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# Research on fundamental frequencies and dynamic characteristics of pre-stressed concrete beams based on experiment and numerical simulation

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## 1. Introduction

In recent years, pre-stressed concrete structures have been widely applied in many civil engineering structures gradually by virtue of their characteristics including economic efficiency, practicability, strong carrying capacity, durability and so on. Among pre-stressed concrete structures, pre-stressed concrete beam structure is the most common. Pre-stressed concrete structures have the advantages of improving the crack resistance and stiffness of components, saving materials, reducing weight and enhancing the anti-fatigue performance of components. Due to these advantages, pre-stressed concrete structures have been widely applied in civil engineering including bridges, building structures, ocean platforms and so on since the 1930s. They are playing an increasingly role in economic life and social development [1-2]. In the recent 20 years, more than 75% of concrete bridges have adopted pre-stressed concrete structures.

Under the situation of design loads or overload, slab bridges may lose a part of pre-stress due to the creep of concrete and the looseness of pre-stress tendons through long-term service. If pre-stress is seriously less than the design value, the normal service ability of bridge structure will be affected. Security issues will appear if the situation is serious. At present, studies on pre-stressed concrete bridges have made many achievements. Miyamoto [3] studied the performance of unbonded pre-stressed concrete beams, provided the formula of beam modal frequency in the case of a given pre-stress value and used the results of experiments and tests for correction. Abraham [4] tested changes in the natural frequency of beams and found that the frequency of beam presented a rising trend with the decrease of pre-stress, but pre-stress had little influence on the vibration modal of the beam. He [5] studied PRC beam through vibration tests and obtained a conclusion that the natural vibration frequency of PRC beam would decrease with the increase of pre-stress in the case of unchanged constraint conditions, which was consistent with the research conclusion of Abraham. Liu [6] studied the relationship between the fundamental frequency of T-type beam and pre-stress. The experimental result showed that the pre-stress had an influence on the vibration frequency of concrete beams. Vibration frequency increased with the increase of pre-stress values, instead of that vibration frequency decreased with the increase of pre-stress value according to the computation of theoretical formulas. Liu [7] further carried out research and obtained a conclusion that the regulation of influence of pre-stress on the fundamental frequency of simply supported beam was related to the way of constraint and the variation of the fundamental frequency was depended on cross-section form, material characteristics and pre-stressed eccentricity. Lv [8] studied the response of free vibration and forced vibration and reached a conclusion that axial pre-stress had a greater influence on the low-order frequencies of beams than the high-order frequencies of beams and the frequency of beams gradually decreased with the increase of pre-stress. In the meanwhile, Lv pointed out that it was because pre-stress played a role in compressing and softening beam and equivalently reduced the stiffness of beams. The influence of pre-stress on the dynamic performance and response of beams also drew the attention of people. References [9-11] supposed pre-stress increment was proportional to the vibration displacement of beam midpoint, and analyzed the dynamic characteristics of pre-stressed beams. References [12-14] adopted the modal perturbation method of complex dynamic analysis to analyze the dynamic characteristics of pre-stress beams and conducted a comparative analysis on the obtained result and experimental data. Regarding the relationship between pre-stress and natural vibration frequency, reference [15] thought an axial pressure applied externally would lead to the transverse softening of beams. Namely, axial pressure increased transverse deformation and reduced the natural vibration frequency of beams. Reference [16] thought the natural vibration frequency of pre-stressed beams in tests increased with the increase of pre-stress, which was opposite to the conclusion obtained by isotropic materials under the action of axial force of classical dynamics.

In summary, studies on the pre-stressed beam and fundamental frequency are mostly based on single experiments or analyzed through adopting finite element method [5-16], and there are few studies combining experiments model and finite element method. This paper adopted the hammering method to excite concrete beams with different pre-stress values, eccentricities and counterweights based on two kinds of methods including experiment and numerical simulation, processed and analyzed the collected midspan acceleration of beams to quantitatively analyze the relationship between the fundamental frequency of beams and pre-stress. Finally, this paper obtained the deflection of pre-stressed concrete beams under the variable load through experiments and finite element method. This research took into account comprehensive and systematic factors. In addition, the research result was very reliable.

## 2. Theoretical basis

At present, analysis on the fundamental frequency of pre-stressed concrete beams mostly considers the pre-stress in the beam as the external force of beams. Based on this analytical theory, the stress system of pre-stressed simply supported beam was simplified as follows. The simplified result was shown in Fig. 1.


Fig. 1 a) Schematic diagram of simply supported beam. b) Schematic diagram for the force of pre-stressed simply supported beam

The following assumptions were made in the case of analyzing pre-stressed beam: 1. Follow Euler-Bernoulli beam theory, namely without considering shear deformation; 2. Consider pre-stress tendons as bonded pre-stress and consistent deformation of pre-stress tendons and beam body and without considering the influence of bond slip; 3. Without considering initial upper arch, namely the straight axis of beam body under the action of weight and pre-stress; 4. Without considering the influence of damping. These assumptions were appropriate to analyze natural vibration characteristics and then obtain the vibration equation of simply supported beam:

$$EI\frac{\partial^4 y(x,t)}{\partial x^4}+T\frac{\partial^2 y(x,t)}{\partial x^2}+\bar{m}\frac{\partial^2 y(x,t)}{\partial x^2}=0. \quad (1)$$

Saiid simplified pre-stress into the axial force applied on both ends of the beam and obtained a conclusion that the natural vibration frequency of pre-stressed simply supported beam decreased with the increase of pre-stress. He built motion differential equation through a simply supported beam under axial force and obtained the expression formula of natural vibration frequency of pre-stressed concrete simply supported beam under the action of pre-stress as follows:

$$\omega_n=\sqrt{\left(\frac{n\pi}{l}\right)^4\frac{EI}{\bar{m}}-\left(\frac{n\pi}{l}\right)^2\frac{T}{\bar{m}}}. \quad (2)$$

This theoretical simplification did not take into account the interaction between pre-stress tendons and concrete. It was improper to only take pre-stress tendons as a concentrated force applied to beam ends. Based on the above theory, the formula of natural vibration frequency of concrete simply supported beam was modified as follows:

$$\omega_n^2=\frac{n^4\pi^4\left\{E\left[I_0+e_0^2(1-\eta)^2bh\right]+E_SA_S\eta^2e_0^2\right\}}{mL^3}, \quad (3)$$

wherein $n$ was positive integer; $L$ referred to the span of beams; $e_0$ stood for the eccentricity of pre-stress tendons; $E_S$ and $A_S$ were elasticity modulus and sectional area of pre-stress tendons; $E$ represented the elasticity modulus of concrete; $I_0$ was the inertia bending moment of resistance; $m$ was the total mass of beam body; $\eta$ was the eccentric correction coefficient of cross-section.

It could be seen from the above formula that the stiffness of tensioned pre-stress tendons just offset the effect of "stress softening" brought by the pressure of beam end. In bonded pre-stressed simply supported beam, pre-stress tendons had the same influence on natural vibration characteristics with ordinary tendons and only played a role in strengthening the cross-section. The above formula was analyzed to found that the size of pre-stress basically had no obvious influence on the natural vibration frequency of concrete beams and the natural vibration frequency of concrete beams increased with the increase of elasticity modulus, area and eccentricity of pre-stress tendons.

The relationship between the fundamental frequency of pre-stressed beam and pre-stress value could not be well explained based on the theory of classical mechanics. Therefore, this paper would adopt the method of laboratory model experiments to analyze this problem in detail.

## 3. Experiments of the fundamental frequency of pre-stressed beams

### 3.1. Experimental proposal

At the bottom of the beam, there were 2 tendons with a diameter of 14 mm and a total area of 580 mm$^2$. The centre of tendons was 342 mm away from the top of beam. There were hoops vertical to the beam, which adopted HPB400 tendons and had a diameter of 6 mm. The thickness of the protection layer was $c$ = 20 mm. The spacing of hoops was 150 mm. According to experimental design, the yield strength of tendons was 400 MPa. Longitudinally, the yield strength and elasticity modulus of tendons were 500 MPa and 200 GPa. The yield strength and elasticity modulus of hoops were 475 MPa and 200 GPa respectively. The experiment adopted C40 concrete. The design value of tensile strength was 1.89 MPa. Elasticity modulus was 34.5 GPa. At appropriate time after casting, the post-tensioning method was used for tensioning. To prevent stress concentration at the pre-stress end, a steel plate with a volume of 100 mm $\times$ 100 mm $\times$ 5 mm was buried here. The model size and reinforcements of rectangular beam were shown in Fig. 2.

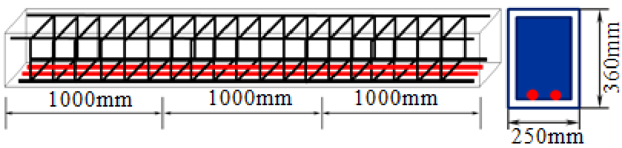


Fig. 2 Sizes of pre-stressed concrete beams (mm)

Generally speaking, experimental modal analysis starts from measuring the input and output of the tested structure. Related links are introduced from three aspects, including structure to be tested, data acquisition system and data analysis method.

Firstly, it is structure to be tested. After structure specimen is completed, it should be supported in an appropriate way in the right place. Excitation point and response point are distributed in the tested structure. Acceleration sensor is installed in the position of response point.

Secondly, data acquisition system is connected and debugged and digital signals in the test are recorded. Digital signals are time-domain signals of force and acceleration. Transformation function is obtained through FFT (Fast Fourier Transform). Finally, modal analysis methods are chosen to obtain the modal parameters of the structure.

Impulse signals shock the excitation point through the force hammer. Force sensor transforms force signals into electrical signals. The acceleration sensor is fixed at the response point. Response signals transform acceleration value into electrical signals through the acceleration sensor. Charge-type sensor connects with the signal acquisition equipment through charge amplifier. Voltage-type sensor connects with the signal acquisition equipment through constant flow source. All signals are recorded by the signal acquisition instrument in the form of voltage values and saved in a computer. Dynamic experiment signal acquisition system is shown in Fig. 3.

Single-point excitation is the most simple, common and reliable way of excitation. The way of single-point excitation is effective because of high reliability and validity of excitation, direct and simple excitation process as well as good controllability of excitation tests. For small and medium-sized structures, single-point excitation can obtain satisfactory results. Pre-stressed concrete beam studied in this paper was a small and medium-sized structure. Therefore, this paper considered using the hammering method for single-point excitation, which not only saved time, but also saved cost. As shown in Fig. 2, at the bottom of concrete beams, there were 2 pre-stress tendons. To test the relationship between different pre-stress values and the fundamental frequency of beams through experiments, this paper adopted jack to tension pre-stress tendons and used an oil pressure gauge to control the tensioning force to obtain the required pre-stress value. As shown in Fig. 3, a vibration acceleration sensor was installed in the center of concrete beams, and the sensor was equipped with an extremely low frequency response and high sensitivity, which showed remarkable effect in safety test of the beam because the analyzed frequency is mainly in the low frequency. After pre-stress was applied, the force hammer was used to hammer one section of beams. In this way, vibration signals of beams would be obtained by data acquisition equipment through the acceleration sensor and inputted into the computer for processing. In each experiment, hammering was repeated three times. The average value of three results was taken as the final experimental data. In the whole experimental process, pre-stress values were set as 0 kN, 50 kN, 100 kN, 150 kN and 200 kN respectively. Therefore, this experiment had 5 working conditions. Fig. 4 showed the test process of the fundamental frequency of pre-stressed concrete beams.

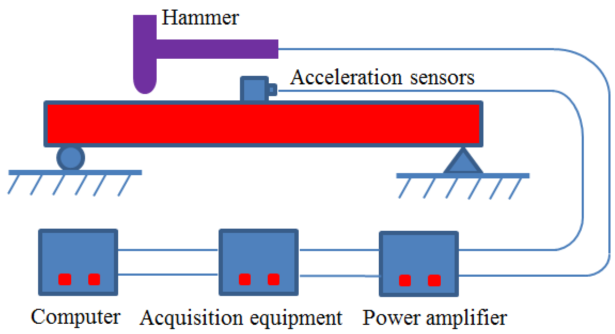


Fig. 3 Modal experiment of pre-stressed beam

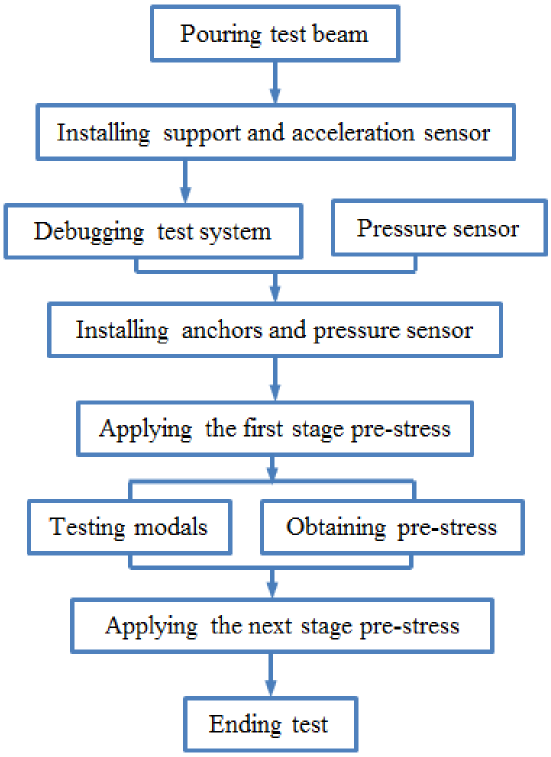


Fig. 4 Modal experimental process of pre-stressed beams

### 3.2 Analysis of experimental results

Vibration acceleration of pre-stressed concrete beams in the time domain could be obtained according to the process of the above experimental proposal. FFT was conducted for time-domain signals to obtain corresponding frequency-domain signals, as shown in Fig. 5.

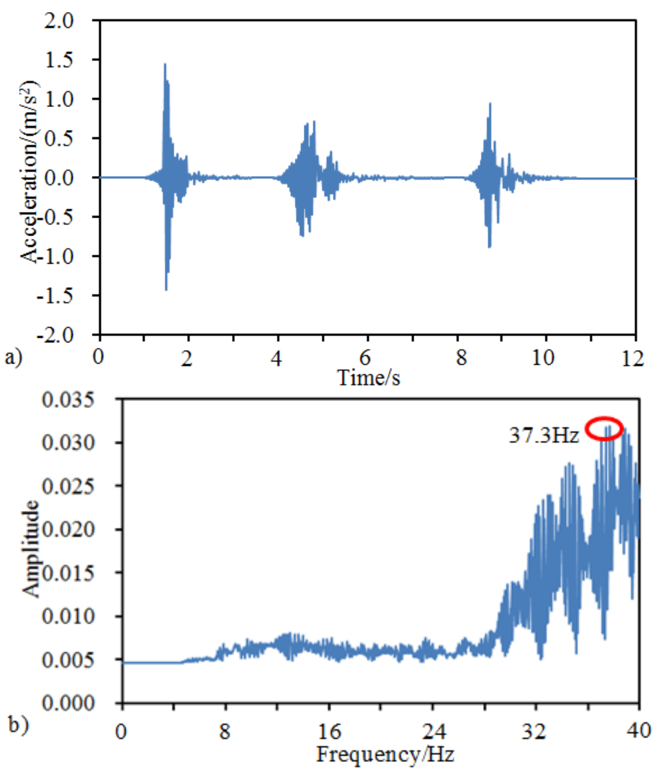


Fig. 5 Acceleration of mid-span of beams under the pre-stress value of 50 kN: a) acceleration in time domain, b) acceleration in frequency domain

It could be seen from Fig. 5 that vibration acceleration had drastic fluctuations after the force hammer applied instantaneous excitation to concrete beam. In 0s to 1.5 s, vibration acceleration was in a steady state mainly because the excitation of the force hammer was still not delivered to the acceleration sensor through concrete beams and the acceleration sensor did not receive effective signals. In 1.5 s to 2.5 s, vibration acceleration had drastic fluctuations and was basically symmetric around 0. The maximum value was 1.5 m/s$^2$ while the minimum value was -1.5 m/s$^2$. In 2.5 s to 4 s, vibration acceleration signals became steady again because vibration nodes would appear due to the special vibration mode of beams. Vibration would become very weak and find it difficult to be captured by the acceleration sensor in the position of vibration nodes. In 4 s to 6 s, vibration acceleration again presented drastic fluctuations. At this time, the maximum value was 0.7 m/s$^2$ while the minimum value was -0.7 m/s$^2$. Fluctuations of vibration acceleration at this stage were not as severe as those at the previous stage. In 6 s to 8.5 s, vibration acceleration again became steady, which had the same cause with the previous stage. However, vibration acceleration again presented drastic fluctuations in 8.5 s to 9.5 s. The maximum value 1.0 m/s$^2$ was while the minimum value was -1.0 m/s$^2$. Later, the process of the previous stage was repeated. FFT was conducted for the vibration acceleration of the above time domain to obtain frequency-domain signals. From frequency-domain vibration acceleration, it could be noticed that vibration acceleration basically became steady before 28 Hz. The fundamental frequency of concrete beams was 37.3 Hz.

Fig. 5 only displayed time-domain and frequency-domain vibration accelerations when pre-stress value was 0 kN. Similarly, time-domain and frequency-domain vibration accelerations could be obtained through experiments when pre-stress value was 50 kN, 100 kN, 150 kN and 200 kN. The change trend of vibration acceleration under these pre-stress values was similar to that under 0 kN. Therefore, a detailed explanation could not be made. Corresponding fundamental frequencies could be obtained through conducting a frequency-domain analysis on concrete beams under various pre-stress values. Fundamental frequencies were 37.8 Hz, 37.96 Hz, 38.81 Hz and 38.99 Hz respectively, as shown in Fig. 6. As shown in Fig. 6, the fundamental frequency of concrete beams showed no obvious changes, which was consistent with the conclusion of Saiid [17] obtained through experiments. It indicated that the experimental result in this paper was credible.

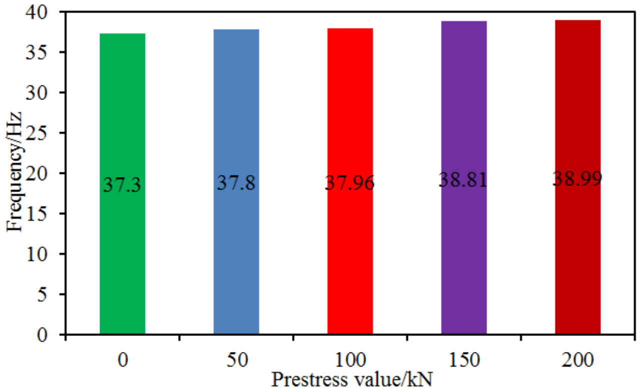


Fig. 6 Relationship between the pre-stress value and fundamental frequency of beams

## 4. Numerical analysis of the fundamental frequency of pre-stressed beams

From the description and analysis of the experimental process, it could be seen that experimental efficiency was low and high cost would be produced. Therefore, this paper considered adopting the finite element method to analyze the dynamic characteristics of pre-stressed concrete beams.

### 4.1. Finite element model

In the case of conducting finite element analysis, the more dense meshes were, the more accurate computational result would be. However, concrete materials were different. Due to the complexity and inhomogeneity of concrete materials, the more dense elements were, the higher the degree of strain concentration might be, which led to the result of non-convergence. Therefore, the density of meshes should be divided appropriately. Large mesh size would result in the decline of computational accuracy and difficulty in coordinating element displacement. Small mesh size would increase element number, long computation time and possible difficulty in convergence. According to general experience, element size should be equal to or greater than the largest bone grain size of several times of concrete. 50mm was used in this paper. C50 concrete used solid elements for modeling. The sectional dimension and material property of concrete were the same with those of model in the test. Hoops used Truss element for modeling, whose connection with concrete of 3D solid elements could adopt public nodes, establish constraint equation and use spring elements so as to consider the bond slip and no slip between tendons and concrete. Under normal circumstances, using Truss element to simulate tendons in concrete could satisfy accuracy. In addition, convergence was relatively good. Most of models in the material library could be used. To avoid large stress concentration at the cross-section of beam end in the case of applying pre-stress and make it convenient to apply pre-stress, pre-stress tendons adopted Rebar element and applied pre-stress through setting initial strain value. In the end, the model contained 5355 elements. Boundary conditions were processed as follows: Point constraints were applied to two supports. The situation of simply support on both ends was simulated through point displacement constraining the direction of x, y and z on one end and point displacement constraining the direction of y and z on another end. The overall finite element model of the structure was shown in Fig. 7.

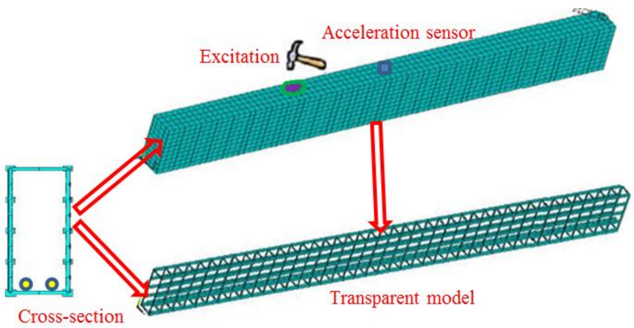


Fig. 7 Finite element model of pre-stressed beams

To analyze the fundamental frequency of pre-stressed concrete beams based on finite element method and compare with the experimental results, the method adopted in the test was used for reference here to apply the load to finite element model. Namely, FFT was conducted for signals to obtain the fundamental frequency information of the structure after simply supported beam model was hammered three times to extract mid-span acceleration response signals of concrete beams. In the process of actual tests, each hammer force was different. To conduct a comparative analysis more accurately, the average value of three experimental results was taken as final data. Fig. 8 displayed the time-domain signals of vibration acceleration of concrete beams when pre-stress value was 0 kN and 200 kN.

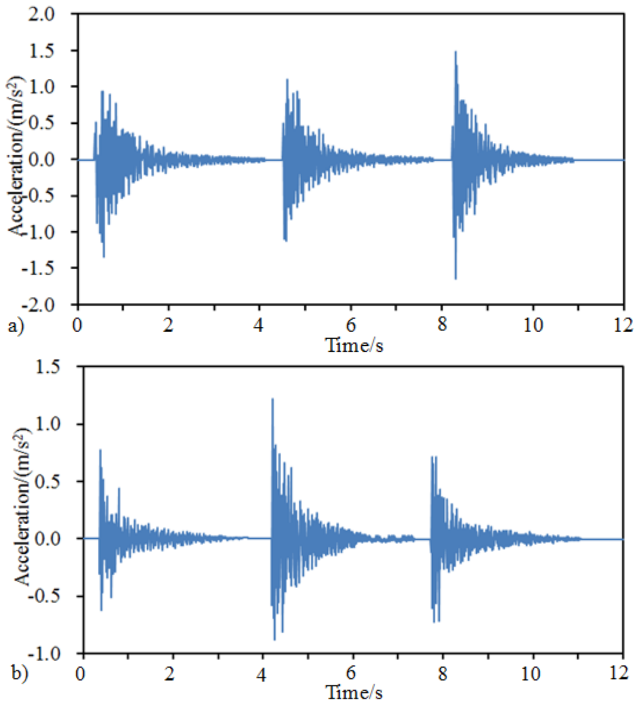


Fig. 8 Vibration acceleration of pre-stressed beam under different pre-stress values: a) acceleration of pre-stressed beam under $T$=0 kN, b) acceleration of pre-stressed beam under $T$=200 kN

As shown in Fig. 8, vibration acceleration signals computed by numerical simulation showed drastic fluctuations many times in the whole analysis time. In 0 s to 0.5 s, vibration acceleration signals were in a steady state mainly because the instantaneous excitation applied at this time was not delivered to the position of observation point through concrete beams. In 0.5 s to 2 s, vibration acceleration signals had severe fluctuations. The maximum value was 1.0 m/s$^2$ while the minimum value was -1.5 m/s$^2$. In 2 s to 4 s, vibration acceleration signals presented a trend of attenuation. There were not drastic fluctuations. In 4 s to 4.5 s, vibration acceleration signals tended to be steady. This cause was similar to the above experimental process mainly because concrete beams would have nodes in the process of vibration and vibration at nodes was weak. In 4.5 s to 6 s, vibration acceleration again showed drastic fluctuations. The maximum value was 1.2 m/s$^2$ while the minimum value was -1.2 m/s$^2$. In 6 s to 8 s, vibration acceleration again decreased. In 8.5 s to 10 s, vibration acceleration again showed serious fluctuations. The maximum value 1.5 m/s$^2$ was while the minimum value was -1.7 m/s$^2$. Therefore, vibration acceleration of concrete beam showed increasingly large peak value of fluctuation over time, which was different from experimental results. When pre-stress value was 200 kN, the change situation of vibration acceleration was similar to that under 0 kN.

### 4.2. Research on the relationship between pre-stress value and the fundamental frequency

The fundamental frequency of concrete beams computed by finite element method was compared with experimental results. The histogram was shown in Fig. 9. As shown in Fig. 9, the relative errors of the fundamental frequency of beams obtained through experiments and finite element were within 1%. In addition, the fundamental frequency of beams and pre-stress value had the same change regulation. Namely, fundamental frequencies showed slight changes with the increase of pre-stress value. The pre-stress value increased from 0 kN to 200 kN. The fundamental frequency of the tested model had increased by 4.5% while the fundamental frequency of the finite element model increased by 5.3%. Under other levels of pre-stressed loads, the tested model and numerical model showed the same order of magnitude of growth trend in the fundamental frequency. The above analysis showed that the real vibration signals of concrete beams could be well collected and the exact relationship between the fundamental frequency and pre-stress value of pre-stressed beams could be obtained after the hammering method was introduced into the numerical model.

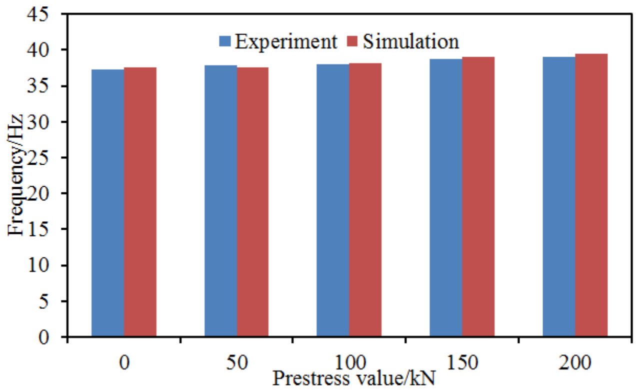


Fig. 9 A comparison of experimental and simulation histogram of pre-stress value

To analyze the relationship between the pre-stress value and fundamental frequency of concrete beams more visually and quantitatively, a comparison of experimental and numerical simulation results was made in the form of curve diagram, as shown in Fig. 10. As shown in Fig. 10, the

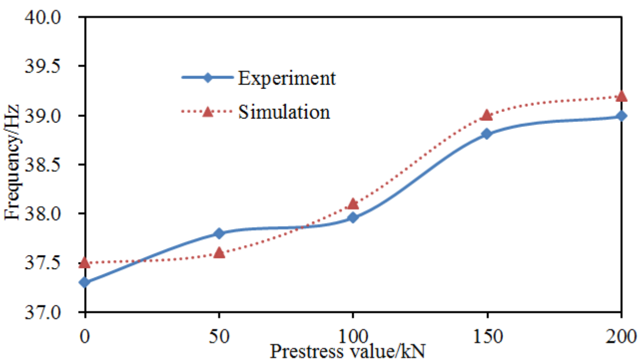


Fig. 10 A comparison of experiment and simulation curve of pre-stress value

change trend and difference value of experiment and simulation were approximate, which indicated that the finite element model in this paper was reliable.

The modal of finite element model was extracted when pre-stress value was 0 kN. The result was shown in Fig. 11. It could be seen from the figure that pre-stressed concrete beam mainly showed bending modes. From the first order mode to the fourth order mode, it could be found that the position of severe vibration of pre-stressed concrete beams extended from the center to both ends. As the mode of concrete beams under other pre-stress values was exactly the same with that under 0 kN, corresponding modes were not presented here. The mode was mainly related to the material parameters and properties of structure. As a result, the mode of concrete beams showed no changes according to different pre-stresses.

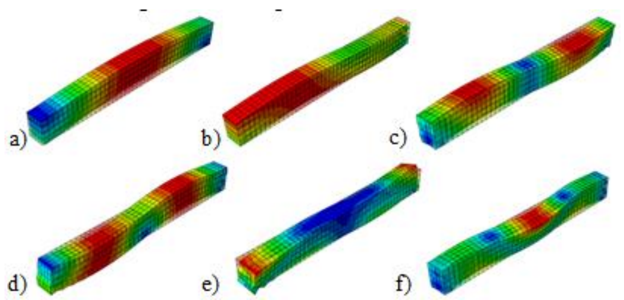


Fig. 11 Modal of pre-stressed concrete beams: a) first order, b) second order, c) third order, d) fourth order, e) fifth order, f) sixth order

4.3. Research on the relationship between eccentricity, pre-stress value and the fundamental frequency of beams

The analysis showed that the finite element model of pre-stressed concrete beams was reliable. Therefore, the verified finite element model in Fig. 7 studied the relationship between pre-stressed concrete beam and eccentricity. As shown in Fig. 12, 2 pre-stress tendons were installed in concrete beams. The distances between tendons and the center were 0 mm, 90 mm and 180 mm respectively. The fundamental frequencies of pre-stressed concrete beams for three different distances were computed. The result was shown in Fig. 13.

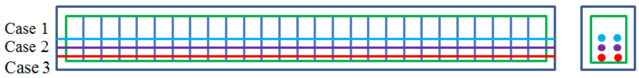


Fig. 12 Pre-stressed beams under different eccentricities

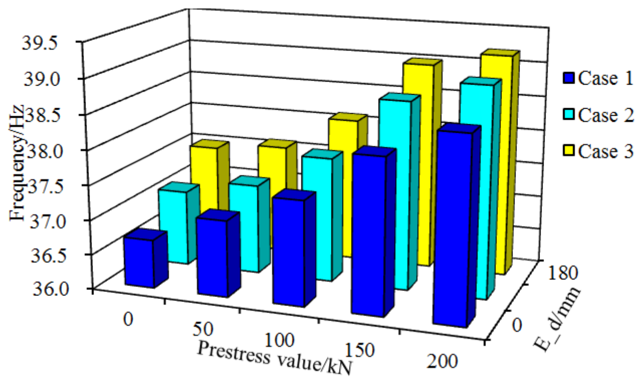


Fig. 13 Relationship between the eccentricity and fundamental frequency of beams

As shown in Fig. 13, the fundamental frequency of pre-stressed concrete beam showed no obvious changes with the increase of pre-stress value under the same eccentricity. When the pre-stress value increased from 0 kN to 200 kN, the fundamental frequency of concrete beams only rose by about 5%. Under the same pre-stress value, the fundamental frequency of concrete beams gradually increased with the increase of the eccentricity. This phenomenon could be explained by the effective stiffness of beam. In the case of certain pre-stress value, the increase of eccentricity would increase the bending stiffness of beam. From the perspective of stress, the bottom of pre-stressed concrete beams was in a state of tension when external vertical loads were applied to the pre-stressed concrete beams. Namely, the position where eccentricity was large was in a state of tension. Based on plane cross-section assumption, the axis part of beams was neutral axis. Therefore, it was the most reasonable to arrange pre-stress tendons at the bottom of the beam body. Modeling and analysis were conducted for various working conditions in order to comparatively analyze the influence of eccentricity of pre-stress tendons on the mode of beams. An explanation was made through extracting the mode of beams under three kinds of eccentricities when pre-stress value was 200 kN, as shown in Fig. 14 to Fig. 16.

Fig. 14 showed the mode of concrete beams when eccentricity was 0mm. It could be seen from the figure that the position of the severe vibration in the top five orders extended from the center to both ends. Pre-stressed concrete beams mainly showed bending modes. Only the fifth order mode showed torsional vibration. The second, fifth and sixth order modes indicated that the vibration of pre-stressed concrete beam was severe mainly on the top.

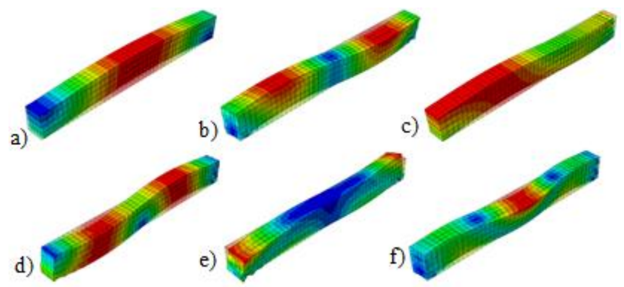


Fig. 14 Modal of pre-stressed beams under the eccentricity of 0 mm: a) first order, b) second order, c) third order, d) fourth order, e) fifth order, f) sixth order

Fig. 15 showed the mode of concrete beams when eccentricity was 90 mm. As shown in the figure, the mode of concrete beams was different from that under the eccentricity of 0 mm. The first, fourth, fifth and sixth order modes

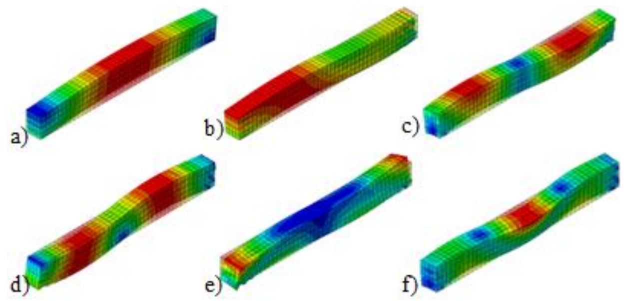


Fig. 15 Modal of pre-stressed beams under the eccentricity of 90 mm: a) first order, b) second order, c) third order, d) fourth order, e) fifth order, f) sixth order

of pre-stressed concrete beams under the eccentricity were consistent with the corresponding mode under the eccentricity of 0 mm. The second order mode was the same with the third order under the eccentricity of 0 mm. The third mode was the same with the second mode under the eccentricity of 0 mm. As well, the fifth order mode showed torsional vibration. Fig. 16 showed the mode of concrete beams when eccentricity was 180 mm. It was totally consistent with that under the eccentricity of 90 mm. So, a detailed explanation would not be made here.

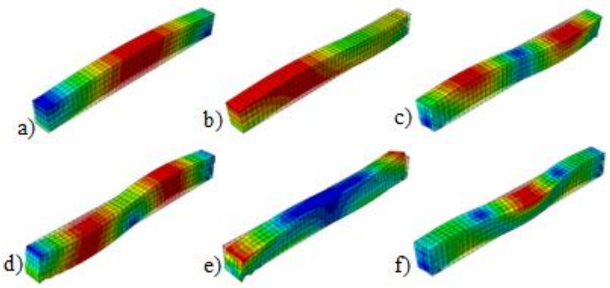


Fig. 16 Modal of pre-stressed beams under the eccentricity of 180 mm: a) first order, b) second order, c) third order, d) fourth order, e) fifth order, f) sixth order

### 4.4. Research on the relationship between counterweight, pre-stress value and fundamental frequency of beams

Vehicles usually pass through the bridge during its use. Therefore, it is necessary to consider the influence of counterweight on the fundamental frequency of pre-stressed concrete beams. As shown in Fig. 17, the counterweight of 0 kN, 30 kN and 60 kN was applied to the position of 1/3 of beams. The eccentricity of pre-stress tendons was 180 mm. The fundamental frequency of pre-stressed concrete beams under different counterweights was computed to make a comparison. The result was shown in Fig. 18.

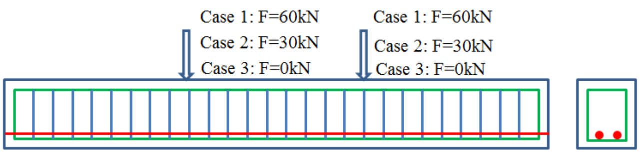


Fig. 17 Pre-stressed beams under different counterweights

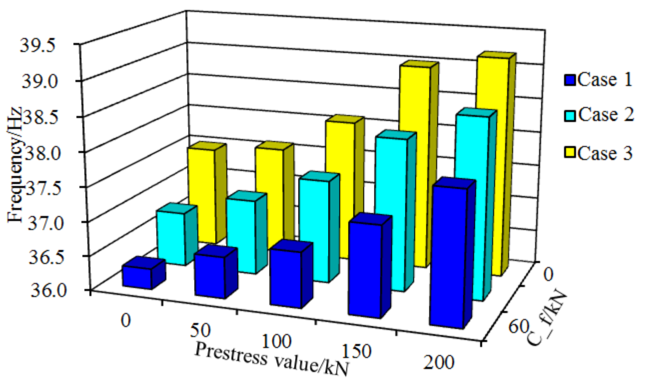


Fig. 18 Relationship between the counterweight and fundamental frequency of beams

As shown in Fig. 18, the fundamental frequency of pre-stressed concrete beams slightly increased with the increase of pre-stress value under the same counterweight. Under the same pre-stress value, the fundamental frequency of concrete beams would decrease with the increase of counterweight. The pre-stress value 200 kN was taken as an example. The fundamental frequency decreased from 39.2 Hz to 37.9 Hz when counterweight increased from 0 kN to 60 kN. This phenomenon could also be explained through increasing the distributed mass of beams. Namely, the existence of concentrated loads was equivalent to the increase of distributed mass of beam body to some degree. In the process of vibration, the upper loads of beam body did not disappear, but motioned accordingly, which thus would affect the stiffness matrix of beam body in the process of vibration and then influence the fundamental frequency of beams. To study the relationship between counterweight and modal, modeling and analysis were conducted for various working conditions. Here, pre-stress value 200 kN was taken as an example to compute the mode of concrete beams under three kinds of counterweight, as shown in Fig. 19 to Fig. 21. Fig. 19 showed the mode of concrete beams when counterweight was 0 kN, which was consistent with that under the eccentricity of 90 mm and 180 mm.

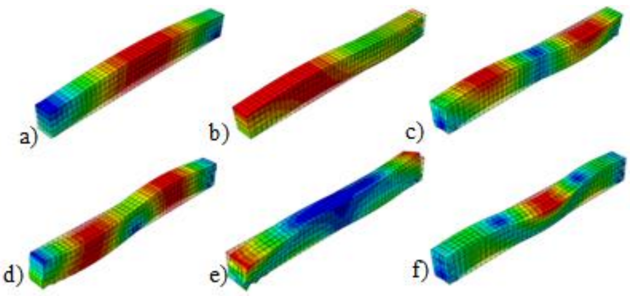


Fig. 19 Modal of pre-stressed beams under the counterweight of 0 kN: a) first order, b) second order, c) third order, d) fourth order, e) fifth order, f) sixth order

Figs. 20 and 21 showed the modes of concrete beams under the working condition with the counterweight of 30 kN and 60 kN. As shown in two figures, the modes of concrete beams under two counterweights were consistent.

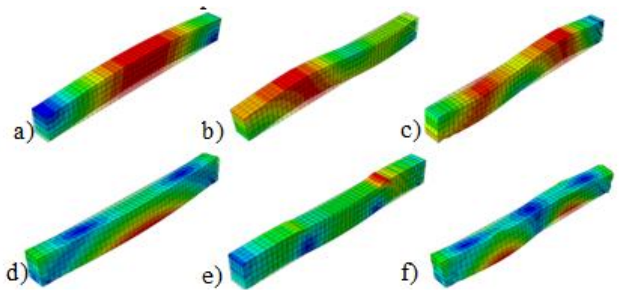


Fig. 20 Modal of pre-stressed beams under the counterweight of 30 kN: a) first order, b) second order, c) third order, d) fourth order, e) fifth order, f) sixth order

Through comparing with Fig. 19, it could be seen that the top two order modes showed no obvious changes after counterweight was applied to pre-stressed beams. However, the latter four order modes changed to a certain degree. Through the comparison of the third order mode, it showed that the position of severe vibration of concrete beams extended from the upper part to the lower part after counterweight was applied. It was because corresponding counterweight was applied here. Through the comparison of the fourth mode, it showed that vibration transferred from both ends to the center. The fifth order mode of vibration transformed from torsional vibration into bending vibration in the middle part. The sixth mode transformed from both ends to the lower part.

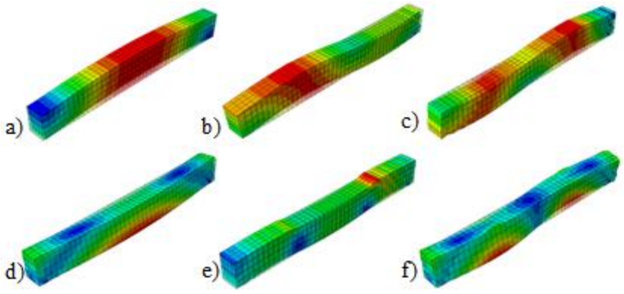


Fig. 21 Modal of pre-stressed beams under the counter-weight of 60 kN: a) first order, b) second order, c) third order, d) fourth order, e) fifth order, f) sixth order

## 5. Analysis of dynamic characteristics of pre-stressed beams

Only the fundamental frequency and modal of pre-stressed concrete beam were studied above. However, the dynamic characteristics of concrete beams were not involved. As shown in Fig. 22, load was applied to the position of 1/3 of concrete beams. The load could turn into a variable load through the jack shown in Fig. 23 in the process of research. The tested content in this experiment included the size of applied load and the mid-span deflection of concrete beams. The deflection of concrete beams was tested by displacement measurement. Jack was used to apply the load. The computer was used to load step by step. The displacement meter was used to measure deformation.

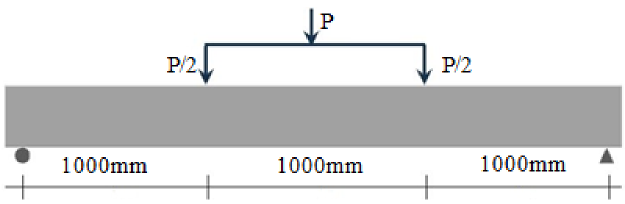


Fig. 22 Schematic diagram for the dynamic loading of pre-stressed beams

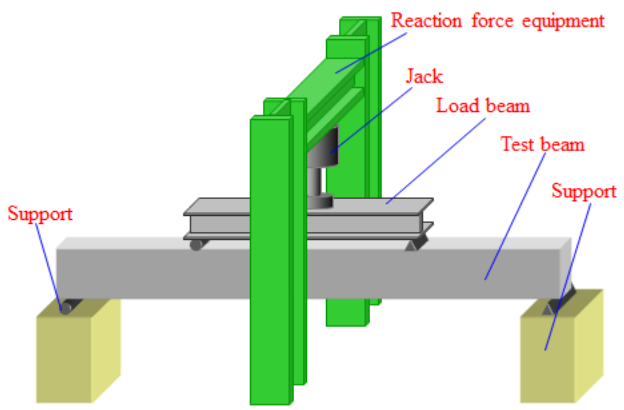


Fig. 23 Dynamic loading experiment of pre-stressed beams

According to the loading process of the experiment, the finite element model of concrete beams was established

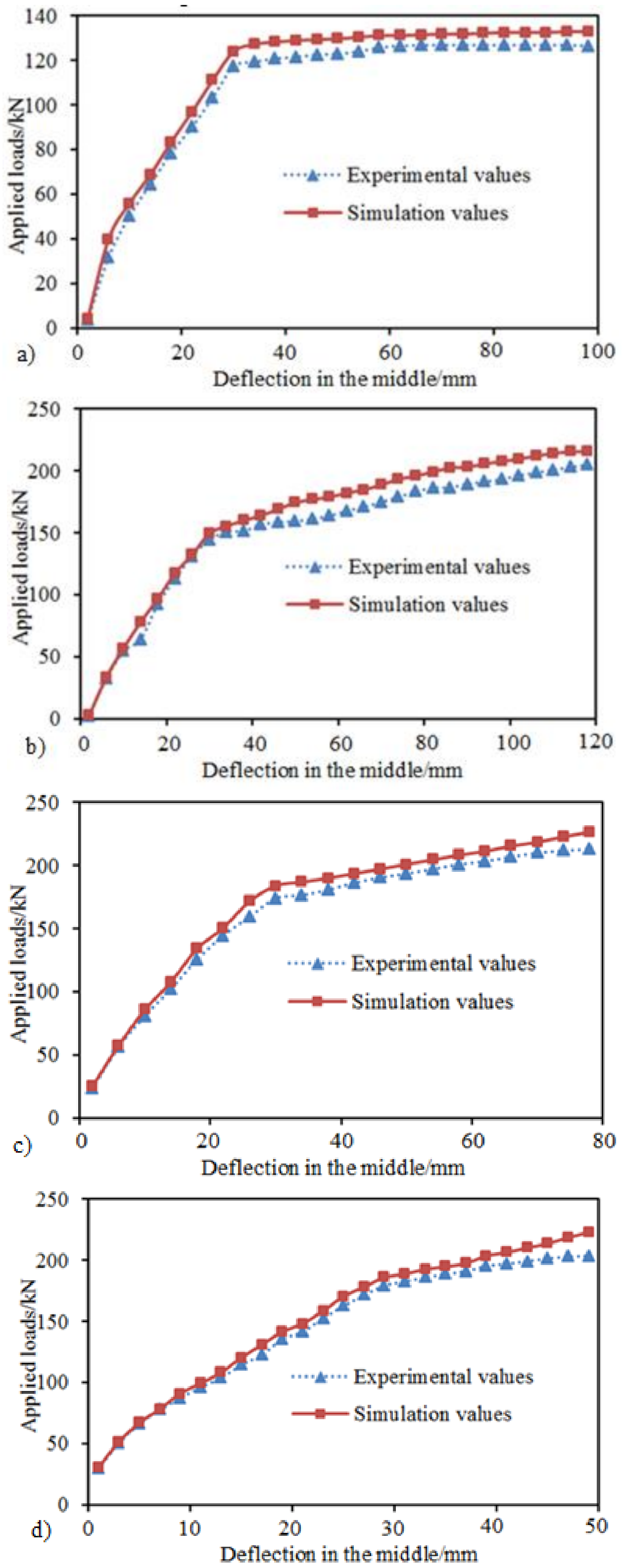


Fig. 24 A comparison between experiment and simulation of deflection of pre-stressed beams: a) beam without pre-stress tendons, b) beam with pre-stress tendons (0 kN), c) beam with pre-stress tendons (100 kN), d) beam with pre-stress tendons (200 kN)

to compute the deflection of corresponding positions and compare with experimental results, as shown in Fig. 24. For concrete beams, applying pre-stress tendons was equivalent to strengthening the beam. According to the comparison of experimental results in Fig. 24, load-deflection curves of various beams at the initial stage of applying the load (before the crack of concrete beam) showed small differences. Load-deflection curves of test pieces basically presented a linear relationship. At this time, the strain of pre-stress tendons was very small, played an insignificant role and made little contribution to the effect of crack resistance. After the crack of concrete beam, the stiffness of concrete beam decreased accordingly due to the reduced effective height of cross-section. The load-deflection curve slope decreased and deflection grew quickly. Before the yielding of tendons, the slope of load-deflection curve of the same test sample constantly decreased with the development of crack, but showed no obvious changes and was mainly manifested in elastic deformation. Load-deflection curve slope of strengthened beam increased to some degree compared with the corresponding slope of un-strengthened beam. As displayed from Fig. 24, a, load-deflection curve reflected that the beam was strengthened inadequately for un-strengthened beam. According to the whole change process of un-strengthened beam, mid-span displacement started to change linearly with the increase of load. Till the appearance of initial crack, the straight slope decreased to some degree till the yielding of tendons. For strengthened beam, mid-span displacement started to change linearly till the appearance of initial crack and then the curvature of curve decreased to some degree till the yielding of tendons. Different from the elastic change period of un-strengthened beam, strengthened beam could still continue to bear the load after the yielding of tendons till the crack of pre-stress tendons. It could be seen from the comparison of experimental and finite element calculation results in each figure that relative error was small, which indicated that it was feasible to use finite element to predict the deflection of concrete beam.

## 6. Conclusions

1. The hammering method was used to test the time-domain vibration acceleration of different pre-stressed concrete beams. FFT was conducted to obtain frequency-domain results and fundamental frequency information. Results showed that pre-stress value had little influence on the fundamental frequency of concrete beams, which was consistent with published research results. The experimental result of this paper was reliable.

2. The finite element model of pre-stressed concrete beams was established. The fundamental frequency of concrete beams under different pre-stress values was computed to compare with experimental results. Relative error was small, which indicated that the finite element model in this paper was effective.

3. Under the same eccentricity, the fundamental frequency of pre-stressed concrete beams showed no obvious changes with the increase of pre-stress values. Under the same pre-stress value, the fundamental frequency of concrete beams gradually increased with the increase of the eccentricity. It was because the increase of the eccentricity would increase the effective stiffness of concrete beams.

4. Under the same counterweight, the fundamental frequency of pre-stressed concrete beams showed no obvious changes with the increase of pre-stress values. Under the same pre-stress value, the fundamental frequency of concrete beams gradually decreased with the increase of counterweight. It was because the increase of counterweight could increase the distributed mass of concrete beams.

5. The deflection of pre-stressed concrete beams under the variable load was obtained through experiments and finite element method. Experimental and numerical simulation results showed good consistency. It was also effective to use the finite element method to study the dynamic deflection of concrete beams. Both experimental and simulation results showed that applying pre-stress tendons to concrete beams could play an important role in crack resistance.

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Yahui Wang

RESEARCH ON FUNDAMENTAL FREQUENCIES AND DYNAMIC CHARACTERISTICS OF PRE-STRESSED CONCRETE BEAMS BASED ON EXPERIMENT AND NUMERICAL SIMULATION

S u m m a r y

To study the relationship between the fundamental frequency and pre-stress value of pre-stressed concrete beams, this paper adopted the hammering method to excite concrete beams under different pre-stress values, eccentricities and counterweights based on two kinds of methods including experiments and numerical simulation, processed and analyzed the collected mid-span acceleration of beams to quantitatively analyze the relationship between the fundamental frequency of beams and pre-stress. Research results showed that pre-stress value had little influence on the fundamental frequency of concrete beams, which was consistent with published research results. The experimental result in this paper was reliable. The experimental and numerical simulation of pre-stress value showed small relative errors. The finite element method in this paper was effective. When the eccentricity is the same, the fundamental frequency of pre-stressed concrete beams showed small changes with the increase of pre-stress value. When the pre-stress value is the same, the fundamental frequency of concrete beams gradually increased with the increase of eccentricity because the increase of eccentricity would improve the effective stiffness of concrete beams. When the counterweight is the same, the fundamental frequency of pre-stressed concrete beams showed small changes with the increase of pre-stress value. When the pre-stress value is the same, the fundamental frequency of concrete beams gradually decreased with the increase of counterweights, which was because counterweights could increase the distributed mass of concrete beams. Finally, the deflection of pre-stressed concrete beams under the variable load was obtained through experiments and finite element method. Experimental and numerical simulation results showed good consistency. It was also effective to use the finite element method to study the dynamic deflection of concrete beams. In addition, experimental and numerical results indicated that applying pre-stress tendons to concrete beams could play an important role in crack resistance. The research in this paper not only further verified existing results, but also provided reference for studying the fundamental frequency and dynamic characteristics of pre-stressed concrete beams.

**Keywords:** pre-stressed concrete beams; fundamental frequency; eccentricity; counterweight; dynamic deflection

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