

Residual modal energy evaluating of fatigue damaged composite structure

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1. Introduction

Structural damage detection based on changes in vibration characteristics has received much attention in recent years. Among all the vibration-based detection methods, the ones based on the changes in natural frequencies or frequency response functions are considered to be the easiest to implement.

The control of vibrations either in passive or active ways of damageable structures, systems and machines is not always done properly. Their mechanical and dynamical characteristics are evaluated and known once a time, habitually in the virgin state of the considered structures. For nondamageable structure this approach is exact and legal, but, for the damageable one this may conduct to disasters. In this study following the state of the structure is quasi-permanent and its characteristics are evaluated state by state in its all progressive damage way.

Damage definitions are proposed in the literature, but, all of them are equivalent about the loss rigidity phenomena. The interested reader will find more details on the models of damage in the synthesis article of Degrieck & Peepgrgem [1]. The loss rigidity phenomenon is considered as the main responsible for the increasing of the damage in the composite beam [2, 3]. Global structural modelling of this damage is achieved by implementing the local damaged elastic law in a global structural bending formulation. The prediction of the damage using Kachanov postulate is adopted.

This paper deals with the evaluation of the residual modal energy in glass/epoxy unidirectional fibrous composite beam at a desired state. In this damaged state, the damaged elementary stiffness matrix is proposed for the evaluation of the global potential energy. The extraction of the global stiffness matrix is done as in classical finite element analysis. Dynamical equation under its matrix form is given and solved to extract the effective modal characteristics at the given dosage of cyclic solicitations.

The efficiency of this approach for damaged structures is shown for its possible implementation in FEM codes to treat more complex structures.

2. Modeling aspect (continuum – discrete media)

The problem to be solved consists of a simply supported beam meshed into n elementary finite element beams of $l_i = L/n$ length and an elementary bending strength $(EI)_i$, (Fig. 1).

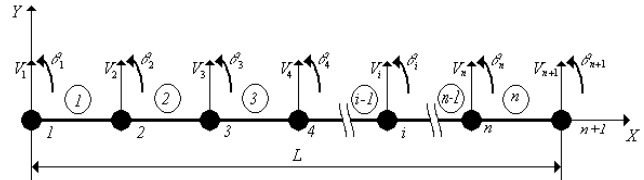


Fig. 1 Beam elements modelling the whole bending beam and their respective DOFs

2.1. Principle of damaged finite element

Familiar reader with finite element modelling knows how stiffness and mass matrices (K^e and M^e) are extracted classically respectively from elementary potential and kinetic energies [4]. In the case of Bernoulli's beam (with neglected shearing force), the simplest element beam in bending is characterized by four degrees of freedom (Fig. 1).

2.2. Elementary and global damaged stiffness matrices

The identified Sidoroff & Subaggio damage evolution law [5] by taking coefficients A , b and c from the experimental tests data is under the following form

$$\frac{dD}{dN} = 0.0035 \frac{(\Delta\varepsilon)^2}{(1-D)^8} \quad (1)$$

where D is the damage variable, dN is the Number of cycles per increment and $\Delta\varepsilon$ incremental strain.

The damage is located at the nodes and introduced into the stiffness matrix and assigned to each degree of freedom; the rotation θ and the translation v .

Evaluating the deformation energy in a given damage state

$$\int_v \tilde{\sigma} d\varepsilon \quad (2)$$

The elementary damaged stiffness matrix is extracted classically [4] and built in the following form

$$K_D^e = \frac{EI}{L} \begin{bmatrix} 12(1-D) & 6L(1-D) & -12 & 6L \\ 6L(1-D) & 4L(1-D) & -6L & 2L \\ -12 & -6L & 12(1-D) & -6L(1-D) \\ 6L & 2L & -6L(1-D) & 4L(1-D) \end{bmatrix} \quad (3)$$

where \tilde{E} , $\tilde{\sigma}$, ε are respectively the effective nodal Young modulus, effective elastic stress and strain.

The global damaged stiffness matrix depends on the meshing of this structure; an automatic meshing is performed for this matter and takes the following form

$$K_D^G = B^t K_{DD}^G B \quad (4)$$

where K_{DD}^G is the unassembled global stiffness matrix, B and B^t are respectively the Boolean matrix (transition from the local DOF to the global ones) and its transposed one. The mass matrix M has exactly the usual form in the bending case with two degrees of freedom at each element node [4].

3. Modal analysis

3.1. Dynamical characteristics of nondamaged structure

The dynamic analysis of a continuous linear structure meshed into elements, with M^G and K^G are respectively the global mass and stiffness matrices, can be reduced to the resolution of a second order system of differential equations in $x(t)$ as

$$M^G \ddot{x} + K^G x = 0. \quad (5)$$

3.2. Dynamical characteristics of damaged structure

According to the study of the evolution of the damage made paragraph 2, the damage takes maximum values into the external fibres under locally tension on transversal faces of the beam. These specific values of damage were introduced systematically into the elementary matrices of rigidities for each section.

By taking $x(t) = X_D e^{i\omega_D t}$ in Eq. (5), the eigenvalue problem of the damaged beam becomes

$$\left[K_D^G \omega_D^2 M^G \right] \{ X_D \} = \{ 0 \} \quad (6)$$

where K_D^G is global stiffness Matrix at a given state of damage, ω_D is eigenpulsation at a given state of damage.

Eigenvalues for the system are obtained by solving the following characteristic equation

$$\det \left[K_D^G - \omega_D^2 M^G \right] = 0. \quad (7)$$

3.3. Residual modal energy evaluating: extraction of loss modal energies factors

In a cyclic loading test of a unidirectional composite material, the dissipation of energy can be characterized by the follow-up of the loss of modal energy. This loss is due primarily to the three controlling states of the damage mechanisms of bending fatigue [6], depending on the stage of damage.

The modal deformation energy of the nondamaged beam is given by

$$U_n = \frac{1}{2} [\varphi_n]^T [K^G] [\varphi_n] \quad (8)$$

where K^G is stiffness matrix for nondamaged case and φ_n is eigenvector matrix for nondamaged case.

The modal deformation energy of the beam at fixed stage of damage is given by

$$U_{nD} = \frac{1}{2} [\varphi_{nD}]^T [K_D^G] [\varphi_{nD}] \quad (9)$$

where K_D^G is stiffness matrix at fixed stage of damage and φ_{nD} is eigenvector matrix at fixed stage of damage.

The loss of rigidity in the studied beam is considered here as the main reason of the loss of internal elastic energy, hence, the residual modal energy in the studied fibrous composite structure under cyclic loading. So, the ratio of the dissipated energy ΔU_n in a given step of damage to the internal energy U_n can be expressed by:

$$\zeta_{nD} = \frac{\Delta U_n}{U_n} = \frac{U_n - U_{nD}}{U_n} \quad (10)$$

where U_n is modal strain energy for nondamaged case and U_{nD} is modal strain energy for damaged case.

It depends on the focused step needed for the determination of the loss factor for the fixed residual internal energy allowed.

The well known damping loss factor η_{nD} can be evaluated by dividing the given value of ζ_{nD} by the number of cycles matching with the desired cycle at the desired damage level. In another term, it represents the energy loss factor. Eq. (10) furnishes the cumulus of damping until designed step denoted N . Then, η_{nD} can be evaluated as follows

$$\eta_{nD} = \zeta_{nD} N^{-1} \quad (11)$$

where N is the number of cycles of fatigue loading.

3.4. Damaged and nondamaged responses in time and frequency

The damaged and nondamaged free responses in time of the beam at the mid length ($x = L/2$) are given classically by

$$w(x, t) = \sum_{n=1}^{\infty} (A_n \cos \omega_n t + B_n \sin \omega_n t) \sin \frac{n\pi}{L} x \quad (12)$$

The damaged and nondamaged responses of the studied forced beam in frequency at a given time can be expressed as

$$w(x, t) = \sum_{n=1}^{\infty} \frac{2P}{\rho S L} \frac{1}{(\omega_n^2 - \omega^2 + 2j\xi_n \omega_n \omega)} \times \sin \frac{n\pi}{L} x_0 \sin \frac{n\pi}{L} x \cos \omega t \quad (13)$$

where $j^2 = -1$, ω_n is eigenpulsation, ω is pulsation of the harmonic excitation $P \cos \omega t$ applied at the mid length of the beam $x_0 = L/2$, P is magnitude of excitation force, ξ_n is the given modal damping factor and ρ, L, S are respectively density, length, transversal area of the beam.

Materials and geometrical properties [5]

Length	Width	Height	Glass initial Young Modulus	Glass Density	Initial Local Rigidity	Initial Global Rigidity
$L = 0.4\text{m}$	$b = 0.04\text{m}$	$h = 0.02\text{m}$	$E_0 = 72400\text{MPa}$	$\rho = 2540\text{ kg/m}^3$	$H_0 = E_0 I_{GZ} = E_0 \frac{bh^3}{12}$	$K_0 = 48 E_0 \frac{bh^3}{L^3}$

4. Results and discussions

All results presented here are made with a considered glass/epoxy simply supported beam. Materials and geometrical properties are shown in Table 1.

Figs. 2 and 3 show respectively the evolution of local rigidity in the length of studied glass/epoxy beam and stress evolution in the median section ($x = L/2$) depending of the level of considered cycles of sollicitations. They show the decreasing rigidity. Figs. 4 and 5 give the residual modal energy and its respective loss energy factors versus the level of loading in cycles for the first mode. Figs. 6 and 7 give the residual modal energy and its respective loss energy factors versus the level of loading in cycles for the second mode.

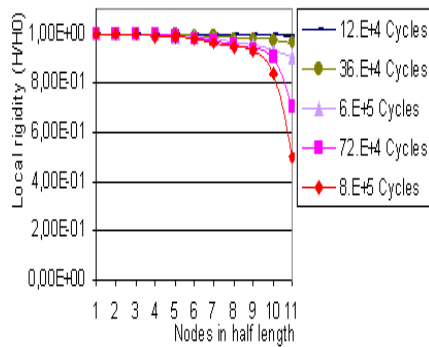


Fig. 2 Local rigidity evolution in length-number of cycles

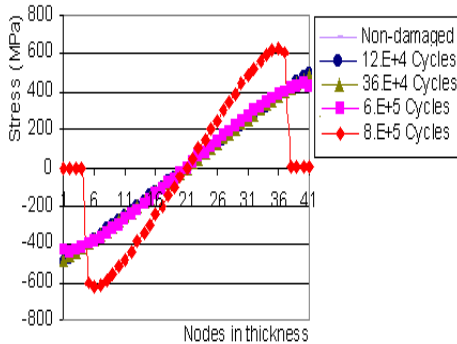


Fig. 3 Stress evolution in the median section

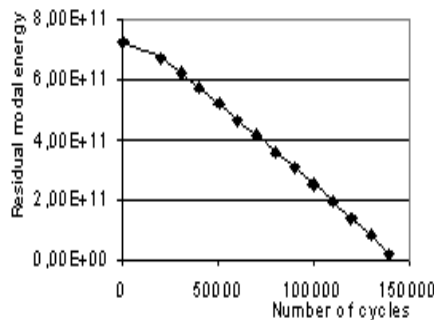


Fig. 4 Residual modal energy-number of cycles, first mode

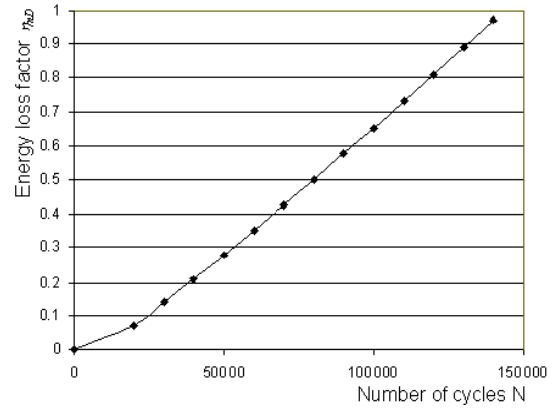


Fig. 5 Energy loss factor - number of cycles, first mode

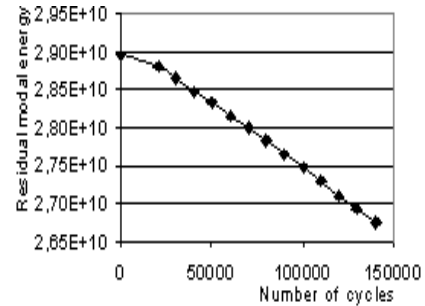


Fig. 6 Residual modal energy - number of cycles, second mode

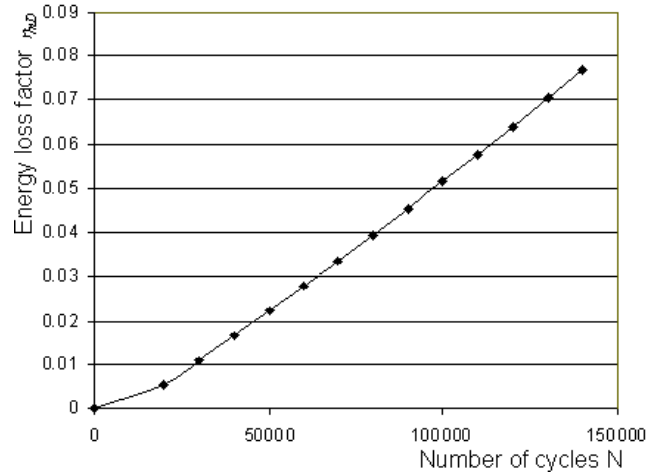


Fig. 7 Energy loss factor - number of cycles, second mode

It can be remarked that vanishing of eigenfrequencies is concluded in the same level of the maximum damage and hence in the maximum stress state in a given section.

Responses in time of the beam at the mid length are plotted in Fig. 8 and show the disparity between the damaged and nondamaged ones. A shifting is clearly seen on these curves to demonstrate the influence of damage on the responses of the studied beam. On Fig. 9, responses in

frequencies show the affectation of the amplitude. Damping caused by the damage is clearly shown. The shifting also in frequency as in time appears consequently.

It is clear that decreasing of eigenfrequencies values affect significantly the handling of any future tests (experimental and/or numerical tests) to be done on such damageable structures.

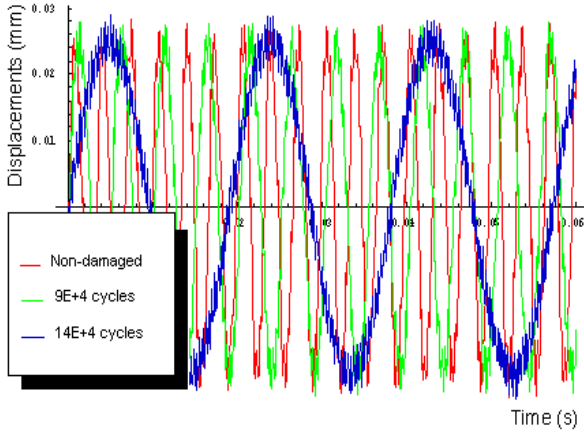


Fig. 8 Reponses in time at mid-length

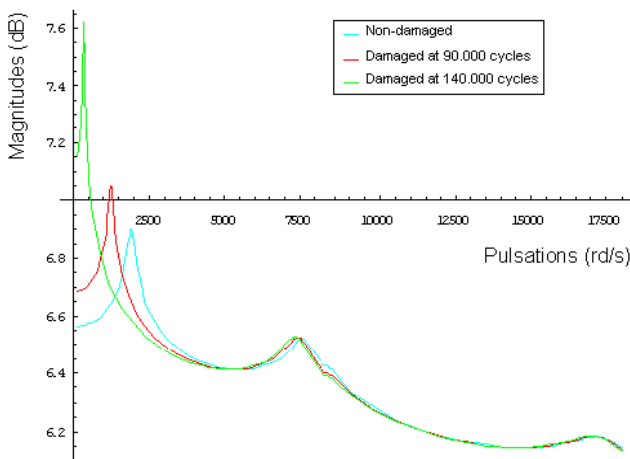


Fig. 9 Reponses in frequency at $x=L/2$, with $\xi_n=0.054$

5. Conclusion

The damping caused qualitatively by the beginning of cracks and quantitatively by the loss of energy in fibers is then appreciable and should be taken into account and included in all modeling, optimizing and passive and/or active control of such type of structures.

This study shows an efficiency to determine the variation of dynamic characteristics especially the modal energy remaining; this should have a determinant utility:

- to avoid disaster during tests when performed, in experimental dynamic cycling loading;
- in both active and passive modal control;
- in optimizing design, physical constraints should be updated depending on the current state of the optimized structure;
- in structural health monitoring, the proposed model is introduced in existent structural package or finite elements one;
- as a nondestructive test (NDT), both to recommend frequency range to perform dynamic test with

composite specimen safely and for damage detection.

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NUOVARGIO PAŽEISTŲ KOMPOZICINIŲ
KONSTRUKCIJŲ ĮVERTINIMAS LIEKAMAJO
FORMOS PASIKEITIMO ENERGIJA

R e z i u m ė

Šis straipsnis skirtas liekamosios formos pasikeitimo energijos įvertinimui tam tikros būsenos stiklo ir epoksidinio tiesaus pluošto kompozito sąlygoje. Pasiūlyta elementari pažeisto standumo matrica bendrai potencinei energijai įvertinti. Globali standumo matrica yra sudaryta taikant klasikinį baigtinių elementų metodą. Pažeidimui prognozuoti panaudota Kačanovo prielaida. Nustatyta, kad standumo praradimas yra pagrindinė kompozito sąlygos pažeidimo padidėjimo priežastis. Bendras struktūrinis pažeidimo modeliavimas yra sukurtas įdiegus vietinį tamprumo pažeidimą bendroje konstrukcijos lenkimo formuluotėje. Dinamikos lygtys pateiktos matricų pavidalu ir išspręstos įvertinant efektyvias formos kitimo energijos charakteristikas pagal tam tikrus ciklinius reikalavimus. Šio pasiūlymo efektyvumas tiriant pažeistas konstrukcijas rodo, kad jį galima taikyti ir nagrinėjant sudėtingesnes konstrukcijas baigtinių elementų metodu.

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RESIDUAL MODAL ENERGY EVALUATING OF
FATIGUE DAMAGED COMPOSITE STRUCTURE

S u m m a r y

This paper deals with the evaluation of the residual modal energy in glass/epoxy unidirectional fibrous composite beam at a desired state. In this damaged state, the damaged elementary stiffness matrix is proposed for the evaluation of the global potential energy. The extraction of the global stiffness matrix is done as in classical finite element analysis. The prediction of the damage using Kachanov postulate is adopted. The loss rigidity phenomenon is considered as the main reason of increasing of the damage in the composite beam. Global structural modeling of this damage is achieved by implementing the local damaged elastic law in a global structural bending formulation. Dynamical equation under its matrix form is given and solved to extract the effective modal characteristics at the given dosage of cyclic solicitations. The efficiency of this approach for damaged structures is shown for its possible implementation in FEM codes to treat more complex structures.

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ОЦЕНКА УСТАЛОСТНОГО ПОВРЕЖДЕНИЯ
КОМПОЗИТНЫХ КОНСТРУКЦИЙ С
ИСПОЛЬЗОВАНИЕМ ОСТАТОЧНОЙ ЭНЕРГИИ
ФОРМОИЗМЕНЕНИЯ

Р е з ю м е

В представленной статье рассматривается, как оценить остаточную энергию формоизменения балки в заданном состоянии. Балка изготовлена из композита прямолинейных волокон стекло-эпоксид. В этом поврежденном состоянии, для оценки глобальной потенциальной энергии, предложена поврежденная элементарная матрица жесткости. Глобальная матрица жесткости создана при использовании классического метода конечных элементов. Для прогнозирования повреждения использован постулат Качанова. Потеря жесткости определена как основная причина увеличения повреждения в балке композита. Общее структурное моделирование повреждения упругости в общей конструкционной трактовке изгиба. Уравнения динамики представлены в виде матриц и решены с применением эффективных характеристик энергии формоизменения учитывая данные циклические условия. Эффективность этого предложения для поврежденных конструкций подтверждает возможность его применением в МКЭ в более сложных конструкциях.

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