result of stress-strain state of laminated micro-corrugated cardboard components, it was derived mathematical models from the Hertz contact bodies' theory and Winkler hypotheses that take into account the value of bending moment of wave corrugated forces, depending of its deflection in longitudinal and transverse section and the pressure, that helps to determine the optimal ratio of the contact zone between the rollers of the adhesive system and wave corrugated radius.
2. It was established that the ratio radiuses *R/r* does not change the maximum of corrugated deflection, but only increases the length of the contact zone.
3. It was investigated the dependence of wave corrugated deformation in the transverse and longitudinal directions and two main dimensionless parameters: the ratio radiuses *R/r* and the relative immersion the adhesive roller in the wave corrugated d_{max} / r and established that change of it maximum deflection dictated only by parameter d_{max} / r .

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S. HAVENKO, V. BERNATSEK, S. KHADZHYNOVA

INVESTIGATION OF STRESS-STRAIN STATE OF MICRO-CORRUGATED CARDBOARD COMPONENTS IN THE PROCESS OF ITS LAMINATING AND MANUFACTURING PACKAGING

S u m m a r y

It was investigated the influence of mechanical factors on the restructuring of micro-corrugated cardboard based on physical models. We considered typical cases of stress-strain state of micro corrugated cardboard during laminating, particularly when micro-corrugated cardboard, as a resilient layer of a certain thickness, compressed by efforts of two rigid cylinder of the same radius. Deformation of micro corrugated cardboard associated with the change of bandwidth of adhesive layer. It was established that when the corrugated wave has a initial sinusoidal shape, the contact bandwidth increases with contact pressure increasing and, consequently, the surface deflection, due to bending deformation of wave corrugated form and limited to loss of elastic properties at extreme deflection. Elastic deformation (compression) of trapezoidal wave corrugated form does not changing the width of the contact zone, as connected only with the phenomenon of its side faces loss stability. It was established that the ratio of radiuses and maximum of wave corrugated deflection are major factors that influence on the deformation of micro-corrugated cardboard during lamination in longitudinal and transverse direction.

Keywords: wave corrugations form, micro-corrugated cardboard, lamination, stress-strained state, modelling.

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