ISSN 1392–1207. MECHANIKA. 2020 Volume 26(1): 42–50

Research on Optimum Design of Variable-lead Spiral Pair Based on Contact Stress and Elasto-hydrodynamic Lubrication State

He FEI, Xu XIE, Guo NING, Zhang BENTAO, Yang KE
Nanjing University of Science and Technology, Mechanical Engineering College, Jiangsu Nanjing 210094, China, E-mail: 15757856473@163.com

crossref http://dx.doi.org/10.5755/j01.mech.26.1.20436

1. Introduction

With the increasing demand for stability, reliability and safety of modern machinery, the traditional spiral transmission structure can no longer meet the engineering requirements. The variable lead spiral transmission stands out for its high stability, zero-lead self-locking and elasto-hydrodynamic lubrication. It is widely used in extrusion dehydration institutions, filling production line transmission devices, subway door transmission devices and so on.

2. An overview of the variable lead spiral transmission

2.1 The current research status of the variable lead spiral transmission

Compared with the traditional spiral transmission, the variable lead spiral transmission can realize the variable speed linear motion of the nut and also has the characteristics of zero-lead self-locking.

In terms of the design of the variable lead helix, Tang Chun [1] applied the variable lead conic spiral to the design of the cooling tank shape of the rocket engine, and designed a variable inclination variable lead spiral groove. Liu Lixin [2] put forward the design of varying pitch screw according to the transmission device of the filling production line. Shan Jihong [3] analyzed the law of cylindrical spiral motion and established the parametric equation of the variable-lead cylindrical helix and helical surface. Zhang Wei [4] adopted circular transition curve, polynomial curve and a variation transition curve to fit the variable-lead helix.

In terms of the research on the performance and motion characteristics of variable lead spiral mechanism, Ming J.T [5] analyzed the motion of variable-lead spiral mechanism with a conical helical roller to optimize the geometric parameters of the tapered roller by improving the transmission quality and reducing the sliding speed. Shen-Tarng Chiou [6] analyzed the motion characteristics of variable-lead spiral mechanism (VLSM) and obtained the influence of mechanism input speed on output torque and power. Wei Zhang [7] analyzed the influence of different variable lead helical lines on the elastohydrodynamic lubrication state of the spiral pair mechanism and obtained the conditions of forming the lubricant film with variable lead spiral transmission mechanism. Chen Xiaofen [8] carried out a kinematic analysis of the movement of the roller and the screw rod in the spiral drive, obtained the relative sliding speed of the roller relative to the screw rod and analyzed the factors that influenced the relative sliding. BN Zotov [9] utilized both the experimental and simulation calculations to analyze the pressure characteristics of the invariable lead screw rod and the variable lead screw rod. HS Yan [10] and other researchers analyzed the meshing characteristics of variable-pitch spiral pair of different helical rollers and the geometric surface geometry of the helical groove.

Some scholars chose the most suitable lead function for different application situations. HB. Qiao [11] adopted fuzzy comprehensive evaluation to comprehensively evaluate the reliability of the elevator braking system in mine, demonstrating the reliability of the whole braking system. Gao Jianghong [12] applied fuzzy comprehensive evaluation to the design of motion of the cam mechanism follower and established a two-level evaluation system to evaluate the motion of follower from the aspects of speed, load, cost, noise and precision in order to select the most reasonable motion law of cam mechanism follower.

Based on the analysis of the motion characteristics of the variable-lead helix and the roller, the parameter equation of the variable-lead helix is designed firstly. Then based on the analysis of the contact stress and the state of the elasto-hydrodynamic lubrication, the optimum design method of the roller is studied.

2.2. The analysis of motion characteristics of variable lead spiral transmission

Schematic diagram of variable lead spiral drive mechanism is shown in Fig. 1, where *1* is the variable lead screw rod, *2* is the nut, *3* is the spiral groove, and *4* is the screw roller. The nut meshes with the spiral groove of the variable lead screw rod through the screw roller. The screw roller guides the nut to move along the spiral groove track in the spiral groove, and the nut performs the change speed linear motion according to the axial direction of the screw rod. Compared with the traditional spiral mechanism, variable lead spiral mechanism has the characteristics as follow:

1) Stability: The lead of the variable lead screw drive continuously changes. There is no sudden change of speed during the movement, which eliminates the impact caused by the sudden change of speed and makes the roller in spiral groove own higher stability in the process of movement;

2) Zero-lead self-locking: When the roller moves to the zero lead of the screw rod, since the lead angle is zero, the spiral angle must be less than the friction angle. The self-locking occurs when the roller moves to the zero-lead position of the screw rod, which means that when the roller acts as an active part and the screw rod acts as a follower, free transmission cannot be achieved, that is the

zero-lead self-locking.

3) Elasto-hydrodynamic lubrication: In the variable-lead spiral pair, the contact between the surface of roller and the spiral groove will generate great contact pressure. If the lubrication condition is good, the contact between them will produce lubricating film and realize elasto-hydrodynamic lubrication (EHL).

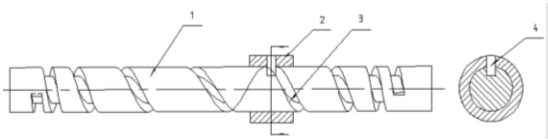


Fig. 1 Schematic diagram of the variable lead spiral drive mechanism

3. Analysis and design of the variable lead screw

The variable lead screw rod can be divided into two types: equal slot wide variable lead screw and equal crown width variable lead screw. This paper mainly studies the equal slot wide variable lead screw.

3.1. The mathematical model of the variable lead screw

Spiral motion is a combination of rotational and linear motion, which can be regarded as a linear movement which a moving point moves along a cylindrical bus-bar and a circular motion along a cylindrical axis. The velocity of linear motion is $v(t)$, a function of t and the angular velocity function for circular motion is $w(t)$. When $w(t)$ and $v(t)$ are constant at the same time, the spiral motion is invariable lead spiral motion; if one of them is not a constant function, it turns into a variable lead spiral motion.

$$\begin{cases} x = R\cos\left(\int_0^t w dt\right) \\ y = R\sin\left(\int_0^t w dt\right). \\ z = vt \end{cases} \tag{1}$$

Eq. (1) is the parameter equation of invariable lead screw parameter, where w represents the angular velocity of the screw movement and v is speed of the linear motion. In the invariable lead bit $z_0=v(t_0)$, t_0 represents the time of one rotation of the screw, and R is the radius of the screw rod.

$$\begin{cases} x = R\cos\left(\int_0^t w(t)dt\right) \\ y = R\sin\left(\int_0^t w(t)dt\right). \\ z = \int_0^t v(t)dt \end{cases} \tag{2}$$

Eq. (2) is a general parametric equation for a variable-lead screw, where $w(t)$ and $v(t)$ are variables. $v(t)$ is the velocity of linear motion and $w(t)$ is the angular velocity function for circular motion. The common variable lead screw motion is that the rotating speed of the screw is constant, that is, $w(t)$ is a constant w; nut lead change $v(t)$ is a variable, then the same time period the lead $z_0 = \int_0^t v(t)dt$, since $v(t)$ is variable. Thus the formed screw is a variable-lead screw.

Fig. 2 shows the unfolded drawing of invariable lead and variable lead screw. Unfolded drawing of a constant lead screw is a straight line, and the unfolded drawing of a variable lead screw is a curve. The angle between unfolded line of the invariable lead screw and the horizontal axis is the helix angle, and the angle between tangent line and the horizontal line at each point on the unfolding curve of the variable lead screw is the helix angle at this position.

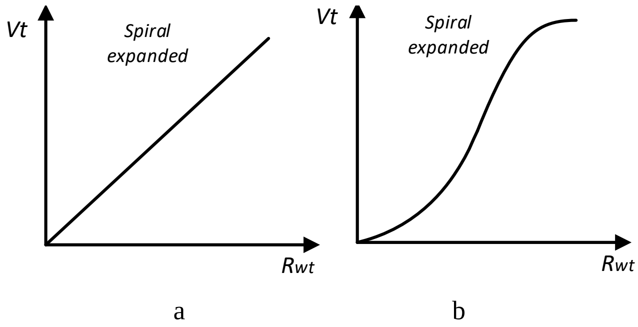


Fig. 2 Unfolded drawing of variable lead screw and invariable lead screw

3.2. Settings of variable lead screw parameter

3.2.1. Design of lead function

The lead function of the variable lead helix is the same as that of the follower nut. The main parameters to be considered include the movement time t of the driven mechanism, the movement distance S, and the movement velocity v and acceleration a at each position. There are two different lead functions in Fig. 3.

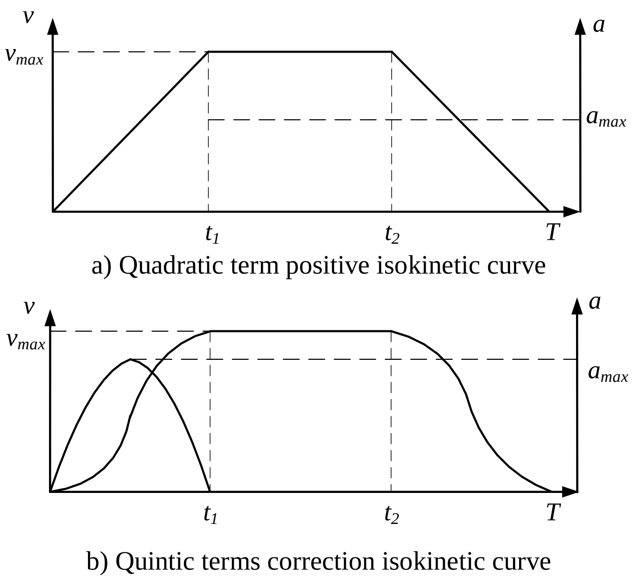


Fig. 3 The curve form of two kinds of lead functions

3.2.2. The determination of the angular velocity of the screw

According to the change of the lead range, the whole variable lead screw is divided into two sections: the lead change section and the invariant section. The lead of the screw in the invariant section is recorded as L. The equation for calculating the angular velocity of the screw rod is Eq. (3).

$$\begin{cases} \bar{v} = \dfrac{S_0}{t_0} \\ n = \dfrac{\bar{v}}{L} \\ w = \dfrac{2\pi n}{60} \end{cases}. \tag{3}$$

Among them, $\bar{v}$, mm/min represents the average speed, S_0, mm is the length of the screw rod, t_0, min is the time that the nut takes to run the screw rod, n, r/min is the rotational speed of the screw rod, L is the lead of the screw rod in the constant lead phase, and w is the angular speed.

3.2.3. The design of the radius of the screw rod

The influence of the pressure angle should be considered in the setting of the radius of the screw rod of the variable guide. In order to realize the greater transmission efficiency, the factor of the helix angle φ should be taken into consideration. The transmission efficiency η of the screw pair is calculated as:

$$\eta = \frac{tan\varphi}{tan(\varphi + \rho_v)}. \tag{4}$$

In Eq. (4), φ is the helix angle, and ρ_v is equivalent friction angle of the screw pair.

The efficiency of the helix angle φ in the range of 20º-60º is highest. The relationship between the helix angle and the diameter of the screw rod d can be listed as follows:

$$tan\varphi = \frac{L}{\pi d}. \tag{5}$$

The relationship between pressure angle and screw radius is shown in Eq. (6), where the allowable pressure angle [α] is in the range of 25º-35º.

$$R \geq \frac{\left|\dfrac{ds}{d\varphi}\right|_{max}}{tan[\alpha]}. \tag{6}$$

The R is the radius of the screw rod, d_s is the derivation of the linear displacement of the roller, and $d\varphi$ is the derivative of the rotational angle of the roller.

According to the fourth strength, the calculated stress of dangerous cross-section is calculated to check the strength of the screw.

$$\sigma = \sqrt{\left(\frac{4F}{\pi d^2}\right)^2 + 3\left(\frac{T}{0.2d^3}\right)^2} < [\sigma]. \tag{7}$$

The d is the diameter of the screw rod. The T is the torque that drives the screw rod, and the [σ] is the allowable stress. F is the maximum axial load for the motion of the screw rod, which is calculated as Eq. (8).

$$F = \begin{cases} F_1 = \mu \cdot mg + f + m\alpha \\ F_2 = \mu \cdot mg + f \\ F_3 = \mu \cdot mg + f - m\alpha \\ F_4 = -\mu \cdot mg - f - m\alpha \\ F_5 = -\mu \cdot mg - f \\ F_6 = -\mu \cdot mg - f + m\alpha \end{cases}, \tag{8}$$

where: α is the acceleration; m is the screw delivery quality; μ is the friction coefficient of the guide rail; and f is the guide surface resistance; F_1 is axial load at forward acceleration; F_2 is axial load at constant forward speed; F_3 is axial load at forward deceleration; F_4 is axial load at returning acceleration; F_5 is axial load at constant returning speed; F_6 is axial load at returning deceleration.

The maximum of the F is within the range of F_1 or F_3.

Select the appropriate helix angle, and calculate the radius R of the screw rod according to the Eq. (5). Then put it into the Eqs. (6) and (7) to verify whether it meets the requirements of the pressure angle and strength. Finally, the designed parameter variables are put into Eq. (2) to obtain the parameter equation of the variable lead helix.

In addition to the above parameters, the width and depth of the spiral groove are determined by the spiral roller, and the design of the spiral roller in the variable lead spiral pair is described below.

4. Analysis of contact stress of spiral rollers

4.1. Analysis of the motion characteristics of spiral rollers

There are several types of rollers, such as cylindrical rollers, tapered rollers, ball cone rollers, and drum rollers. In order to ensure that the contact between the roller and the spiral groove is a line contact, the cylindrical roller is selected for research.

The motion analysis of the spiral roller is mainly to determine whether the movement of the cylindrical roller in the screw spiral groove is pure rolling or rolling accompanied by sliding.

So the contact point between the cylindrical roller and the spiral groove is selected to study the motion characteristics of spiral rollers, and the screw is defined as the dynamic reference system.

The angle speed of the variable lead screw is w_1, the outer diameter of the spiral groove is r_1, and the inner diameter is r_2. The self-propagation angular velocity of the cylindrical roller is w_2, and the radius of is r_3, as shown in Fig. 4.

The velocity of the spiral groove at the outer diameter is $v_1=r_1\times w_1$, the velocity at the inner diameter is $v_2=r_2\times w_1$, and the velocity of the surface of the roller is $v_3=r_3\times w_2$.

If the relative motion between the roller and the spiral groove is required to be pure rolling, then the speed of the contact point of the roller and the spiral groove should be equal, namely:

$$v_3 = v_1, \;\; v_3 = v_2, \tag{9}$$

where: v_1 is the velocity of the spiral groove at the outer diameter; v_2 is the velocity at the inner diameter; and v_3 is the velocity on the surface of the roller.

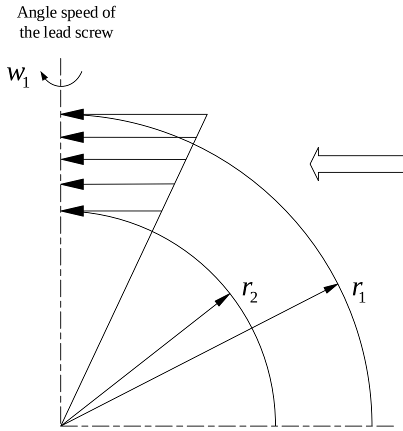


Fig. 4 Roller in the spiral groove velocity diagram

It is obvious that $v_1 \neq v_2$, so there is no pure rolling between the cylindrical roller and the spiral groove, and there must be sliding. Then the sliding friction may aggravate the friction and wear and reduce the service life of the roller.

Therefore, it is necessary to analyze the contact stress and elasto-hydrodynamic lubrication of the screw pair. The spiral roller can be designed by decreasing the contact stress and increasing the thickness of the oil film. The design method can improve the life of the roller.

4.2. Contact analysis of the screw pair

Contact type between cylindrical roller and spiral groove can be equivalent to the two cylinders' line contact. According to Hertz contact theory, the maximum contact stress between the two is the formula [13]:

$$\sigma_H = \sqrt{\frac{F_n}{\pi L}\frac{\frac{1}{\rho}}{\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}}}. \tag{10}$$

In the formula: L is contact line length, mm; ρ is radius of curvature of the composite, mm; μ_1, μ_2 is Poisson ratio of the roller and the screw material, E_1, E_2 are the elastic modulus of the roller and screw material, MPa, and F_n is the normal load between the roller and the spiral groove.

To calculate the maximum contact stress, the above parameters are calculated separately.

4.2.1. Synthetical curvature radius ρ

The formula for calculating the synthetical curvature radius of a spiral pair mechanism is:

$$\frac{1}{\rho} = \frac{1}{R_1} + \frac{1}{R_2}, \tag{11}$$

where: ρ is the synthetical curvature radius, R_1 is the normal curvature radius of the cylindrical roller, and the R_2 is the curvature radius of the spiral groove.

The normal curvature radius of the cylindrical roller is R_1, which is also the roller radius, and the length L of the contact line is the length L_0 of the roller bus bar.

The curved surface of the variable lead helix is $G(\theta, z)$, and its curve surface parameter equation [14] is:

$$\begin{aligned} &G(\theta,\mathrm{z}) = (rcos\theta,\, rsin\theta,\; f(t)) = \\ &= (rcoswt,\, rsinwt,\; f(t)), \end{aligned} \tag{12}$$

where: θ is the tangent angle between the roller and the inner part of the spiral groove. The r is the radius of the screw rod, the w is the angular velocity of the circumferential motion of the helix, the t is the time, and the $f(t)$ is the axial distance of the screw.

According to the differential geometry, the first and second basic forms of the spiral surface are as follows:

$$\begin{aligned} &I = r^2 d\theta^2 + df^2 = r^2 w^2 dt^2 + df^2 \\ &II = -rd\theta^2 = -rw^2 dt^2 \end{aligned}. \tag{13}$$

The normal curvature of the variable lead helix is:

$$\kappa_n = \frac{II}{I} = \frac{-rdt^2}{r^2dt^2 + df^2} = \frac{-rw^2}{r^2w^2 + f'^2}, \tag{14}$$

where: k_n is the normal curvature of the variable-lead helix, f' is the reciprocal of f to t.

Thus the normal curvature radius of the spiral groove R_2 is:

$$\begin{aligned} &R_2 = \frac{1}{|\kappa_n|} = \frac{r^2w^2 + f'^2}{rw^2} = \frac{r^2 + \left(\frac{v(t)}{w}\right)^2}{r} \\ &R_{2max} = \frac{1}{|\kappa_n|} = \frac{r^2w^2 + f'^2_{max}}{rw^2} = \frac{r^2 + \left(\frac{v_{max}}{w}\right)^2}{r}. \\ &R_{2min} = r \end{aligned} \tag{15}$$

Finally, the Synthetical Curvature Radius is obtained:

$$\begin{aligned} &\rho = \frac{w^2(R_1 r + r^2) + v_{(t)}^2}{R_1(w^2 r^2 + v_{(t)}^2)} \\ &\rho_{max} = \frac{w^2(R_1 r + r^2) + v_{max}^2}{R_1(w^2 r^2 + v_{max}^2)}. \\ &\rho_{min} = \frac{R_1 r}{R_1 + r} \end{aligned} \tag{16}$$

4.2.2. Modulus of elasticity and Poisson ratio of materials

The material used for roller and screw is generally 40Cr and 45 # steel, all of which are made of steel. The modulus of elasticity is $2.06\times10^5\,\mathrm{MPa}$ and Poisson ratio is 0.3.

4.2.3. Normal load

As shown in Fig. 5, F_Q is the integrated work load on the roller. Because there is only one roller, so it is close to the axial load for the motion of the screw rod in the Eq. (8). F_n is the normal force and F_m is the tangential force on the roller.

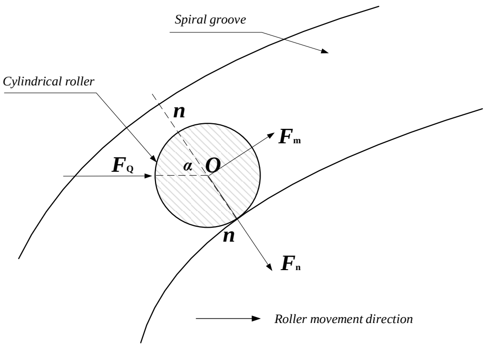


Fig. 5 A schematic diagram of the force of a roller in a spiral groove

The formula between them:

$$F_n = F_Q cos\alpha$$
$$\alpha = arctan\frac{\left|\frac{ds}{d\varphi}\right|}{r} = arctan\frac{v}{wr}, \quad (17)$$

where: α is the pressure angle between the spiral groove and the roller. ds are the derivative of the linear displacement of the roller, and $d\varphi$ is the derivative of the roller rotation angle. r is the radius of the screw rod, w is the angular velocity of the circular motion of the helix, and v is the linear velocity of the roller.

5. Optimal design of roller based on elasto-hydrodynamic lubrication

5.1. Analysis of Elasto-hydrodynamic lubrication characteristics of screw pair

In order to improve the service life of the variable lead screw pair, in addition to reducing the contact stress between the spiral roller and the spiral groove, we also need to improve the lubrication condition of the screw pair. Lubrication can not only reduce friction and wear of variable lead screw pair mechanism, but also decrease the movement temperature of screw pair and prevent corrosion. It is very important to study the lubrication of screw pair.

Considering that the movement time of the whole screw pair is much longer than that of the lubricating oil flowing into the Hertz contact area, the quasi-steady state Elasto-hydrodynamic model is used to calculate the minimum oil film thickness between the roller and the spiral groove. The formula for calculating the minimum oil film thickness is based on the Dowson-Higginson’s line contact minimum oil film thickness calculation formula [15]**:**

$$h_{min} = \frac{2.65a^{0.54}\left(\eta_0 u\right)^{0.7} R^{0.43}}{E'^{0.03}W^{0.13}}, \quad (18)$$

where: α is the pressure-viscosity coefficient of the lubricating oil, m^2/N; η_0 is dynamic viscosity of lubricating oil at normal temperature and pressure, Ns/m^2, u is entrainment speed of lubricating oil, m/s, R is the equivalent curvature radius, m, E' is equivalent elastic modulus, Pa and W is maximum load on unit contact length, N/m.

5.1.1. Equivalent curvature radius

$$R = \frac{R_1 R_2}{R_1 \pm R_2}, \quad (19)$$

where: R_1 and R_2 in the formula is the normal curvature radius of the roller and the spiral groove at the contact point respectively. When the lead is increased, “-” sign is taken in the formula; when the lead is decreased, “+” sign is taken in the formula.

5.1.2. Equivalent elastic modulus

According to the Hertz contact theory, the formula for calculating the equivalent elastic modulus (E') of the screw pair is:

$$E' = \frac{1}{2}\left(\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}\right), \quad (20)$$

where: μ_1, μ_2 are Poisson's ratio of roller and screw rod, E_1, E_2 are elastic modulus of roller and screw rod.

5.1.3. The entrainment speed of the lubricating oil

Because the viscosity of the lubricating oil used in the screw pair structure is relatively high, even a very small sliding can transfer the shear of the spiral groove of the screw rod. The reel speed of the lubricating oil between the spiral groove and the roller is as follows:

$$u = u_1 + u_2 \quad (21)$$

where: u_1 is the relative sliding speed of the roller and the spiral groove; u_2 is the absolute speed of the spiral groove in the sliding direction in the overall coordinate system.

$$u_1 = -vsin\theta + v\frac{d\varphi}{dv}ucos\theta, \quad (22)$$

where: v is the moving speed of the roller in the direction of the screw axis; φ is the rotational displacement of the screw; θ is the rotational displacement of the roller in the local coordinate system of the roller.

$$u_2 = Rsin\lambda\frac{d\varphi}{dt}, \quad (23)$$

where: R is the screw radius, λ is the screw angle at a certain contact point, and φ is the screw rotation displacement.

5.1.3. Load on unit contact length

The formula for calculating the load per contact length:

$$W = \frac{F_n}{L_0} = \frac{F_Q cos\alpha}{L_0}, \tag{24}$$

where: F_n is the normal load of the roller, F_Q is the integrated work load of the roller, α is the pressure angle between the spiral groove and the roller, L_0 is length of roller's bus.

5.2. Optimal design of spiral roller

It can be seen from the above research that in order to better set the parameters of the roller, it is required to satisfy the following two conditions, namely, satisfying the contact stress requirement and realizing the elasto-hydrodynamic lubrication (EHL).

5.2.1. To meet the requirement of contact stress

Because the main failure form of roller and spiral groove is pitting fatigue, the fatigue strength design of screw pair should be designed according to pitting fatigue. The maximum contact stress between roller and spiral groove contact surface shall not exceed the allowable contact stress of material, namely:

$$\sigma_H \le [\sigma] \quad [\sigma] \le \frac{\sigma_s}{[S]}, \tag{25}$$

where is: σ_H maximum contact stress of screw pair, $[\sigma]$ is the allowable contact stress of a roller or screw groove, σ_s is the yield strength of the material, $[S]$ is the allowable safety factor of the contact stress of the screw pair.

5.2.2. To achieve elasto-hydrodynamic lubrication

According to the elasto-hydrodynamic lubrication theory, the oil film thickness between the roller and the spiral groove refers to the distance between the center lines of the two contact surface distance. Full fluid dynamic lubrication can only be achieved when the minimum oil film thickness can completely cover the surface roughness; otherwise it is partial elasto-hydrodynamic lubrication or boundary lubrication. The index for determining the state of elasto-hydrodynamic lubrication is the ratio of the film thickness λ, which is equal to the ratio of the minimum thick film thickness to the root mean square deviation of the surface roughness. The expression of the ratio of the film thickness λ is [16]:

$$\lambda = \frac{h_{min}}{\sqrt{\sigma_1^2 + \sigma_2^2}}, \tag{26}$$

where: σ_1, σ_2 are respectively the surface roughness of two contact surfaces.

When $\lambda \le 1$, it represents the boundary lubrication conditions and has abrasion.

When $1 < \lambda < 3$, it represents the partial elastohydrodynamic lubrication. Whether abrasion occurs should be judged according to the specific circumstances.

When $\lambda \ge 3$, it represents the full elastohydrodynamic lubrication without abrasion.

6. Case study metro gate's variable lead spiral transmission

6.1. The advantages of variable lead screw pair in metro gate design

Due to the frequent opening and closing and high-speed operation of the subway door, it needs to have high stability and high precision, especially self-locking. Therefore, the traditional constant lead screw has some shortcomings in reliability and service life; The variable lead screw can realize the variable linear motion of the nut or rotor under the condition of keeping the lead screw at constant speed, and has the feature of zero-lead self-locking, which can meet the requirement of frequent opening and safety of the subway door.

So as to achieve the synchronized movement state of the two doors. When the metro door opens and closes, it can slow down or increase the speed of the metro door by using variable lead screw rod instead of traditional screw rod, which will greatly reduce the impact brought by the abrupt speed change of the screw on the metro door system and make it run more smoothly.

6.2. Design of the variable lead helix

6.2.1. Design of lead function

The nut in the subway door system is connected with the metro door through the gantry, so the lead function of the helix is also the motion equation of the metro gate. The design parameters of the metro door are shown in Table 1. The selected door opening is 1400 mm, the single moving distance is 700 mm, and the opening and closing time of the door is 3 s.

Table 1

Structural parameters of the metro door

Parameters	Numerical value
door opening, mm	$1400 \pm_{0}^{4}$
door height, mm	1860
supply air pressure, bar	5
service voltage, V	DC110
door opening and closing time, s	3 ± 0.5
Adjustment range of opening and closing time, s	1.5-4.5

The motion equations for designing metro gates are given in Eq. (26).

$$S = \begin{cases} 280t^2 & [0 \le t \le 0.5] \\ 280t - 70 & [0.5 < t \le 2.5] \\ 630 + 280(t-2.5) - 280(t-2.5)^2 & [2.5 < t \le 3] \end{cases}, \tag{27}$$

where: S is the movement distance, t is the movement time.

6.2.2. Setting of the angular velocity of the screw

The motion distance of the subway gate S_0 is 70 mm, and the total time of the movement t_0 is 3 s. The guide can be selected from 60-80 mm and L=70 mm is chosen as the lead of the invariant guide stage of the screw from the perspective of standardization. The angular velocity is calculated by the Eq. (3). The angular velocity is w=29.3 rad/s.

6.2.3. Design of the screw radius

After the spiral line of the screw is unfolded on the cylindrical surface, the expansion line of the spiral line is the displacement curve of the roller, which has the consistency. The x axis represents the circumferential direction of the screw, and the y axis represents the movement direction of the nut. Helix angle φ is the angle between the tangent and the horizontal line of the expansion line, pressure angle α the angle between the normal and the y axis. It can be seen from Fig. 5 that the helix angle φ is equal to the pressure angle α, and the best choice of pressure angle and helix angle is 30º-35º. When $\varphi=\alpha=30º$ is selected in the Eq. (5), the screw radius R=9.3 mm can be obtained. When round number R=10 mm is taken in Eq. (6), it is correct.

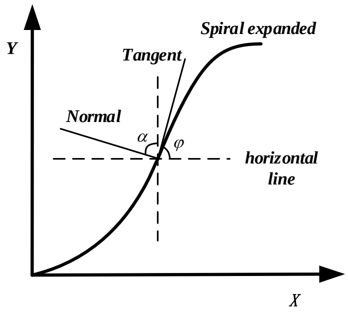


Fig. 6 Unfolded-drawing of helix

Select 45 steel as the material of the screw rod, and its yield strength is: σ_s=355 MPa. Rated torsion is T=5000 Nmm, and the maximum axial force is F=288.4 N, then put them into Eq. (7):

σ_s=19.15 MPa<[σ].

The strength of the screw meets the requirement, so the screw radius is finally determined to be 10 mm.

6.2.4. Determination of the parameter equation of the variable lead helix

Finally, the parameters of the lead function, angular velocity and radius are put into (2), and the parametric equation of the variable lead helix can be obtained:

$$\begin{cases} x = 10cos(29.3t) \\ y = 10sin(29.3t) \\ z = \begin{cases} 280t^2 \\ 280t - 70 \\ 630 + 280(t-2.5) - 280(t-2.5)^2 \end{cases} \end{cases} . \quad (28)$$

6.3. Design of variable lead screw roller

According to the contact stress formula and the minimum oil film thickness, the radius and bus's length of the cylindrical roller directly affect the basic stress and the lubricant film thickness of the screw pair mechanism, thus affecting the life and motion accuracy of the screw pair. The cylindrical roller radius and bus length will be designed from the perspectives of two factors, contact stress and oil film thickness.

The material of the roller is 40Cr after modulation and the mechanical parameters of the material are shown in Table 2. Safety factor [S] is selected as 1.5. According to the Eq. (25), allowable contact stress of the roller is [σ] = =235 MPa.

Table 2

Mechanical parameters of screw pair structural

	Mechanical strength σ_b, MPa	Mechanical strength σ_s, MPa	Mechanical strength σ_{-1}, MPa	Hardness, HB
40Cr	1000	800	485	≤255

6.3.1. Design of roller basic parameters based on contact stress

First, the primary range of the roller's radius is given $R_1(\text{mm}) \in (3,8)$, the primary range of the length of the roller bus is $L(\text{mm}) \in (4,7)$.

To ensure that the other parameters are constant, the length of the roller's bus is respectively 4, 5, 6, 7 mm, and the change of the roller radius from 3 to 8 mm is analyzed. The change of contact stress of the screw pair mechanism is shown in Fig. 7.

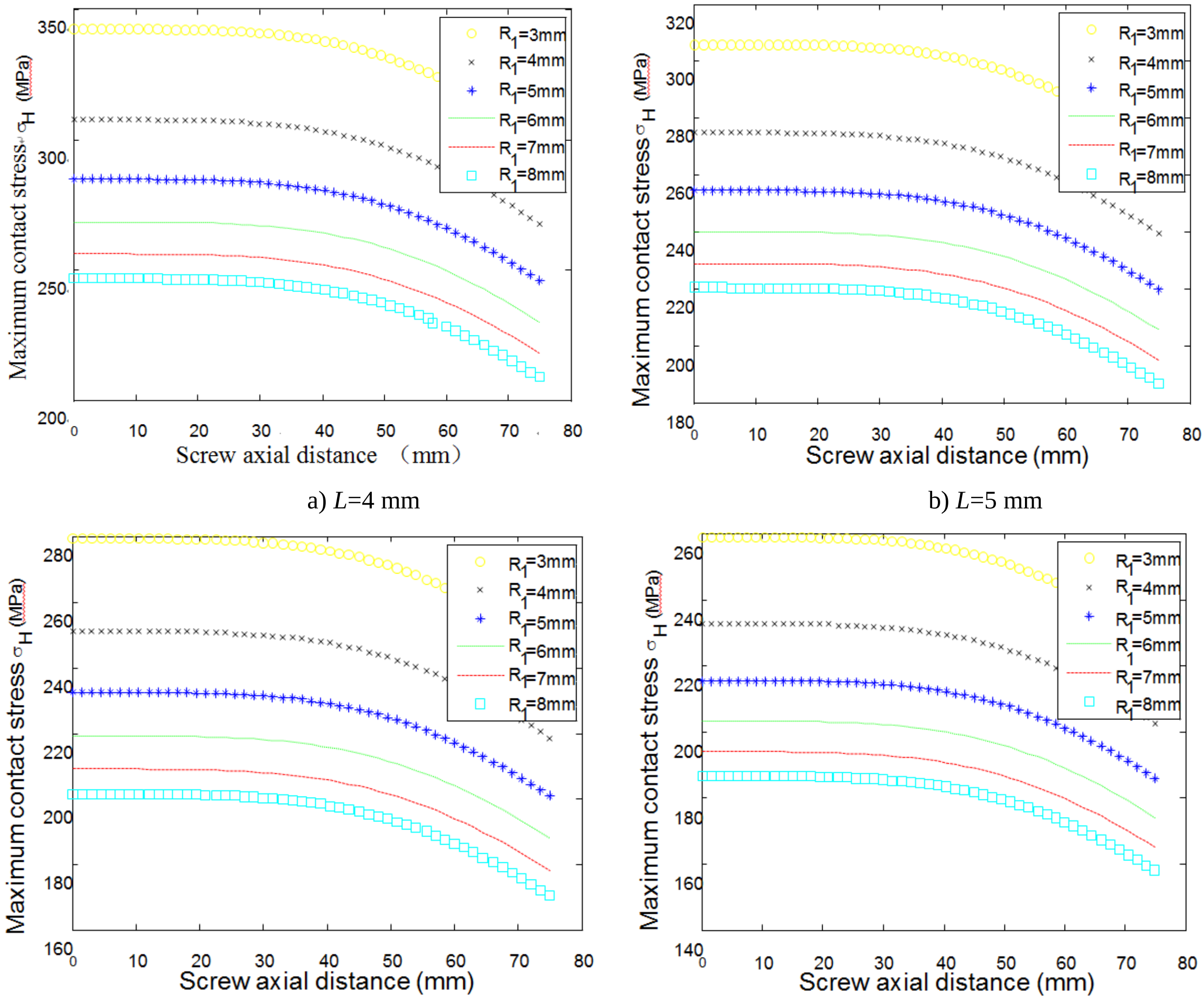
According to Fig. 7, a-d, the contact stress decreases with the increase of R_1, and decreases with the increase of the length of the bus. According to a, when L=4 mm, R_1=3 mm, the maximum contact stress σ_H is greater than the allowable contact stress 235 MPa, so the L must be ≥ 5 mm.

According to Fig. 7, c-d, to ensure that the maximum contact stress is within the allowable range, when L=5 mm, R_1≥7 mm; when L=6 mm, R_1≥5 mm; when L=7 mm, R_1≥4 mm.

6.3.2. Design of basic roller parameters based on oil film thickness

The surface roughness of the roller and the screw pair is set to level II, which are respectively 0.125 and 0.125 mm, then σ=0.178 mm. In order to realize partial elasto-hydrodynamic lubrication, the minimum oil film thickness is h_{min}>0.178 mm. The two parameters (L=4 mm) or R_1=3 mm are excluded by analyzing the contact stress. Other parameters remain unchanged. The change of roller's radius from 4 to 8 mm and the minimum oil film thickness of screw pair mechanism are analyzed respectively under the condition that roller's bus length is 5, 6 and 7 mm.

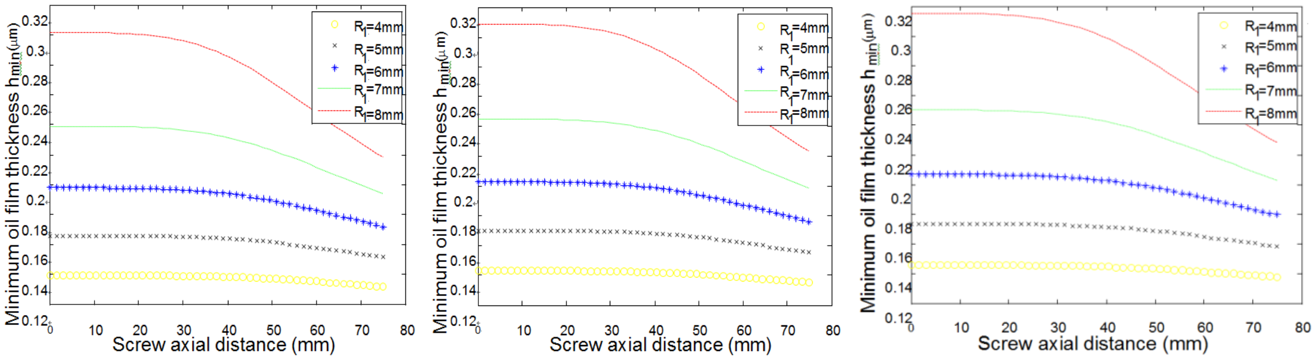
According to Fig. 8, the minimum oil film thickness increases with the increase of the radius of the cylindrical roller and increases with the increase of the roller bus length, but the magnitude of the later one is negligible. In order to realize the elasto-hydrodynamic lubrication state of the variable lead screw pair, the roller's radius R_1≥6 mm.



a) L=4 mm b) L=5 mm

c) L=6 mm d) L=7 mm

Fig. 7 Maximum contact stress at different roll radii



a) L=5 mm b) L=6 mm c) L=7 mm

Fig. 8 Minimum oil film thickness at different roller radii

6.4. The setting of roller parameters

In order to ensure the contact stress is smaller than the allowable contact stress of the variable lead screw pair and realize elasto-hydrodynamic lubrication, the value range of the bus length $L \in [5,6]$ and radius of the cylindrical roller $R_1 \in [6,8]$ can be obtained through the analysis of the contact stress and elastohydrodynamic lubrication of the variable lead screw pair. At the same time, the size of the length of the bus also affects the range of the radius, which means that the range of the allowable radius is different in the range of the different length of the bus. The specific value range is shown below.

$$\begin{cases} L \in [5,7], R_1 \in [7,8] \\ L \in [6,7], R_1 \in [6,8] \end{cases}.$$

The value of the bus's length and radius of the concrete roller can be taken in this range based on the actual conditions.

7. Conclusions

The optimal design idea and method of variable lead screw pair based on contact stress and elastohydrodynamic lubrication state are presented in this paper. The design of variable lead helix includes the design of lead function, the setting of screw angular velocity and screw radius. The spiral angle and pressure angle of screw pair have great influence on the determination of screw radius. Cylindrical line is used to analyze contact stress and elastohydrodynamic lubrication of roller and spiral groove. The differential geometry theory is used to calculate the curvature radius of each point of the spiral groove. The influence of the radius of the cylindrical roller on the maximum contact stress and the minimum oil film thickness of the screw pair mechanism are studied to determine the value range of the radius of cylindrical roller and the length of bus. Then the elastohydrodynamic lubrication (EHL) of the screw pair is realized. Finally, the variable lead screw pair obtained through optimization design increases the stability and life of variable lead screw transmission.

Acknowledgements

This project is funded by Jiangsu Natural Sci ence Foundation (surface) Project (BK201511488).

References

1. **Tang. C.** 2006. Mathematical Modeling and Processing Method of Variable Lead Angle Variable Lead Helical Groove, Dalian University of Technology.
2. **Liu, L.**; **Gao, Y.** 1998. Variable pitch spiral design method, Packaging and Food Machinery (6): 10-12.
3. **Shan, J.; Sun, Y.; Li, K.** 2005. Research on variable lead helicoid algorithm and modeling method, Journal of Donghua University Natural Science Edition 31(4):92-93. http://dx.doi:10.3969/j.issn.1671-0444.2005.04.023.
4. **Zhang, W.** 2015. Research on the Mechanism of Variable-lead Spiral Coupling Transmission and Related Technologies, Nanjing University of Science and Technology.
5. **Ming, J. T.; Jan-Shiung, S.; Jan-Chung, C.** 1996. Kinematic design optimization of the variable lead screw mechanism with cone meshing element, Mechanism & Machine Theory 31(8):1081-1093.
6. **Chiou, S. T.; Chen, F. Y.** 1998. Kinetostatic analysis of variable lead screw mechanisms with 3° conic frustum meshing elements, Mathematical & Computer Modelling 27(1):17-30.
7. **Zhang, W.; Shi, X.; Li, D.** 2013. Research on contact fatigue of variable lead screw system, International Journal of Digital Content Technology & Its Applications 7(10):1-9.
8. **Chen, X.; Li, D.; Shi, X.** 2013. Study on the mechanism of relative sliding between roller and screw in roller screw drive, Machine Tools and Hydraulics 41(9):54-57. http://dx.doi:10.3969/j.issn.1001-3881.2013.09.015.
9. **Zotov, B. N.** 2015. Comparison of computed and experimental characteristics of a variable-lead screw, Chemical and Petroleum Engineering 51(3):1-4.
10. **Yan, H. S.; Cheng, H. Y.** 1997. The generation of variable pitch lead screws by profiles of pencil grinding wheels, Mathematical & Computer Modelling 25(3): 91-101.
11. **Qiao, H. B.; Pang, A. S.; Xu, H. L.** 2012. The application of fuzzy comprehensive evaluation method to the assessment of ascension brake system reliability, 490-495: 698-705.
12. **Gao, J.** 2006. Application of fuzzy comprehensive evaluation in mechanical design, Mechanical design and manufacturing (7): 141-143. http://dx.doi:10.3969/j.issn.1001-3997.2006.07.065.
13. **Xin, H.; Shepherd, D.; Dearn, K.D.** 2012. PEEK (Polyether-ether-ketone) based cervical total disc arthroplasty: contact stress and lubrication analysis, Open Biomedical Engineering Journal 6(6):73-79.
14. **Zhang, W.; Shi, X.; Li, D.** 2013. Mathematical modeling of variable lead helix and design of transition curve in VLSM, Mechanika19 (3):332-335.
15. **Yao, L.**; **Shangxin, L. L.** 1991. The application of elastic fluid dynamic lubrication theory for the plane enveloped circular worm and gear, Journal of Daqing Petroleum Institute 15(1): 73-77.
16. **Wen S.** 1992. Elastic fluid dynamic lubrication, Tsinghua University Press.

He Fei, Xu Xie, Guo Ning, Zhang Bentao, Yang Ke

RESEARCH ON OPTIMUM DESIGN OF VARIABLE-LEAD SPIRAL PAIR BASED ON CONTRACT STRESS AND ELASTO-HYDRODYNAMIC LUBRICATION STATE

S u m m a r y

The design of variable-lead spiral pair mainly includes spiral wire and spiral roller. Based on the motion characteristics of the variable lead screw drive and nut connection mechanism, the variable lead helix function is designed. At the same time, the angular velocity of spiral wire rod and the radius of spiral wire are determined according to the range of screw lead angle and pressure angle, and the parameter equation of helix can be obtained. Then, the contact and lubrication characteristics of the variable-lead helical pair are analyzed to design the helical roller with the goal of achieving the minimum contact stress and the elasto-hydrodynamic lubrication state. The optional range of the radius and bus length of the variable lead screw is obtained. Finally, the application of variable-lead spiral pair in the subway door system is used as an example to illustrate the optimized design of the variable lead spiral pair.

Keywords: variable-lead spiral pair; helix; helical roller; contact stress; elasto-hydrodynamic lubrication.

Received May 12, 2018
Accepted February 03, 2020